FORMULAS ESTRUCTURALES

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PROLOGO

En primer término, debemos reconocer que, con referencia a los problemas de pórticos, existe una gran variedad de métodos para las soluciones, los que en ocasiones resultan incomprensibles para los estudiantes o simplemente difíciles para optar por determinado método. Por otro lado, no siempre las diferentes formas de solución se encuentran reunidas en un sólo libro, sino que, para los diversos estilos de pórticos debemos consultar varios, con la consiguiente pérdida de esfuerzo y tiempo.

En consideración a lo explicado, en el presente trabajo, se trata de presentar una cuidadosa selección de fórmulas prácticas de las diferentes formas de pórticos, más generalizados.

Los datos provienen principalmente de los tratados de Mecánica Racional, Resistencia de Materiales y Estructuras, escritos por consagrados maestros en la materia, tanto del Japón como de Alemania, tales como los Doctores Okamura Masao, Umemura Hazime, Takabeya Fukuhei, Kleionlogel, Gehler y el Dr. Müller-Breslau.

Si mediante el presente trabajo, se evita la inconveniencia de la búsqueda de determinadas fórmulas a través de las consultas de diferentes libros, y se facilita a los estudiantes, la solución de los pórticos de una manera sencilla y práctica, todos nuestros esfuerzos estarán compensados.

Finalmente, hay muchas personas a quienes deseo testimoniar mi agradecimiento, pero por el temor de omitir algunos, dado lo extenso de la lista, prefiero que el tiempo me permita testimoniárselo a ellas, personalmente.

Ing. Fernando Oshiro Higa
Lima, del día en que florecieron las dalias

Primero se llevaron a los comunistas
pero a mi no me importó
porque yo no era.

Enseguida se llevaron a unos obreros
pero a mi no me importó
porque yo tampoco era.

Despues detuvieron a los sindicalistas
pero a mi no me importó
porque yo no soy sindicalista.

Luego apresaron a unos curas
pero como yo no soy religioso
tampoco me importó.

Ahora me llevan a mí
pero ya es tarde.

Bertolt Brecht

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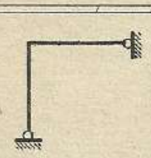
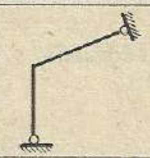
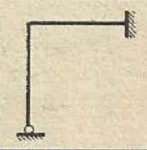
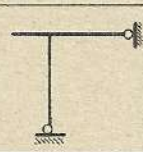
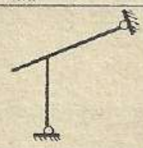
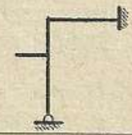
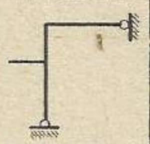
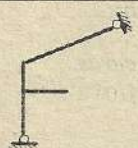
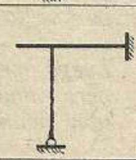
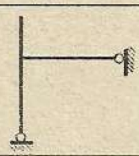
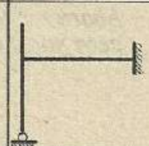
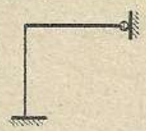
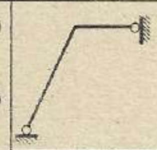
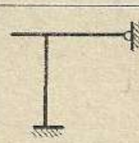
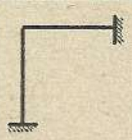
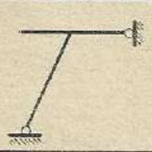
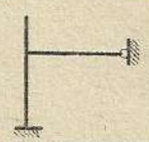
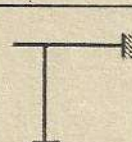
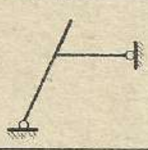
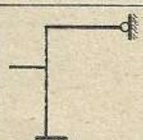
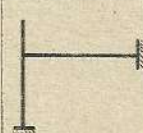
(1) — (15)		1 — 21	(40) — (50)		34 — 36	(72) — (83)		49 — 55
(16) — (19)		22 — 23	(51) — (52)		37	(84) — (85)		55 — 56
(20) — (21)		24 — 24	(53)		37	(86) — (87)		56 — 57
(22)		25	(54)	 Esfuerzo por temperatura.	37	(88)		57
(23)	 Esfuerzo por temperatura	25	(55) — (66)		38 — 44	(89)	 Esfuerzo por temperatura	58
(24) — (36)		30 — 33	(67)		45	(90) — (96)		61 — 63
(37)		33	(68)		45	(97)		63
(38)		33	(69) — (70)		46	(98)		64
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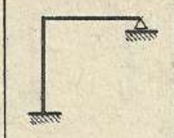
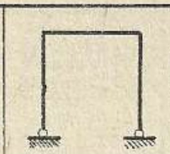
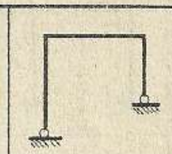
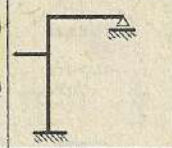
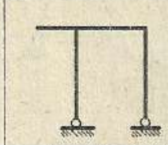
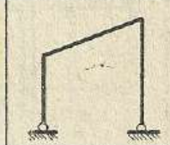
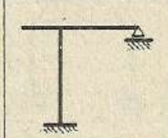
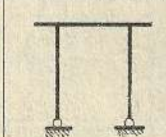
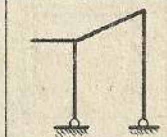
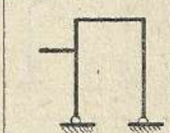
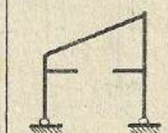
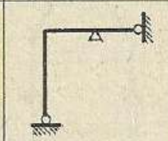
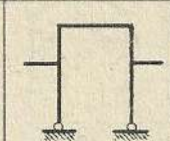
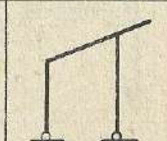
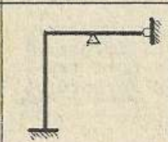
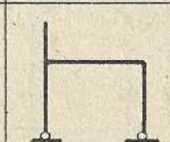
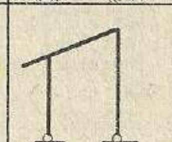
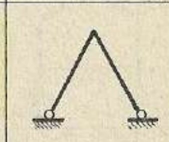
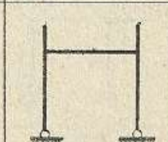
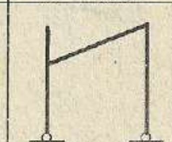
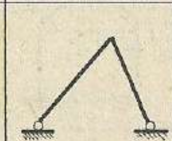
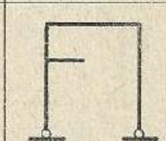
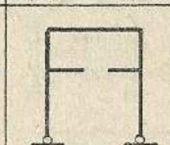
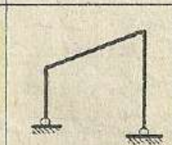
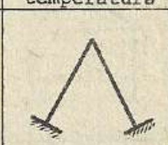
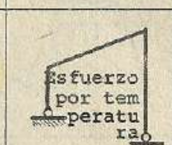
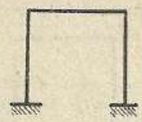
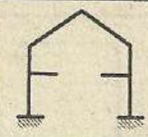
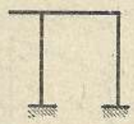
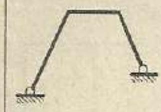
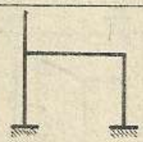
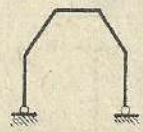
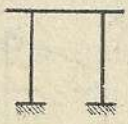
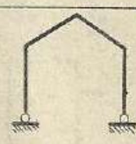
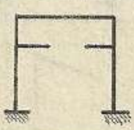
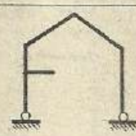
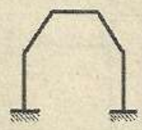
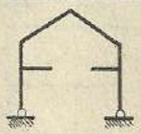
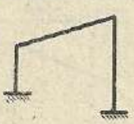
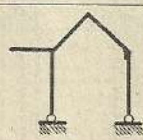
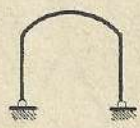
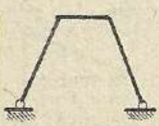
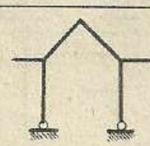
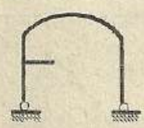
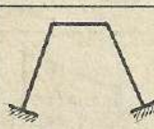
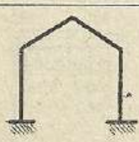
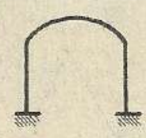
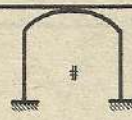
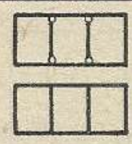
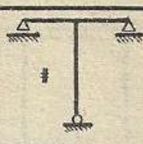
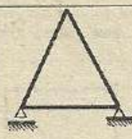
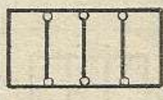
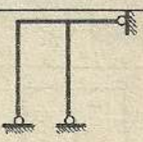
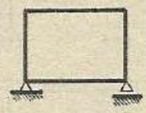
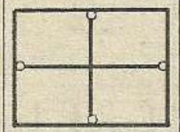
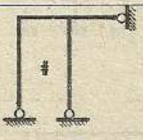
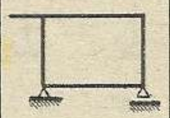
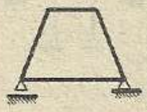
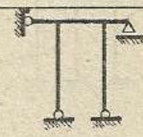
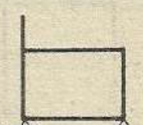
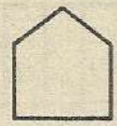
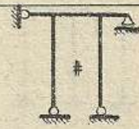
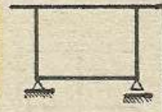
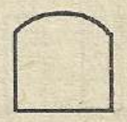
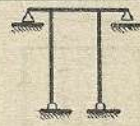
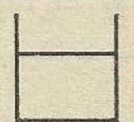
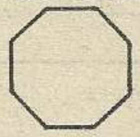
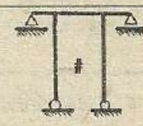
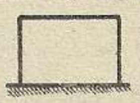
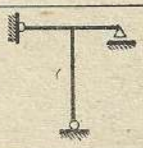
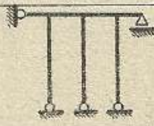
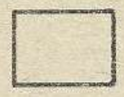
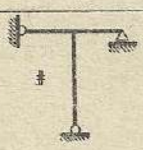
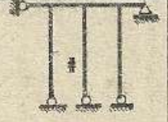
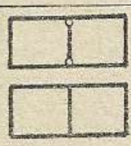
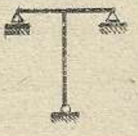
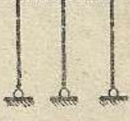
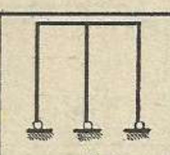
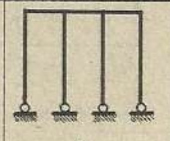
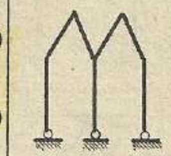
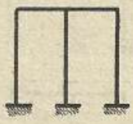
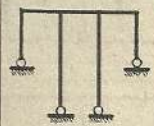
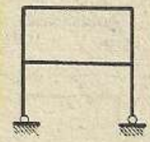
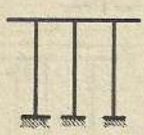
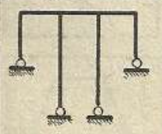
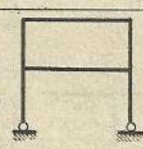
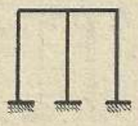
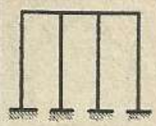
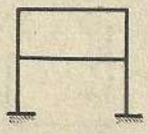
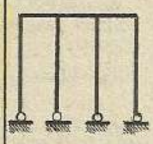
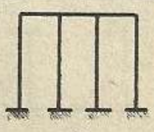
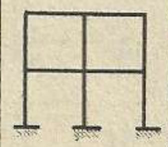
(100)–(108)		65–69	(139)–(156)		79–89	(175)–(178)		101–102
(109)–(110)		69–70	(157)–(163)		89–92	(179)–(188)		103–106
(111)–(112)		70–71	(164)–(165)		93	(189)		106
(113)		71	(166)–(167)		94	(190)		107
(114)–(115)		74	(168)–(169)		95–96	(191)		107
(116)–(117)		74	(170)		96	(192)		107
(118)–(125)		74–75	(171)		97	(193)		108
(126)–(131)		76–78	(172)		97	(194)		108
(132)		78	(173)		98	(195)–(202)		108–112
(133)–(138)		78–79	(174)		98	(203)		112

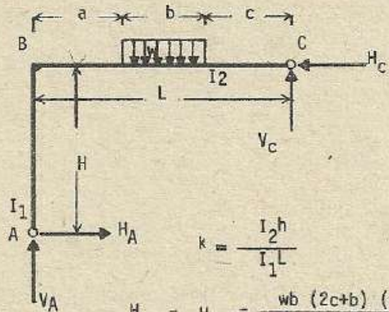
(204)–(215)		113–120	(248)		136	(295)		158
(216)–(217)		120–121	(249)–(257)		136–139	(296)		159
(218)–(219)		121–122	(258)		139	(297)–(303)		160–162
(220)–(221)		122–123	(259)–(276)		139–148	(304)		163
(222)		123	(277)		148	(305)–(311)		163–166
(223)		124	(278)		148	(312)		167
(224)–(232)		126–130	(279)		149	(313)–(319)		167–170
(233)–(241)		131–133	(280)		149	(320)		171
(242)		134	(281)		149	(321)		171
(243)–(247)		134–135	(282)–(294)		151–158	(322)–(328)		173–177

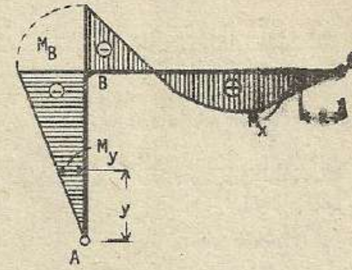
(329)		177	(364)–(365)		195	(405)		213
(330)–(335)		178–180	(366)		196	(406)–(408)		213–214
(336)–(347)		180–184	(367)		196	(409)		215
(348)		185	(368)–(381)		199–203	(410)–(418)		215–219
(349)		185	(382)–(383)		203–204	(419)		220
(350)–(351)		186	(384)–(386)		204–205	(420)–(429)		220–225
(352)–(353)		186–187	(387)		205	(430)		225
(354)–(359)		189–192	(388)–(394)		206–209	(431)–(440)		226–232
(360)–(361)		192–193	(395)		209	(441)		232
(362)–(363)		194–195	(396)–(404)		210–212	(442)–(455)		233–244

(456)	 #	244	(481)	 #	270	(497) — (498)		282
(457) — (463)		246 — 251	(482) — (488)		271 — 275	(499) — (506)		283 — 285
(464) — (466)		252 — 253	(489)	 #	275	(507)	 #	285
(467)	 #	253	(490) — (495)		276 — 280	(508) — (511)		286 — 288
(468) — (480)		255 — 269	(496)	 #	281	(512)		290

: Esfuerzos por temperatura.

(1)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wb\,(2c+b)\,(2L^2 - 2c^2 - 2\,bc - b^2)}{8hL^2\,(k+1)}$$

$$V_A = \frac{wb\,(2c+b)\{\,4L^2\,(k+1)+2L^2 - 2c^2 - 2bc - b^2\}}{8\,L^3\,(k+1)}$$

$$V_C = \frac{wb}{L}\left(a + \frac{b}{2}\right) - \frac{wb\,(2c+b)\,(2L^2 - 2c^2 - 2bc - b^2)}{8L^3\,(k+1)}$$

$$M_B = -\frac{wb\,(2c+b)\,(2L^2 - 2c^2 - 2\,bc - b^2)}{8L^2\,(k+1)}$$

Momento de flexión en la columna

$$M_y = -\frac{wb\,(2c+b)\,(2L^2 - 2c^2 - 2bc-b^2)\;\;y}{8hL^2\,(k+1)}$$

Momento de flexión en la viga

$$M_x = V_C x \qquad x \leq c$$

$$M_x = V_c x - \frac{w}{2}(x-c)^2 \qquad c \leq x \leq b + c$$

$$M_x = V_c x - wb\left(x - \frac{b}{2} - c\right) \qquad x \geq b + c$$

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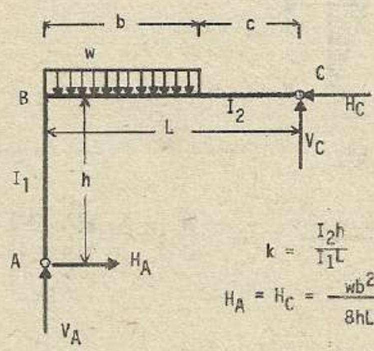
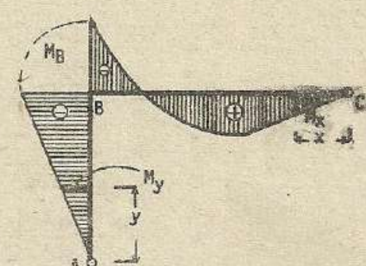
(2)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wb^2\,(L+c)^2}{8hL^2\,(k+1)}$$

$$V_A = \frac{wb\,(L+c)\,\{4L^2\,(k+1)+b(L+c)\}}{8L^3\,(k+1)}$$

$$V_C = \frac{wb^2 \{4L^2 (k+1) - (L+c)^2\}}{8L^3 (k+1)} \qquad M_B = -\frac{wb^2 (L+c)^2}{8L^2 (k+1)}$$

Momento de flexión en la columna

$$M_Y = -\frac{wb^2 \cdot (L+c)^2}{8hL^2 \cdot (k+1)} \cdot y$$

Momento de flexión en la viga

$$M_X = \frac{wb^2 \{4L^2 \cdot (k+1) - (L+c)^2\}}{8L^3 (k+1)} x \qquad x \leq c$$

$$M_X = \frac{wb^2 \{4L^2 \cdot (k+1) - (L+c)^2\}}{8L^3 (k+1)} x - \frac{w}{2} (x - c)^2 \qquad x \geq c$$

En caso de $b = \frac{L}{2}$; $H_A = H_C = \frac{9}{128\, h\, (k+1)} wL^2$

$$V_A = \frac{3\,(16k+19)}{128\,(k+1)} wL \; ; \quad V_C = \frac{(16k+7)}{128\,(k+1)} wL \; ; \quad M_B = -\frac{9}{128\,(k+1)} wL^2$$

Momento de flexión en la columna : $M_y = -\frac{9wL^2}{128h\,(k+1)} y$

Momento de flexión en la viga :

$$M_X = \frac{16\,k+7}{128\,(k+1)} wLx \qquad x \leq \frac{L}{2}$$

$$M_X = \frac{16k+7}{128\,(k+1)} wLx - \frac{w}{2} \left(x - \frac{L}{2}\right)^2 \qquad x \geq \frac{L}{2}$$

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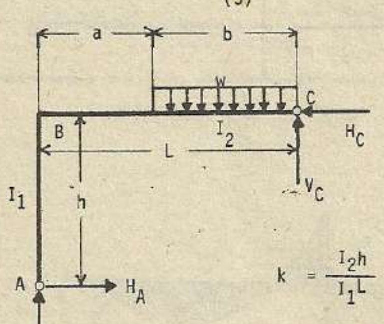
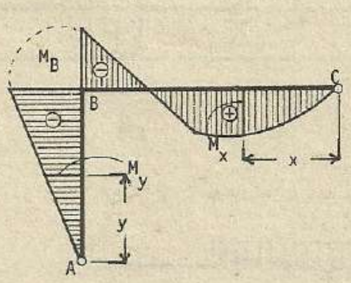
(3)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wb^2 \; (2L^2 - b^2)}{8\,hL^2\,(k+1)}$$

$$V_A = \frac{wb^2 \left[4L^2 (k+1) + 2L^2 - b^2\right]}{8L^3 \; (k+1)} \; ; \quad V_C = \frac{wb}{2L}\left[2L - b - \frac{b(2L^2 - b^2)}{4L^2\,(k+1)}\right]$$

$$M_B = -\frac{wb^2 \; (2L^2 - b)}{8L^2 \; (k+1)}$$

Momento de flexión en la columna : $M_y = -\frac{wb^2 \; (2L^2 - b^2)}{8hL^2 \; (k+1)} y$

Momento de flexión en la viga :

$$M_x = V_C x - \frac{wx^2}{2} \qquad x \leq b$$

$$M_x = V_C x - wb\left(x - \frac{b}{2}\right) \qquad x \geq b$$

En caso donde $b = \frac{L}{2}$; $H_A = H_C = \frac{7}{128h\,(k+1)} wL^2$

$$V_A = \frac{(16k + 23)}{128\,(k+1)} wL \; ; \quad V_C = \frac{48K + 41}{128\,(k+1)} wL \; ; \quad M_B = \frac{7}{128\,(k+1)} wL^2$$

Momento de doblado o flexión en la columna : $M_y = -\frac{7}{128h\,(k+1)} wL^2$

Momento de flexión en la viga :

$$M_x = \frac{48k+41}{128(k+1)} wLx - \frac{wx^2}{2} \qquad x \leq \frac{L}{2}$$

$$M_x = \frac{48k+41}{128(k+1)} wLx - \frac{wL}{2}\left(x - \frac{L}{4}\right) \qquad x \geq \frac{L}{2}$$

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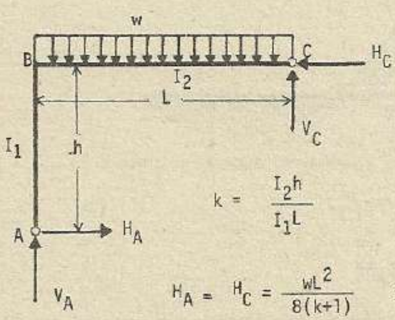
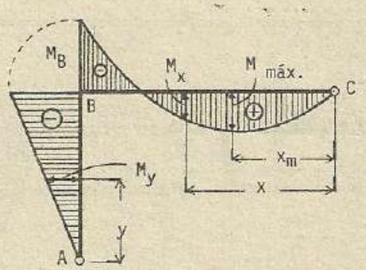
(4)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{wL^2}{8(k+1)} \; ; \quad V_A = \frac{4k+5}{8\,(k+1)} wL$$

$$V_C = \frac{4k+3}{8(k+1)} wL \; ; \quad M_B = -\frac{1}{8\,(k+1)} wL^2$$

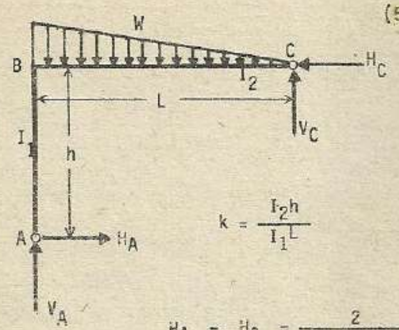
Momento de flexión en la columna : $M_y = -\frac{wL^2 y}{8(k+1)h}$

Momento de flexión en la viga :

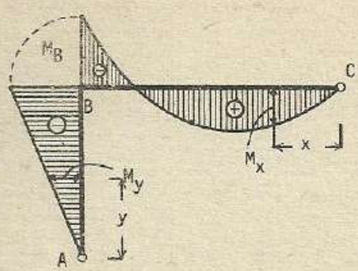
$$M_x = \frac{4k+3}{8\,(k+1)} \cdot wLx - \frac{wL^2}{2}$$

$$x_m = \frac{4k+3}{8\,(k+1)} L \; ; \quad M_{máx.} = \frac{(4k+3)^2}{128\,(k+1)^2} wL^2$$

(5)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{2}{15\ (k+1)} W \frac{L}{h} \quad : \quad V_A = \frac{2\ (5k+6)}{15\ (k+1)} W$$

$$V_C = \frac{5k+3}{15\ (k+1)} W \quad : \quad M_B = -\frac{2}{15\ (k+1)} WL$$

Momento de flexión en la columna :

$$M_y = -\frac{2}{15\ (k+1)} \cdot WLy$$

Momento de flexión en la viga :

$$M_x = \frac{5k+3}{15(k+1)} \cdot Wx - \frac{Wx^3}{3L^2}$$

$$x_m = L \sqrt{\frac{5k+3}{15\ (k+1)}} \qquad M_{máx.} = \frac{2\ (5k+3)}{45(k+1)} \sqrt{\frac{5k+3}{15(k+1)}} \cdot WL$$

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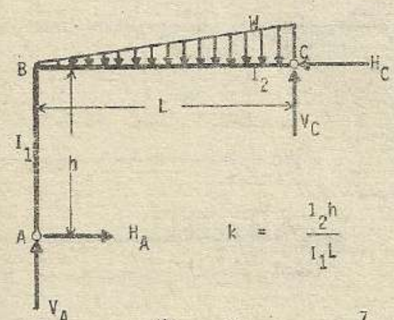
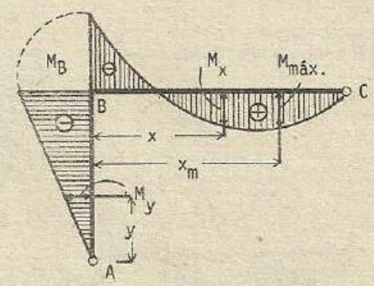
(6)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{7}{60\ (k+1)} W \frac{L}{h} \qquad V_A = \frac{20\ k+27}{60\ (k+1)} W$$

$$V_C = \frac{40k+33}{60\ (k+1)} W \qquad M_B = -\frac{7}{60\ (k+1)} WL$$

Momento de flexión en la columna :

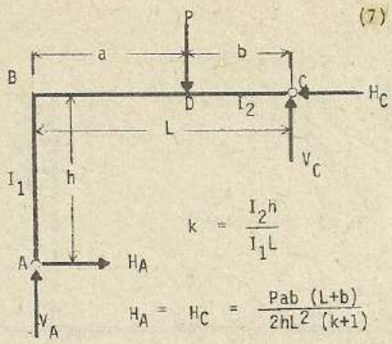
$$M_y = -\frac{7}{60\ (k+1)} WL \frac{y}{h}$$

Momento de flexión en la viga :

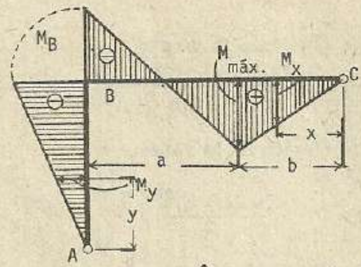
$$M_x = W.\frac{(20k+27)\ x - 7L}{60\ (k+1)} - \frac{Wx^3}{3L^2} \qquad x_m = L\sqrt{\frac{20k+27}{60(k+1)}}$$

$$M_{máx.} = \frac{WL}{3}\left[\frac{(20k+27)}{30(k+1)} \cdot \sqrt{\frac{20k+27}{60\ (k+1)}} - \frac{7}{20\ (k+1)}\right]$$

(7)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{Pab\,(L+b)}{2hL^2\,(k+1)}$$

$$V_A = \frac{Pb\,\{2L^2\,(k+1) + a\,(L+b)\}}{2L^3\,(k+1)}$$

$$V_C = \frac{Pa\,\{2L^2\,(k+1) - b\,(L+b)\}}{2L^3\,(k+1)}\ ; \quad M_B = -\frac{Pab\,(L+b)}{2L^2\,(k+1)}$$

Momento de flexión en la columna : $M_y = -\dfrac{Pab\,(L+b)}{2hL^2(k+1)}\,y$

Momento de flexión en la viga :

$$M_x = \frac{Pb\,\{2L^2\,(k+1) - a\,(L+b)\}}{2L^3\,(k+1)}\,x \qquad x \leq b$$

$$M_x = \frac{Pb\,\{2L^2\,(k+1) - a\,(L+b)\}}{2L^3\,(k+1)}\,x - P\,(x-b) \qquad x \geq b$$

$$M_{máx.} = \frac{pab\,(2kL^2 + 3\,aL - a^2)}{2L^3\,(k+1)} \qquad x_m = b$$

Si $a = b = \dfrac{L}{2}$; $H_A = H_C = \dfrac{3PL}{16\,(k+1)h}$; $V_A = \dfrac{8k+11}{16\,(k+1)}\,P$

$V_C = \dfrac{8k+5}{16\,(k+1)}\,P$; $M_B = -\dfrac{3}{16\,(k+1)}\,PL$; $M_{máx.} = \dfrac{8k+5}{32(k+1)}\cdot PL$

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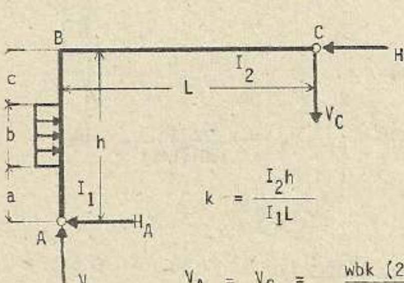
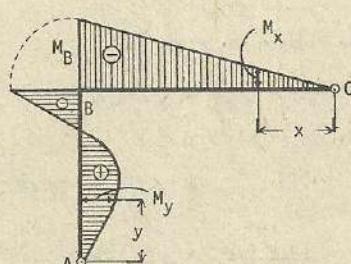
(8)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = V_C = \frac{wbk\,(2a+b)\,(2h^2 - 2a^2 - 2ab - b^2)}{8h^2\,L(k+1)}$$

$$H_A = -\frac{wbk\,(2a+b)\,(2h^2 - 2a^2 - 2ab - b^2)}{8h^3\,(k+1)} + \frac{wb}{h}\left(c + \frac{b}{2}\right)$$

$H_C = wb - H_A$; $M_B = -\dfrac{wbk\,(2a+b)\,(2h^2 - 2a^2 - 2ab - b^2)}{8h^2\,(k+1)}$

Momento de flexión e la viga :

$$M_x = -\frac{wbk\,(2a+b)\,\,(2h^2 - 2a^2 - 2ab - b^2)}{8h^2L\,\,(k+1)}\,x$$

Momento de flexión en la columna :

$M_y = H_A \cdot y$ $\qquad\qquad y \leqslant a$

$M_y = H_A \cdot y - \dfrac{w\,(y-a)^2}{2}$ $\qquad\qquad a \leqslant y \leqslant a+b$

$M_y = H_A \cdot y - wb\,(y - a - \frac{b}{2})$ $\qquad\qquad y \geqslant a+b$

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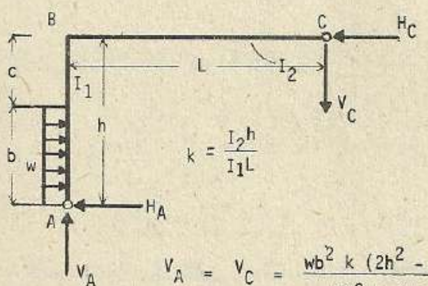
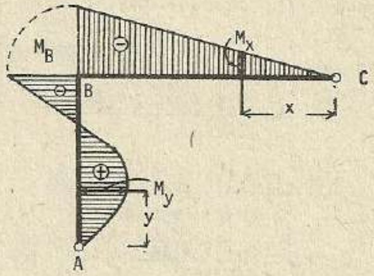
(9)





$$V_A = V_C = \frac{wb^2\,k\,(2h^2 - b^2)}{8h^2\,L\,(k+1)} \quad ; \quad H_A = -\frac{wb^2k\,\,(2h^2 - b^2)}{8h^3\,\,(k+1)} + \frac{wb}{h}\,(c+\frac{b}{2})$$

$$H_C = wb - H_A \quad ; \quad M_B = -\frac{wb^2\,k\,(2h^2 - b^2)}{8h^2\,\,(k+1)}$$

Momento de flexión en la viga : $\quad M_x = -\dfrac{wb^2\,k\,(2h^2 - b^2)}{8h^2L\,\,(k+1)}\,x$

Momento de flexión en la columna :

$M_y = H_A \cdot y - \dfrac{wy^2}{2}$ $\qquad\qquad y \leqslant b$

$M_y = H_A \cdot y - wb\,(y - \frac{b}{2})$ $\qquad\qquad y \geqslant b$

Si $b = c = \dfrac{h}{2}$; $\quad V_A = V_C = \dfrac{7k}{12\,(8k+1)} \cdot \dfrac{wh^2}{L}$; $\quad H_A = \dfrac{41k+48}{128\,(k+1)}\,wh$

$H_C = \dfrac{23\,k + 16}{128\,(k+1)}\,wh \qquad M_B = -\dfrac{7k}{128\,(k+1)}\,wh^2$

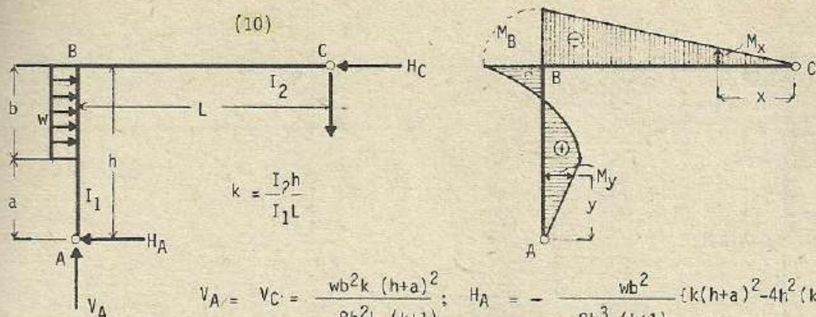
Momento de flexión en la viga : $\quad M_x = -\dfrac{7k}{128\,(k+1)} \cdot \dfrac{wh^2}{L}\,x$

Momento de flexión en la columna :

$M_y = \dfrac{41k + 48}{128\,(k+1)} \cdot why - \dfrac{wy^2}{2}$ $\qquad\qquad y \leqslant b$

$M_y = \dfrac{41\,K + 48}{128\,(k+1)} \cdot why - \dfrac{wL}{2}\,(y - \frac{L}{4})$ $\qquad\qquad y \geqslant b$

(10)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{wb^2 k\ (h+a)^2}{8h^2 L\ (k+1)}\ ; \quad H_A = -\frac{wb^2}{8h^3\ (k+1)} \{k(h+a)^2 - 4h^2(k+1)\}$$

$$H_C = wb - H_A \quad ; \quad M_B = \frac{wb^2\ k(h+a)^2}{8h^2\ (k+1)}$$

Momento de flexión en la viga : $M_x = -\dfrac{wb^2\ k\ (h+a)^2}{8h^2\ L\ (k+1)}\ x$

Momento de flexión en la columna :

$M_y = H_A \cdot y \qquad\qquad y \leq a$

$M_y = H_A \cdot y - \dfrac{w\ (y-a)^2}{2} \qquad\qquad y \geq a$

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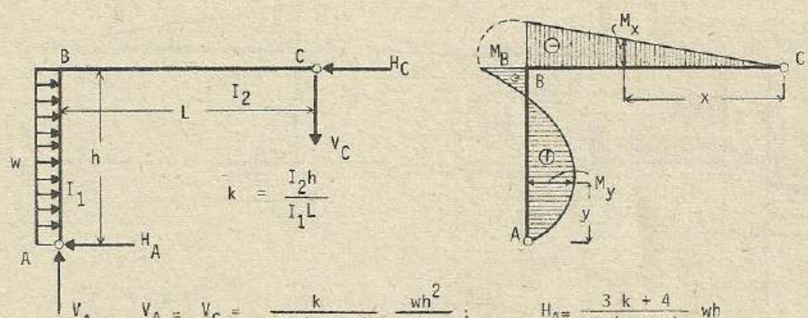
(11)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{k}{8\ (k+1)}\ \frac{wh^2}{L}\ ; \quad H_A = \frac{3\ k + 4}{8\ (k+1)}\ wh$$

$$H_C = \frac{5\ k + 4}{8\ (k+1)}\ wh \quad ; \quad M_B = -\frac{k}{8\ (k+1)}\ wh^2$$

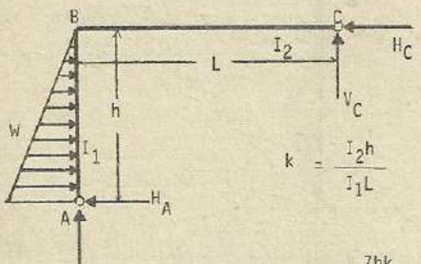
Momento de flexión en la viga : $M_x = -\dfrac{k}{(8\ k+1)\ L}\ wh^2 x$

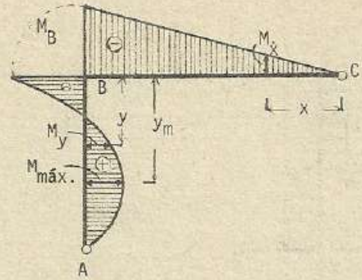
Momento de flexión en la columna :

$$M_y = \frac{\{4\ (h-y) + k\ (3h - 4y)\}}{8\ (k+1)}\ wy\ ; \quad y_m = \frac{3k+4}{8\ (k+1)}\ h$$

$$M_{máx.} = \frac{(3\ k + 4)^2}{128\ (k+1)^2} \cdot wh^2$$

(12)





$$V_A = V_C = \frac{7hk}{60\ (k+1)\ L}\ W \qquad H_A = \frac{33\ k + 40}{60\ (k+1)} \cdot W$$

$$H_C = \frac{27\ k + 20}{60\ (k+1)} \cdot W \qquad M_B = -\frac{7k}{60\ (k+1)} \cdot Wh$$

Momento de flexión en la viga :

$$M_x = -\frac{7k}{60\ (k+1)} \cdot Whx$$

Momento de flexión en la columna :

$$M_y = H_C y - V_C L - \frac{Wy^3}{3h^2}$$

$$y_m = h\sqrt{\frac{3k+5}{15\ (k+1)}} \qquad M_{máx.} = \frac{2\ (3k+5)}{45\ (k+1)}\sqrt{\frac{3k+5}{15\ (k+1)}} \cdot Wh$$

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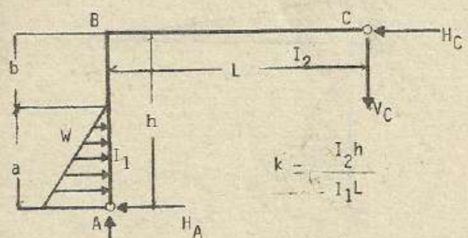
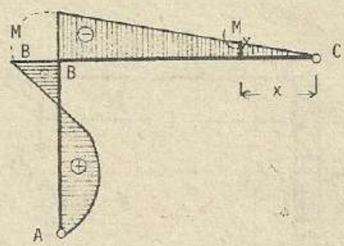
(13)





$$V_A = V_C = \frac{(10h^2 - 3a^2)\ ak}{60\ (k+1)\ h^2 L} \cdot W \ ; \quad H_A = \frac{(3b+2a)}{3h}\ W - V_C \frac{L}{h}$$

$$H_C = -\frac{a}{3h} \cdot W + V_C \frac{L}{h} \ ; \quad M_B = \frac{ak\ (10h^2 - 3a^2)}{60h^2\ (k+1)} \cdot W$$

Momento de flexión en la viga :

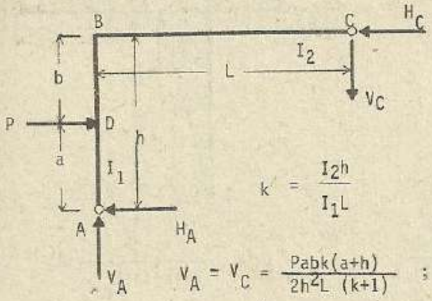
$$M_x = -\frac{(10h^2 - 3a^2)\ ak}{60\ (k+1)\ h^2 L}\ Wx$$

Momento de flexión en la columna :

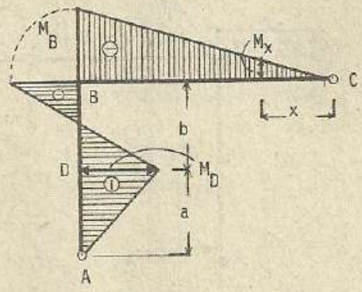
$$M_y = H_C y - V_C L \qquad y \leq b$$

$$M_y = H_C y - V_C L - \frac{W\ (y-b)^3}{3\ a^2} \qquad y \geq b$$

(14)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = V_C = \frac{Pabk(a+h)}{2h^2L\ (k+1)} \quad ; \quad H_A = Pb\,\frac{2h^2\ (k+1) - ak\ (a+h)}{2h^3\ (k+1)}$$

$$H_C = Pa \cdot \frac{2h^2\ (k+1) + bk\ (a+b)}{2h^3\ (k+1)} \quad ; \quad M_B = \frac{Pabk\ (a+h)}{2h^2\ (k+1)}$$

Momento de flexión en la viga :

$$M_x = \frac{Pabk\ (a+h)}{2h^2L(k+1)} \cdot x$$

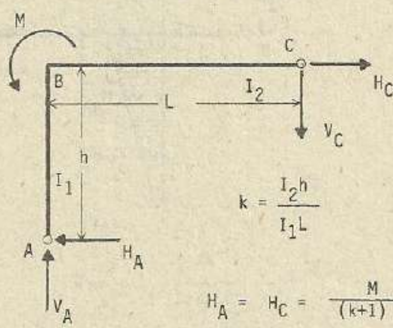
Momento de flexión en la columna :

$$M_y = H_A y \qquad y \leqslant a$$

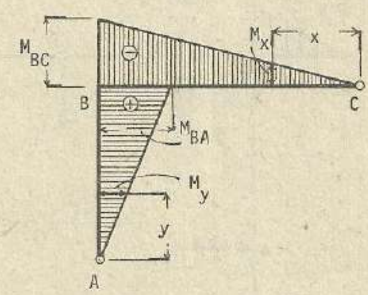
$$My = H_A y - P\ (y - a) \qquad y \geqslant a$$

$$M_{máx.} = \frac{Pab\ \{2h^2 + bk\ (3h-b)\}}{3h^3\ (k+1)}$$

(15)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{M}{(k+1)\ h} \qquad V_A = V_C = \frac{Mk}{(k+1)\ L}$$

$$M_{BA}\ \text{(col.)} = \frac{M}{(k+1)} \qquad M_{BC}\ \text{(viga)} = \frac{Mk}{(k+1)}$$

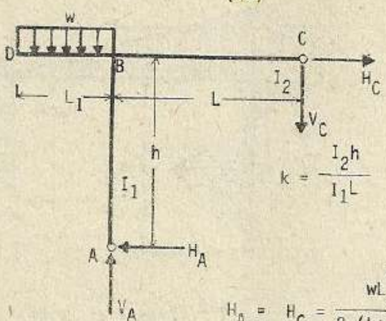
Momento de flexión en la columna :

$$M_y = \frac{M}{(k+1)} \cdot \frac{y}{h}$$

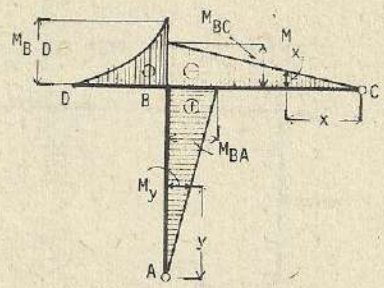
Momento de flexión en la viga :

$$M_x = -\frac{Mk}{(k+1)} \cdot \frac{x}{L}$$

(16)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{wL_1^2}{2\,(k+1)\,h} \quad ; \quad V_A = \frac{wL_1^2\,k}{2\,(k+1)\,L} + wL_1$$

$$V_C = \frac{wL_1^2\,k}{2\,(k+1)\,L} \quad ; \quad M_{BA}\ (\text{col.}) = \frac{wL_1^2}{2\,(k+1)} \quad ; \quad M_{BD}(\text{Cantiliver}) = \frac{wL_1^2}{2}$$

$$M_{BC}\ (\text{viga}) = \frac{wL_1^2\,k}{2\,(k+1)}$$

; Momento de flexión en la columna :

$$M_y = \frac{wL_1^2}{2\,(k+1)\,h}\,.\,y$$

Momento de flexión en la viga :

$$M_x = -\frac{wL_1^2\,k}{2\,(k+1)\,L}\,.\,x$$

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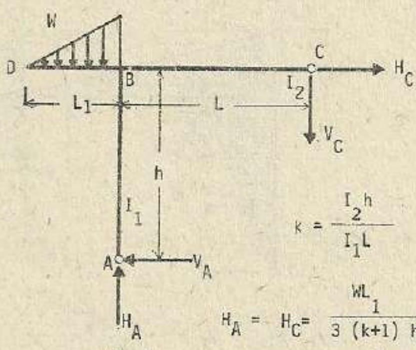
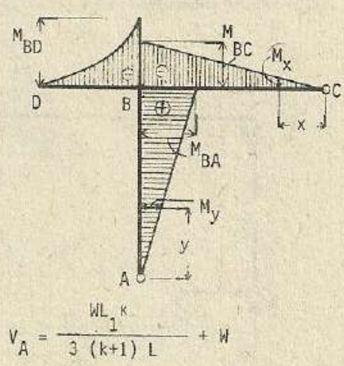
(17)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{WL_1}{3\,(k+1)\,h} \qquad V_A = \frac{WL_1\,k}{3\,(k+1)\,L} + W$$

$$V_C = \frac{WL_1 k}{3\,(k+1)\,L} \quad ; \quad M_{BA} = \frac{WL_1}{3\,(k+1)} \quad ; \quad M_{BD} = -\frac{WL_1}{3} \quad ; \quad M_{BC} = -\frac{WL_1 K}{3\,(k+1)}$$

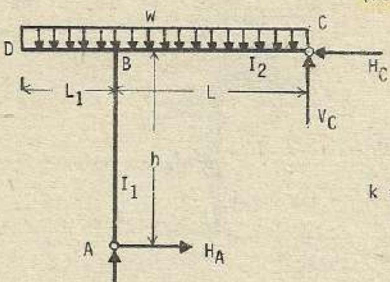
Momento de flexión en la viga :

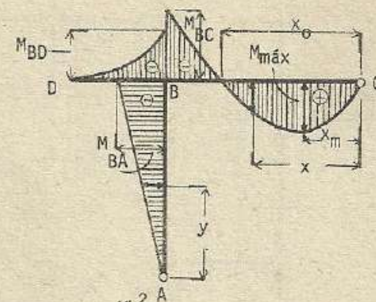
$$M_x = -\frac{WL_1 kx}{3\,(k+1)\,L}$$

Momento de flexión en la columna :

$$M_y = \frac{WL_1}{3\,(k+1)\,h}\,.\,y$$

(18)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wL^2}{8\ (k+1)\ h} - \frac{wL_1^2}{2\ (k+1)h}$$

$$V_A = \frac{(4k+5)}{8\ (k+1)}\ wL + \frac{k}{2\ (k+1)L}\ wL_1^2 + wL_1 \quad ; \quad V_C = w\ (L+L_1) - V_A$$

$$M_{BD} = -w\ \frac{L_1^2}{2} \quad ; \quad M_{BA} = -H_A h \quad ; \quad M_{BC} = V_C L - wL^2/2$$

$$x_m = \frac{V_C}{w} \quad ; \quad M_{máx.} = V_C x_m - w\ \frac{x_m^2}{2}$$

Momento de flexión en la viga : $M_x = V_C x - \frac{wx^2}{2}$; $x_0 = \frac{2V_C}{w}$

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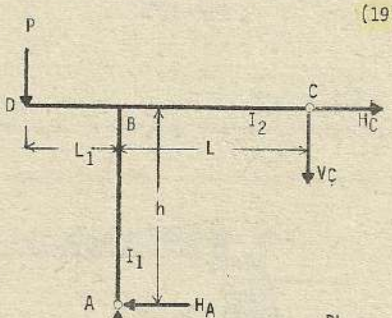
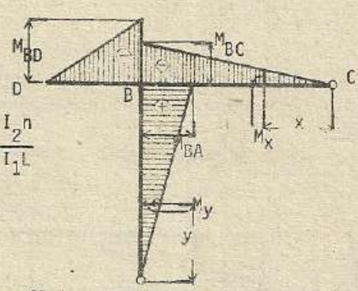
(19)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{PL_1}{(k+1)h} \quad ; \quad V_A = \frac{PL_1 k}{(k+1)\ L} + P$$

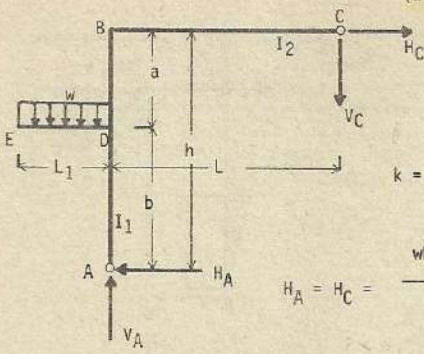
$$V_C = \frac{PL_1 k}{(k+1)L} \quad ; \quad M_{BA} = \frac{PL_1}{(k+1)} ; \quad M_{BD} = PL_1 \quad ; \quad M_{BC} = \frac{PL_1 k}{(k+1)}$$

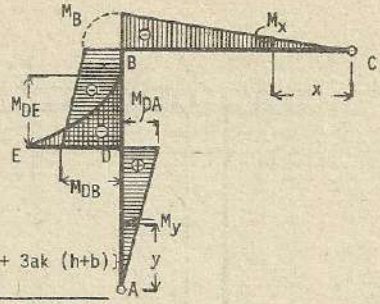
Momento de flexión en la columna :

$$M_y = \frac{PL_1 y}{(k+1)h}$$

Momento de flexión en la viga : $M_x = -\frac{PL_1 kx}{(k+1)\ L}$

(20)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wL_1^2\,\{2h^2 + 3ak\,(h+b)\}}{4h^3\,(k+1)}$$

$$V_A = \frac{wL_1^2 k\,(3b^2 - h^2)}{4h^2 L\,(k+1)} + wL_1 \quad ; \quad V_C = \frac{wL_1^2 k\,(3b^2 - h^2)}{4h^2 L\,(k+1)}$$

$$M_B = -\frac{wL_1^2\,k\,\{2h^2 - 3a\,(h+b)\}}{4h^2\,(k+1)} \quad M_{DE} = -\frac{wL_1^2}{2} \quad ; \quad M_{DA} = \frac{wL_1^2 b\{2h^2+3ka(h+b)\}}{4h^3\,(k+1)}$$

$$M_{DB} = -wL_1^2\left[1 + \frac{b\,\{2h^2+3ka\,(h+b)\}}{2h^3\,(k+1)}\right]$$

Momento de flexión en la viga :

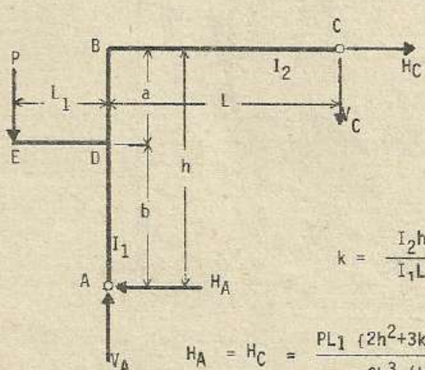
$$M_x = -\frac{wL_1^2 k\,(3b^2 - h^2)}{4\,h^2 L\,(k+1)}\,x$$

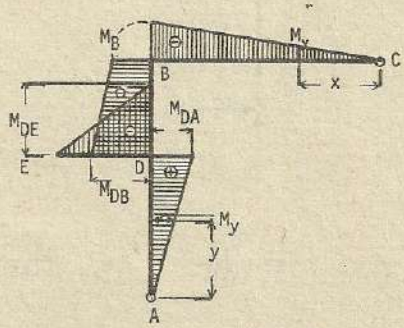
Momento de flexión en la columna :

$$M_y = H_A y \qquad y < b$$

$$M_y = H_A y - \frac{wL_1^2}{2} \qquad y > b$$

(21)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{PL_1\,\{2h^2+3ka\,(h+b)\}}{2h^3\,(k+1)} \quad ; \quad V_A = \frac{PL_1 k\,(3b^2 - h^2)}{2h^2 L\,(k+1)} + P$$

$$V_C = \frac{PL_1 k\,(3b^2 - h^2)}{2h^2 L\,(k+1)} \quad ; \quad M_B = -\frac{PL_1 k\,\{2h^2 - 3a\,(h+b)\}}{2h^2\,(k+1)}$$

$$M_{DE} = PL_1 \quad ; \quad M_{DB} = -PL_1 \left[1 + \frac{b\,\{2h^2 + 3ka\,(h+b)\}}{2h^3\,(k+1)} \right]$$

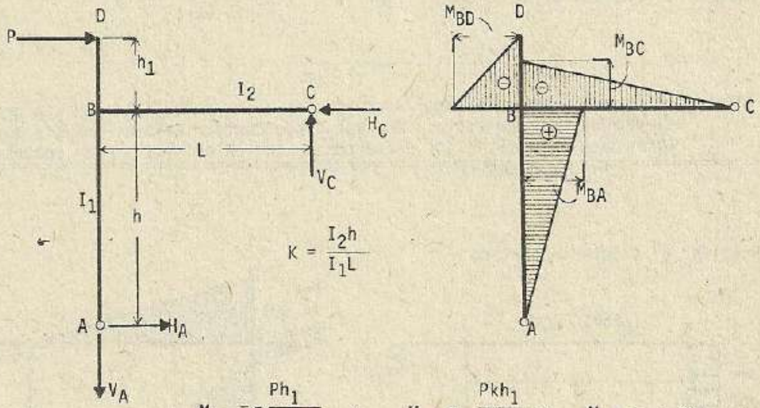
$$M_{DA} = \frac{PL_1 b\,\{2h^2 + 3ka\,(h+b)\}}{2h^3\,(k+1)}$$

Esfuerzo de momento en la viga : $M_x = -\dfrac{PL_1 k\,(3b^2 - h^2)}{2h^2 L\,(k+1)}\,x$

Momento de flexión en la columna :

$$M_y = H_A y \qquad y \leq b$$
$$M_y = H_A y - PL_1 \qquad y \geq b$$

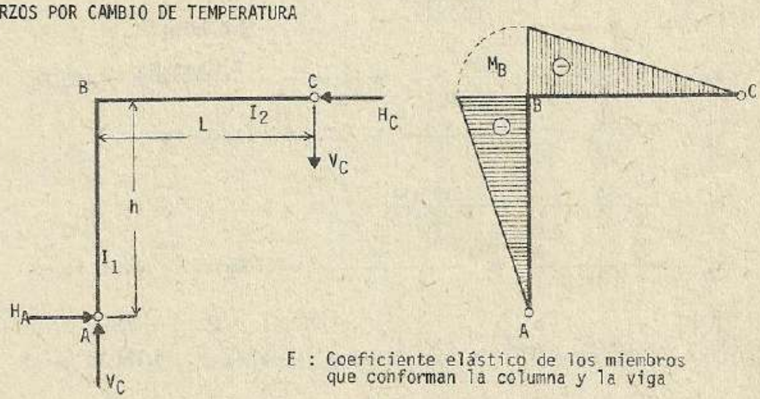
(22)



$$M_{BC} = -\frac{Ph_1}{1+k} \quad ; \quad M_{BA} = \frac{Pkh_1}{1+k} \quad ; \quad M_{BD} = -Ph_1$$

(23)

ESFUERZOS POR CAMBIO DE TEMPERATURA



E : Coeficiente elástico de los miembros que conforman la columna y la viga

ϵ : Coeficiente de expansión

t : Grado de variación de la temperatura

I_1 = Momento de inercia de la columna

I_2 = Momento de inercia de la viga $\qquad k = \dfrac{I_2 h}{I_1 L}$

h = Altura de la columna

L = Luz a largo de la viga

$$H_A = H_C = \frac{3E\varepsilon tI\,(h^2 + L^2)}{L^2h^2(k+1)} \quad ; \quad V_A = V_C = \frac{3E\varepsilon tI\,(h^2 + L^2)}{L^3h\,(k+1)}$$

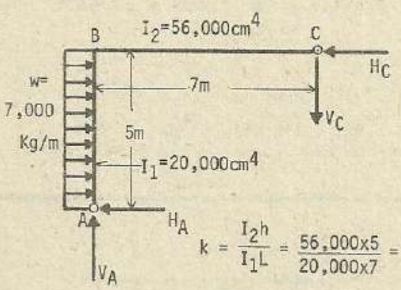
$$M_B = -\frac{3E\varepsilon tI\,(h^2 + L^2)}{L^2h\,(k+1)}$$

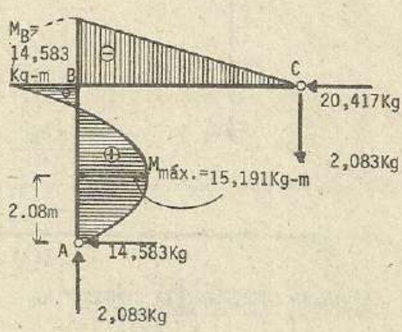
Momento de flexión en la viga : $M_x = -\dfrac{3E\varepsilon tI\,(h^2 + L^2)}{L^3h\,(k+1)}\,x$

Momento de flexión en la columna : $M_x = -\dfrac{3E\varepsilon tI\,(h^2 + L^2)}{L^2\,h^2\,(k+1)}\,y$

Nota : Los resultados presentados corresponden a los casos donde existe una elevación de temperatura. En caso donde contrariamente, exista descenso de temperatura, los esfuerzos se anotan con los signos contrarios.

Resolver el siguiente pórtico





$$k = \frac{I_2h}{I_1L} = \frac{56,000x5}{20,000x7} = 2$$

$$V_A = V_C = \frac{k}{8\,(k+1)} \cdot \frac{wh^2}{L} = \frac{2}{8\,(2+1)} \cdot \frac{7,000x5x5}{7} \doteq 2,083 \text{ Kg.}$$

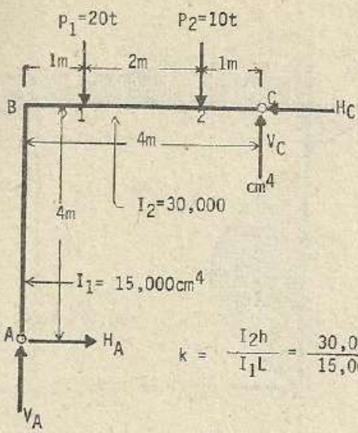
$$H_A = \frac{3k+4}{8\,(k+1)}\,wh = \frac{3 \times 2 + 4}{8\,(2+1)} \times 7,000x5 \doteq 14,583 \text{ Kg.}$$

$$H_C = \frac{5k+4}{8\,(k+1)} \cdot wh = \frac{5x2 + 4}{8\,(2+1)} \times 7,000 \times 5 \doteq 20,417 \text{ Kg.}$$

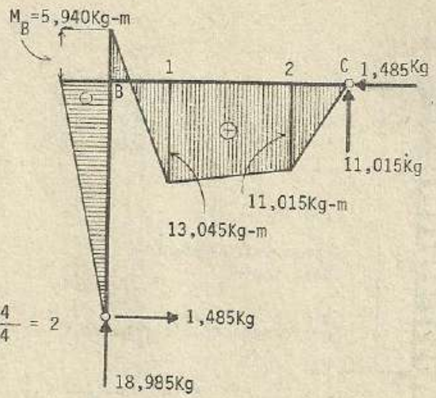
$$M_B = -\frac{k}{8\,(k+1)} \cdot wh^2 = -\frac{2}{8\,(2+1)} \times 7,000x5x5 \doteq -14,583 \text{ Kg -m.}$$

$$M_{max} = \frac{(3k+4)^2}{128\,(k+1)^2}\,wh^2 = \frac{(3x2+4)^2}{128\,(2+1)^2} \times 7,000x5x5 \doteq 15,191 \text{ Kg -m.}$$

$$y_m = \frac{(3k+4)}{8(k+1)} \cdot h = \frac{3x1 + 4}{8\,(2+1)} \times 5 \doteq 2.08 \text{ m.}$$



$$k = \frac{I_2 h}{I_1 L} = \frac{30,000x4}{15,000x4} = 2$$



Debido a (P_1) ; $H_{A'} = H_{C'} = \frac{P_1 ab\ (L+b)}{2hL^2\ (k+1)} = \frac{20x1x3\ (4+3)}{2x4x4x4\ (2+1)} = 1,094$ Kg.

Debido a (P_2) :

$$H_{A''} = H_{C''} = \frac{P_2 ab\ (L+b)}{2hL^2\ (k+1)} = \frac{10x3x1\ (4+1)}{2x4x4x4\ (2+1)} = 391 \text{ Kg.}$$

$\therefore$ $H_A = H_C = 1,094 + 391 = 1,485$ Kg.

$V_A = \frac{Pb}{L} + \frac{H_A h}{L}$ (cuando $\Sigma M_C = 0$)

Debido a (P_1) : $V_{A'} = \frac{P_1 b}{L} + \frac{H_A h}{L} = \frac{20x3}{4} + \frac{1,094\ x\ 4}{4} = 16,094$ Kg.

Debido a (P_2) : $V_{A''} = \frac{P_2 b}{L} + \frac{H_A h}{L} = \frac{10x1}{4} + \frac{0.391x4}{4} = 2,891$ Kg.

$\therefore$ $V_A = 16,094 + 2,891 = 18,985$ Kg.

$V_C = \frac{Pa}{L} - \frac{H_A h}{L}$ (cuando $\Sigma M_B = 0$)

Debido a (P_1) : $V_{C'} = \frac{P_1 a}{L} - \frac{H_A h}{L} = \frac{20x1}{4} - \frac{1.094x4}{4} = 3.906$ Kg.

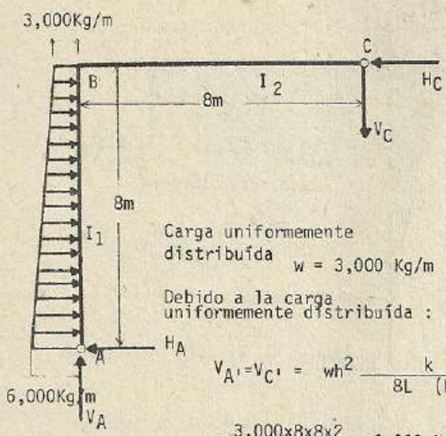
Debido a (P_2) : $V_{C''} = \frac{P_2 a}{L} - \frac{H_A h}{L} = \frac{10x3}{4} - \frac{0.391x4}{4} = 7.109$ Kg.

$\therefore$ $V_C = 3,906 + 7,109 = 11,015$ Kg.

$M_B = -H_A h = -1,485 \times 4 = -5,940$ Kg -m

$M_1 = -M_B + V_A a = -5,940 + 18,985 \times 1 = 13,045$ Kg -m ($M_{máx}$)

$M_2 = V_C b = 11,015 \times 1 = 11,015$ Kg -m.

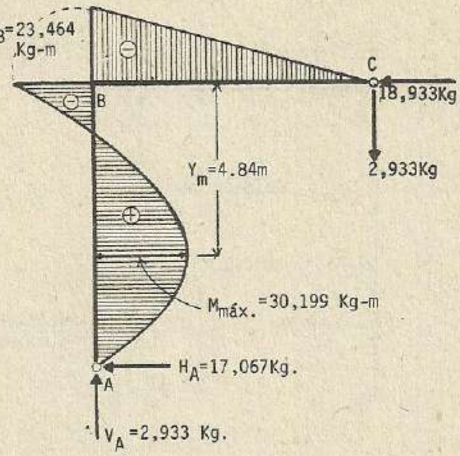


Carga uniformemente distribuída $w = 3{,}000$ Kg/m

Debido a la carga uniformemente distribuída :

$$V_{A'} = V_{C'} = wh^2 \frac{k}{8L\ (k+1)}$$

$$= \frac{3{,}000 \times 8 \times 8 \times 2}{8 \times 8\ (2+1)} = 2{,}000 \text{ Kg.}$$



Debido a la carga triangular :

$$V_{A''} = V_{C''} = wh \cdot \frac{7k}{60L(k+1)} = \frac{12{,}000 \times 8 \times 7 \times 2}{60 \times 8 \times (2+1)} = 933 \text{ Kg.}$$

$$\therefore\ V_A = V_C = 2{,}000 + 933 = 2{,}933 \text{ Kg.}$$

Debido a la carga uniformemente distribuida :

$$H_{A'} = wh \cdot \frac{3k+4}{8(k+1)} = 3{,}000 \times 8 \times \frac{3 \times 2+4}{8\ (2+1)} = 10{,}000 \text{ Kg.}$$

Debido a la carga triangular :

$$H_{A''} = \frac{33k + 40}{60\ (k+1)} \cdot W = \frac{66 + 40}{60(2+1)} \times 12{,}000 = 7{,}067 \text{ Kg.}$$

$$\therefore\ H_A = 10{,}000 + 7{,}067 = 17{,}067 \text{ Kg.}$$

Debido a la carga uniformemente distribuida :

$$H_{C'} = wh - H_{A'} = 3{,}000 \times 8 - 10{,}000 = 14{,}000 \text{ Kg.}$$

Debido a la carga triangular : $H_{C''} = 12{,}000 - 7{,}067 = 4{,}933$ Kg.

$$\therefore\ H_C = 14{,}000 + 4{,}933 = 18{,}933 \text{ Kg.}$$

$$M_B = -V_C L = -2{,}933 \times 8 = -23{,}464 \text{ Kg -m}$$

Para determinar el momento máximo de flexión en la columna, primero obtenemos la distancia (y_m), desde el punto (B), al punto donde la cortante es cero

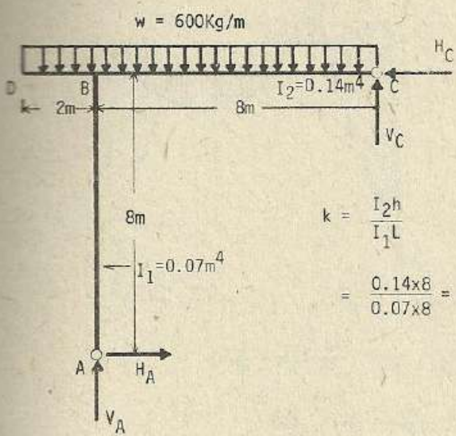
$$V = H_C - wy_m - W \cdot \frac{y_m^2}{h^2} = 0 \quad ; \quad 18{,}933 - 3{,}000\ y_m - \frac{12{,}000}{8 \times 8} \cdot y_m^2 = 0$$

$$750\ y_m^2 + 12{,}000\ y_m - 75{,}732 = 0 \qquad y_m = \frac{-12{,}000 \pm \sqrt{12{,}000^2 + 4 \times 750 \times 75{,}732}}{2 \times 750}$$

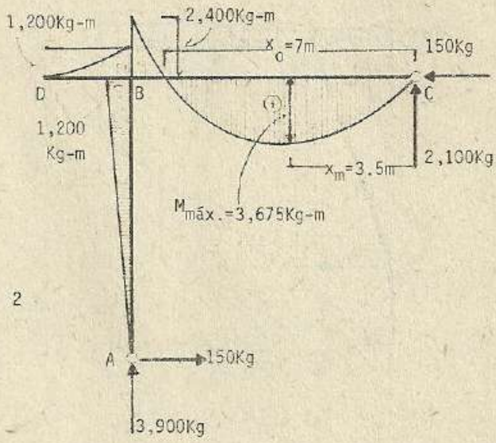
$$= \frac{-12{,}000 + 19{,}266}{1{,}500} \dashrightarrow \text{ desde donde } \quad y_m = 4.84 \text{ m.}$$

$$M_{max.} = H_C y_m - V_C L - \frac{wy_m^2}{2} - W \cdot \frac{y_m^3}{3L^2} = 18{,}933 \times 4.84 - 2{,}933 \times 8 - \frac{3{,}000 \times 4.84^2}{2} -$$

$$\frac{1{,}200 \times 4.84^3}{3 \times 2 \times 8} = 30{,}199 \text{ Kg.}$$



$$k = \frac{I_2 h}{I_1 L} = \frac{0.14\times8}{0.07\times8} = 2$$



$$H_A = H_C = \frac{wL^2}{8(k+1)h} - \frac{wL_1^2}{2\ (k+1)h} = \frac{600\times8\times8}{8\ (2+1)\times8} - \frac{600\times2\times2}{2\ (2+1)\times 8} = 150 \text{ Kg.}$$

$$V_A = \frac{(4k+5)}{8\ (k+1)}\cdot wL + \frac{k}{2\ (k+1)L}\cdot wL_1^2 + wL_1 = \frac{8+5}{8\times\ (2+1)}\times600\times8 + \frac{2}{2(2+1)\times8}\times600\times2\times2 + 600\times2$$

$$= 3{,}900 \text{ Kg.}$$

$$V_C = w\ (L+L_1) - V_A = 600\ (8+2) - 3{,}900 = 2{,}100 \text{ Kg.}$$

$$M_{BD} = -w\frac{L_1^2}{2} = -600\times\frac{2\times2}{2} = -1{,}200 \text{ Kg-m} \quad ; \quad M_{BA} = -H_A h = -150\times8 = -1{,}200 \text{ Kg-m}$$

$$M_{BC} = V_C L - \frac{wL^2}{2} = 2{,}100\times8 - 600\times\frac{8\times8}{2} = -2{,}400 \text{ Kg-m}$$

Considerando como (x_m) la distancia del punto (C) al punto de momento máximo.

$$V_C - wx_m = 0 \dashrightarrow \quad x_m = \frac{V_C}{w} = \frac{2{,}100}{600} = 3.5 \text{ m}$$

$$M_{máx.} = V_C x_m - w\frac{x_m^2}{2} = 2{,}100\times3.5 - 600\times\frac{3.5\times3.5}{2} = 3{,}675 \text{ Kg-m}$$

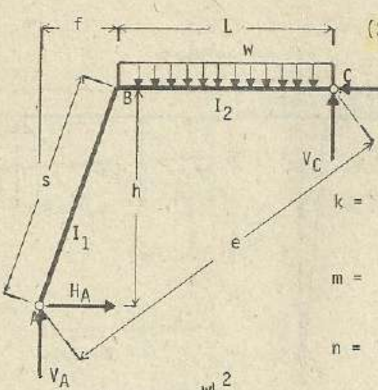
Si hacemos como (x_0) la distancia que media entre el punto (C), al punto donde el momento de flexión es cero :

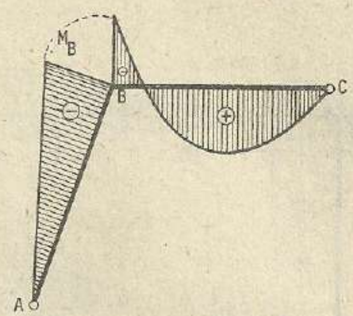
$$V_C x_0 - \frac{wx_0^2}{2} = 0$$

$$x_0\cdot\left(V_C - \frac{wx_0}{2}\right) = 0$$

$$x_0 = 0 \quad ; \qquad x_0 = \frac{2V_C}{w} = \frac{2\times2{,}100}{600} = 7 \text{ m.}$$

(24)





$$k = \frac{I_2 s}{I_1 L}$$

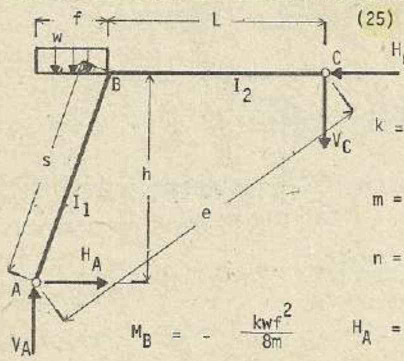
$$m = 1 + k$$

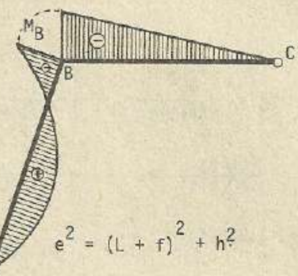
$$n = \frac{f}{L} \qquad e^2 = (L + f)^2 + h^2$$

$$M_B = -\frac{wL^2}{8m} \qquad H_A = H_C = \frac{wL^2}{2h}\left(n + \frac{1+n}{4m}\right)$$

$$V_A = \frac{wL}{2}\left(1 + \frac{1}{4m}\right) \qquad V_C = \frac{wL}{2}\left(1 - \frac{1}{4m}\right)$$

(25)





$$k = \frac{I_2 s}{I_1 L}$$

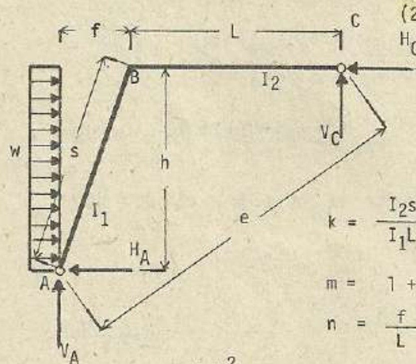
$$m = 1 + k$$

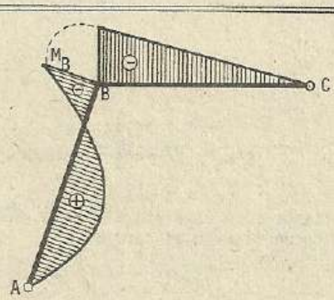
$$n = \frac{f}{L} \qquad e^2 = (L + f)^2 + h^2$$

$$M_B = -\frac{kwf^2}{8m} \qquad H_A = H_C = \frac{wf^2}{2h} - \frac{1+n}{h} M_B$$

$$V_A = wf - \frac{M_B}{L} \quad ; \quad V_C = -\frac{M_B}{L}$$

(26)





$$k = \frac{I_2 s}{I_1 L}$$

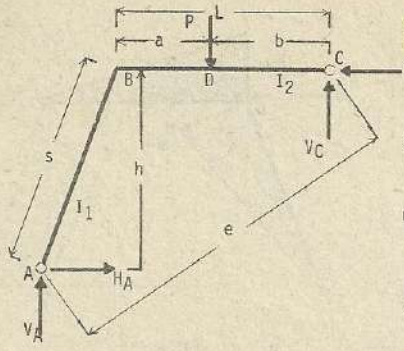
$$m = 1 + k$$

$$n = \frac{f}{L} \qquad e^2 = (L + f)^2 + h^2$$

$$M_B = -\frac{kwh^2}{8m} \qquad H_A = \frac{wh}{2}\left[1 - \frac{(1+n)\,k}{4m}\right]$$

$$H_C = \frac{wh}{2}\left[1 + \frac{(1+n)\,k}{4m}\right] \qquad V_A = V_C = -\frac{M_B}{L}$$

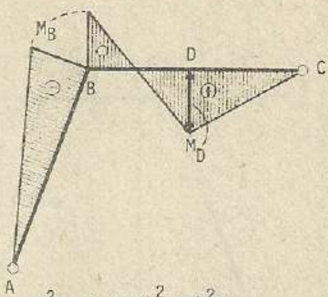
(27)



$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

$$n = \frac{f}{L}$$



$$e^2 = (L + f)^2 + h^2$$

$$M_B = -\frac{Pab}{2L^2 m}(L + b) \qquad H_A = H_C = \frac{nPb - (1 + n)\,M_B}{h}$$

$$V_A = \frac{Pb - M_B}{L} \qquad V_C = \frac{Pa + M_B}{L}$$

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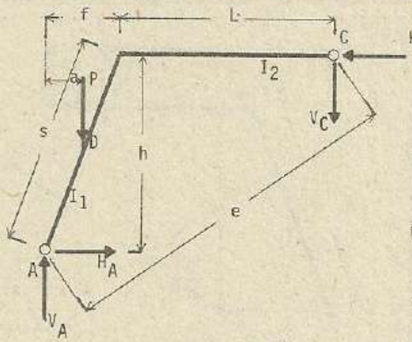
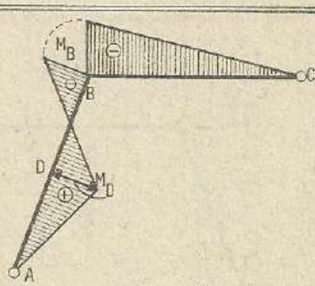
(28)



$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

$$n = \frac{f}{L} \qquad e^2 = (L + f)^2 + h^2$$



$$M_B = -\frac{Pak}{2m}\left[1 - \left(\frac{a}{f}\right)^2\right] \;; \qquad H_A = H_C = \frac{Pa - (1 + n)\,M_B}{h}$$

$$V_A = P - \frac{M_B}{L} \qquad V_C = -\frac{M_B}{L}$$

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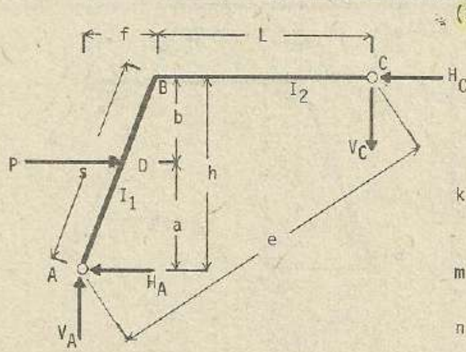
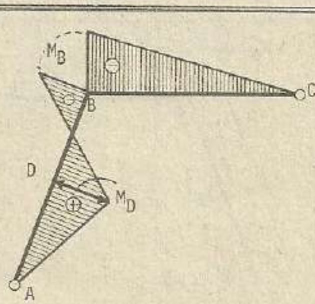
(29)



$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$

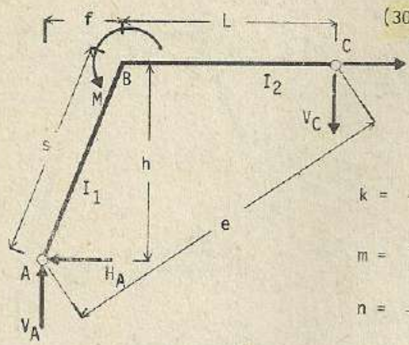
$$n = \frac{f}{L} \qquad e^2 = (L + f)^2 + h^2$$

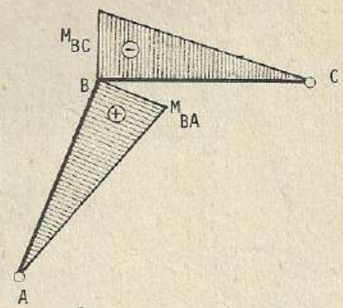


$$M_B = -\frac{Pabk}{2h^2 m}(h + a) \qquad H_A = \frac{Pb + (1 + n)\,M_B}{h}$$

$$V_A = V_C = -\frac{M_B}{L} \qquad H_C = \frac{Pa - (1 + n)\,M_B}{h}$$

(30)





$$k = \frac{I_2 s}{I_1 L}$$

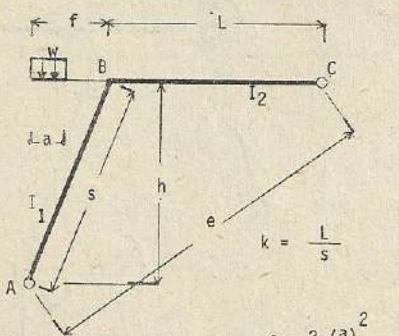
$$m = 1 + k$$

$$n = \frac{f}{L} \qquad e^2 = (L + f)^2 + h^2$$

$$M_{BA} = \frac{M}{m} \quad ; \quad M_{BC} = -\frac{kM}{m}$$

$$H_A = \frac{(1 + n)\, M_{BA} - nM}{h} \qquad V_A = V_C = -\frac{M_{BC}}{L}$$

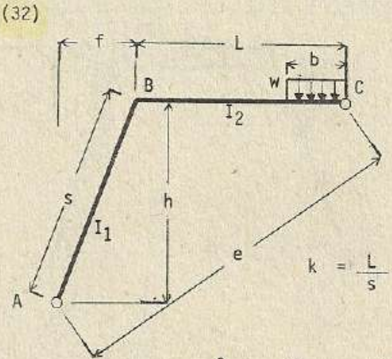
(31)



$$k = \frac{L}{s}$$

$$M_B = -\frac{wa^2}{8}\;\frac{2-\left(\frac{a}{f}\right)^2}{1 + k}$$

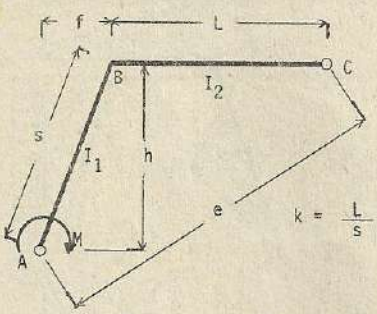
(32)



$$k = \frac{L}{s}$$

$$M_B = -\frac{wb^2}{8}\;\frac{k}{1+k}\left[2 - \left(\frac{b}{L}\right)^2\right]$$

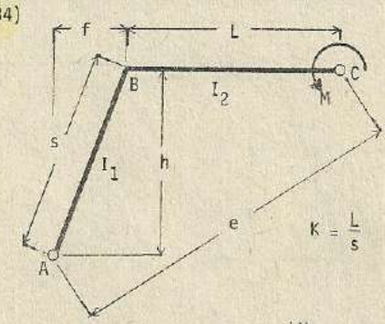
(33)



$$k = \frac{L}{s}$$

$$M_B = -\frac{M}{2\,(1 + k)}$$

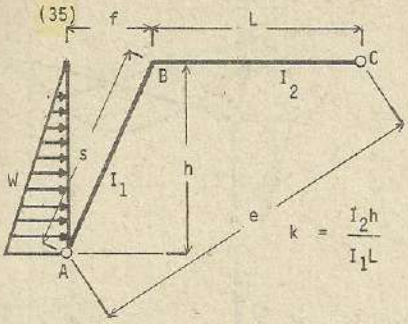
(34)



$$K = \frac{L}{s}$$

$$M_B = -\frac{kM}{2\,(1 + k)}$$

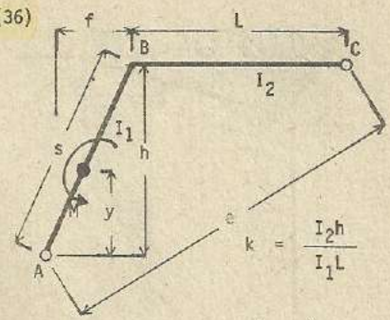
(35)



$$k = \frac{I_2 h}{I_1 L}$$

$$M_B = - \frac{7kWh}{60\ (1+k)}$$

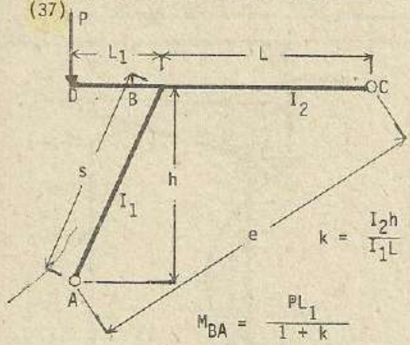
(36)



$$k = \frac{I_2 h}{I_1 L}$$

$$M_B = - \frac{kM\ (3y^2 - h^2)}{2h^2\ (1 + k)}$$

(37)

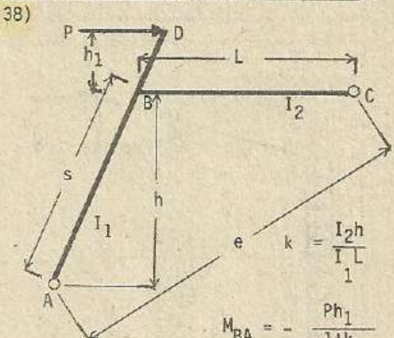


$$k = \frac{I_2 h}{I_1 L}$$

$$M_{BA} = \frac{PL_1}{1 + k}$$

$$M_{BC} = - \frac{PkL_1}{1 + k} \quad ; \quad M_{BD} = - PL_1$$

(38)

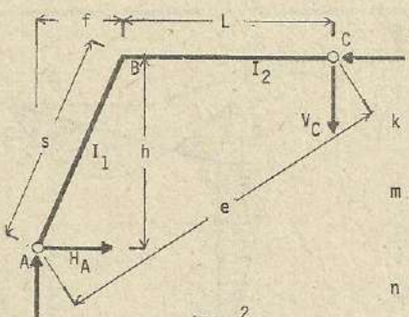


$$k = \frac{I_2 h}{I_1 L}$$

$$M_{BA} = - \frac{Ph_1}{1+k}$$

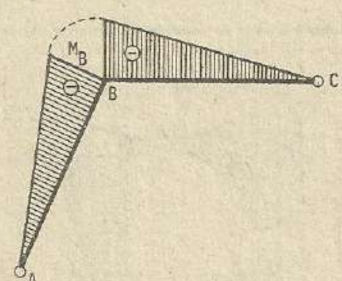
$$M_{BC} = \frac{Pkh_1}{1 + k} \quad ; \quad M_{BD} = Ph_1$$

Esfuerzos por cambio de temperatura (39)



$$k = \frac{I_2 s}{I_1 L}$$

$$m = 1 + k$$



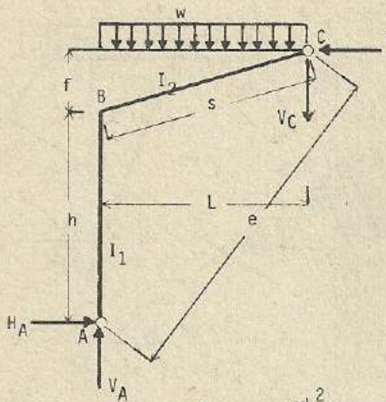
$$n = \frac{f}{L} \qquad e^2 = (L + f)^2 + h^2$$

$$M_B = \frac{3EI_2 e^2\ et}{mL^2 h} \quad ; \quad V_A = V_C = - \frac{M_B}{L}$$

$$H_A = H_C = - \frac{(1+n)\ M_B}{h}$$

Nota : En caso de descenso de la temperatura, los esfuerzos se anotan con los signos contrarios.

(40)

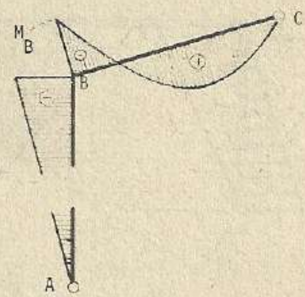


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$n = \frac{f}{s}$$

$$e^2 = L^2 + (h+f)^2$$

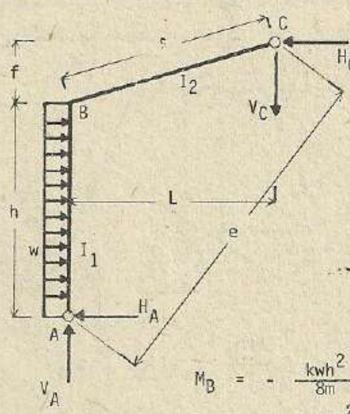


$$M_B = -\frac{wL^2}{8m} \quad ; \quad H_A = \frac{wL^2}{8mh} \quad ; \quad V_A = \frac{wL}{2}\left(1 + \frac{1+n}{4m}\right)$$

$$V_C = \frac{wL}{2}\left(1 - \frac{1+n}{4m}\right)$$

---

(41)

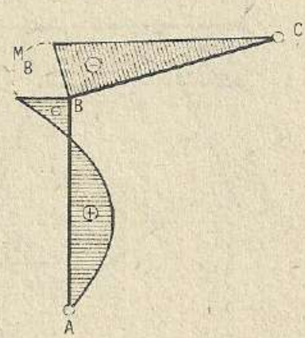


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$n = \frac{f}{s}$$

$$e^2 = L^2 + (h+f)^2$$

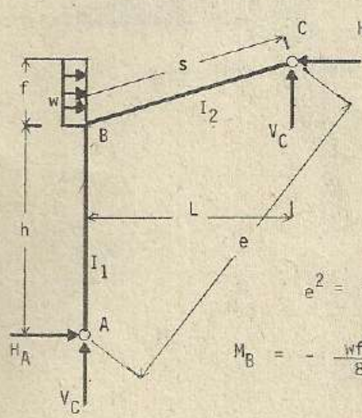


$$M_B = -\frac{kwh^2}{8m} \quad ; \quad H_A = \frac{wh}{2}\left(1 - \frac{k}{4m}\right) \quad ; \quad H_C = \frac{wh}{2}\left(1 + \frac{k}{4m}\right)$$

$$V_C = V_A = \frac{wh^2}{2L}\left[n + \frac{k\,(1+n)}{4\,m}\right]$$

---

(42)

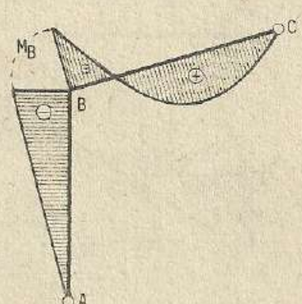


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$n = \frac{f}{s}$$

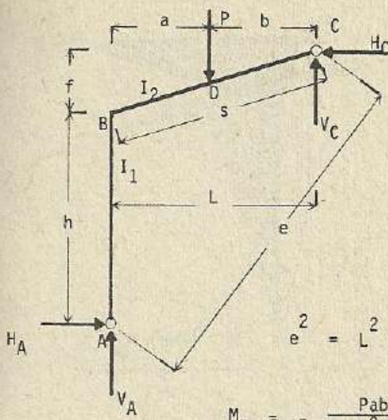
$$e^2 = L^2 + (h+f)^2$$



$$M_B = -\frac{wf^2}{8m} \quad ; \quad H_A = \frac{wf^2}{8mh} \quad ; \quad H_C = wf - H_A$$

$$\frac{wf^2}{2L}\left(1 + \frac{1+n}{[illegible]}\right)$$

(43)

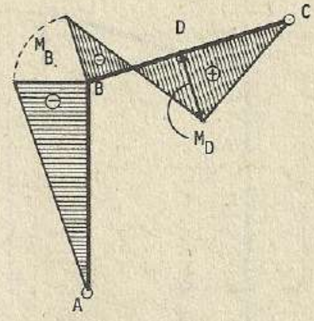


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$n = \frac{f}{s}$$

$$e^2 = L^2 + (h+f)^2$$

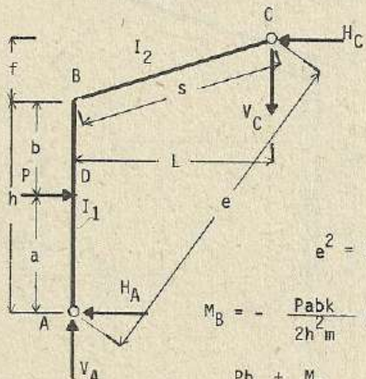


$$M_B = -\frac{Pab}{2L^2 m}(L+b) \quad ; \quad H_A = H_C = -\frac{M_B}{h}$$

$$V_A = \frac{wb - (1+n)M_B}{L} \quad ; \quad V_C = \frac{wa + (1+n)\,M_B}{L}$$

---

(44)

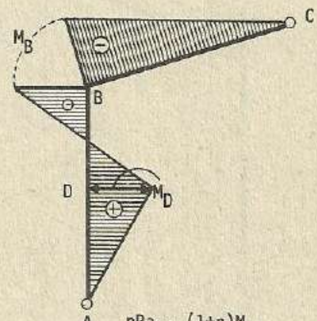


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$n = \frac{f}{s}$$

$$e^2 = L^2 + (h+f)^2$$



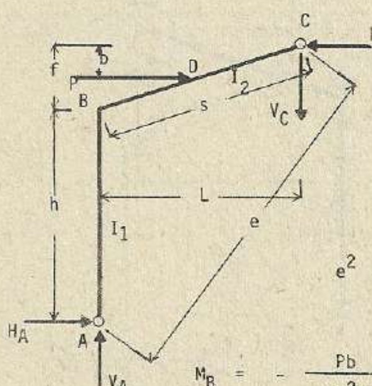
$$M_B = -\frac{Pabk}{2h^2 m}(h+a)$$

$$V_A = V_C = \frac{nPa - (1+n)M_B}{L}$$

$$H_A = \frac{Pb + M_B}{h} \qquad H_C = \frac{Pa - M_B}{h}$$

---

(45)

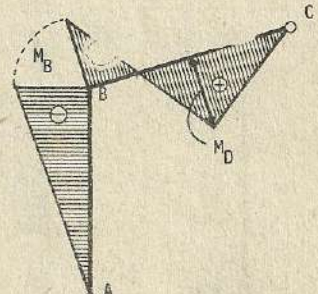


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$n = \frac{f}{s}$$

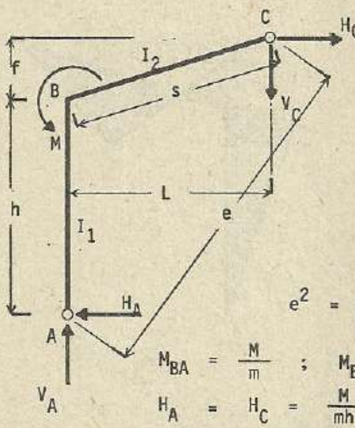
$$e^2 = L^2 + (h+f)^2$$



$$M_B = -\frac{Pb}{2f^2 m}(f^2 - b^2) \quad ; \quad V_A = V_C = \frac{Pa - (1+n)\,M_B}{L}$$

$$H_A = -\frac{M_B}{h} \quad ; \quad H_C = P - H_A$$

(46)

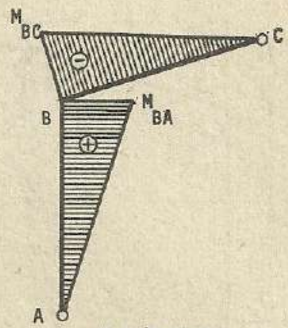


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$n = \frac{f}{s}$$

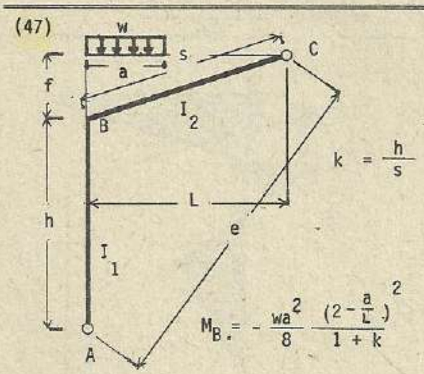
$$e^2 = L^2 + (h+f)^2$$



$$M_{BA} = \frac{M}{m} \; ; \quad M_{BC} = -\frac{kM}{m} \; ; \quad V_A = V_C = \frac{M - (1+n)\,M_{BA}}{L}$$

$$H_A = H_C = \frac{M}{mh}$$

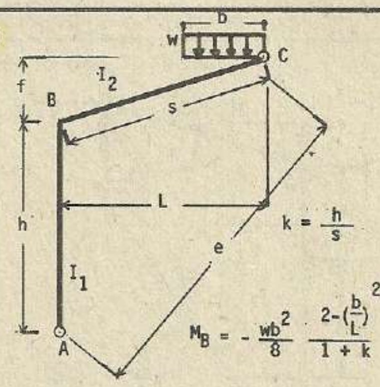
(47)



$$k = \frac{h}{s}$$

$$M_B = -\frac{wa^2}{8}\;\frac{\left(2 - \frac{a}{L}\right)^2}{1 + k}$$

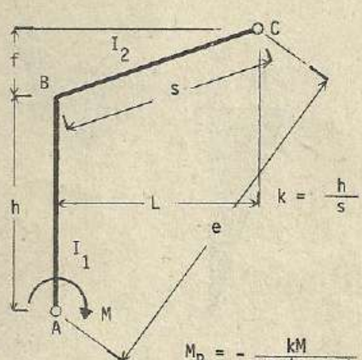
(48)



$$k = \frac{h}{s}$$

$$M_B = -\frac{wb^2}{8}\;\frac{2 - \left(\frac{b}{L}\right)^2}{1 + k}$$

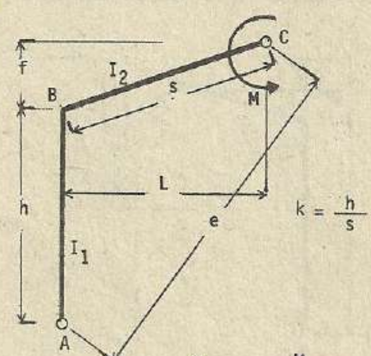
(49)



$$k = \frac{h}{s}$$

$$M_B = -\frac{kM}{2\,(1 + k)}$$

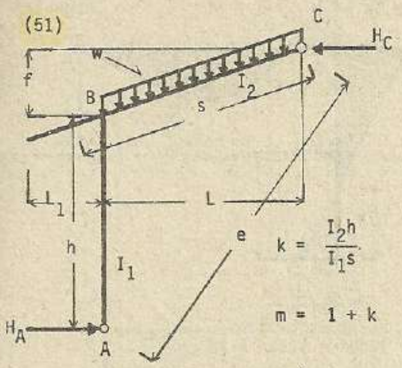
(50)



$$k = \frac{h}{s}$$

$$M_B = -\frac{M}{2\,(1 + k)}$$

(51)

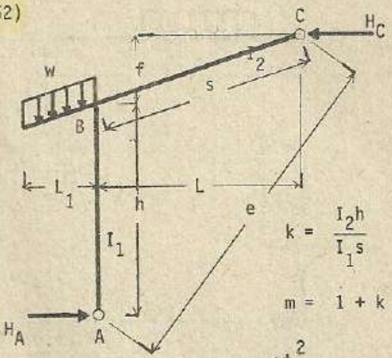


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$H_A = H_C = \frac{wL^2}{8h}\,\frac{1}{m}$$

(52)

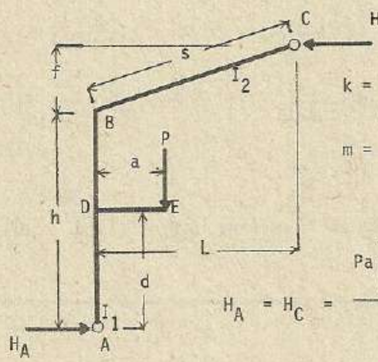


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$H_A = H_C = -\frac{wL_1^2}{2h}\,\frac{1}{m}$$

(53)

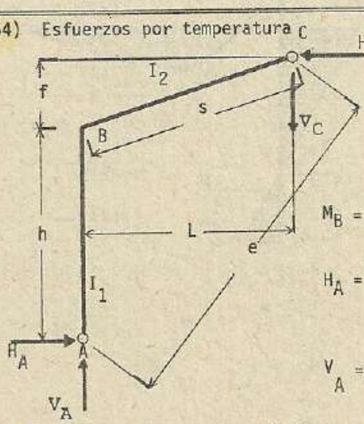


$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1 + k$$

$$H_A = H_C = \frac{Pa\left[3(h^2-d^2)\,\frac{I_2}{I_1} + 2\,hs\right]}{2h^2 s}\,\frac{1}{m}$$

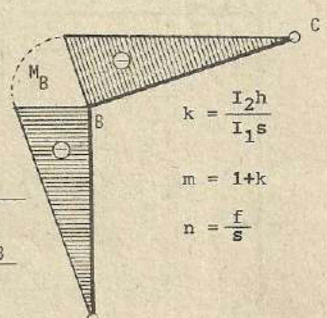
(54) Esfuerzos por temperatura



$$M_B = -\frac{3EI_2 e^2 \varepsilon t}{mLhs}$$

$$H_A = H_C = -\frac{M_B}{h}$$

$$V_A = V_C = -\frac{(1+n)\,M_B}{L}$$



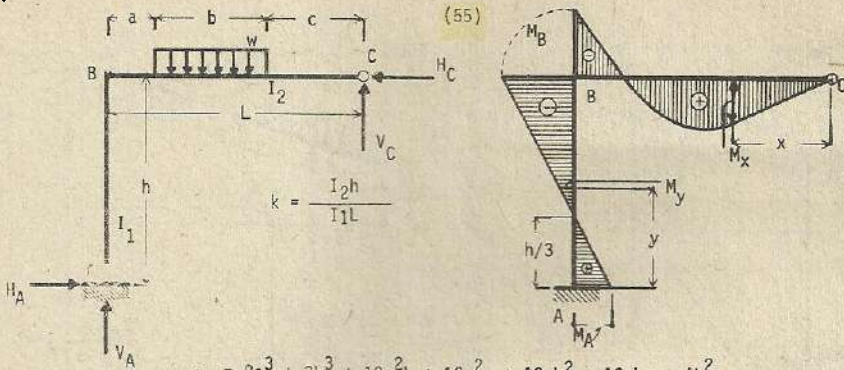
$$k = \frac{I_2 h}{I_1 s}$$

$$m = 1+k$$

$$n = \frac{f}{s}$$

Nota : En caso de descenso de la temperatura, los esfuerzos se anotan con los signos contrarios.

(55)



$$k = \frac{I_2 h}{I_1 L}$$

$$\alpha = 8a^3 + 3b^3 + 18a^2b + 12a^2c + 12ab^2 + 12abc + 4b^2c$$

$$H_A = H_C = \frac{3wb}{4\,(3k+4)h}\left[4\,(2a+b) - \frac{\alpha}{L^2}\right] \quad ; \quad V_C = \frac{wb}{2(3k+4)L}\left[\frac{\alpha}{L^2} + (6a+3b)k\right]$$

$$V_A = \frac{wb}{2\,(3k+4)\,L}\left[2\,(3k+4)L - \frac{\alpha}{L^2} - (6a+3b)k\right] \quad ; \quad M_B = V_C L - \frac{wb}{2}(2a+b)$$

$$M_A = V_C L - \frac{wb}{2}(2\,a+b) + H_C h = -\frac{M_B}{2}$$

Momento de flexión en la viga :

$$M_x = V_C x \qquad x \leqslant c$$

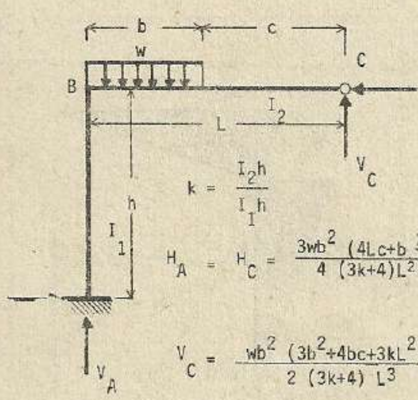
$$M_x = V_C x - \frac{w\,(x-c)^2}{2} \qquad c \leqslant x \leqslant b+c$$

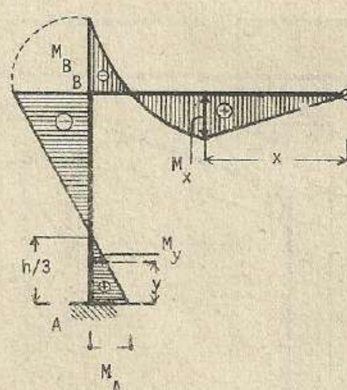
$$M_x = V_C x - wb\,(x - c - \frac{b}{2}) \qquad x \geqslant b+c$$

Esfuerzo por flexión en la columna : $M_y = V_C L - \frac{wb}{2}(2a+b) + H_C y$

---

(56)





$$k = \frac{I_2 h}{I_1 h}$$

$$H_A = H_C = \frac{3wb^2\,(4Lc+b^2)}{4\,(3k+4)L^2\,h}$$

$$V_C = \frac{wb^2\,(3b^2+4bc+3kL^2)}{2\,(3k+4)\,L^3}$$

$$V_A = wb - V_C \quad ; \quad M_B = V_C L - \frac{wb^2}{2}$$

$$M_A = V_C L - \frac{wb^2}{2} + H_C h = -\frac{M_B}{2}$$

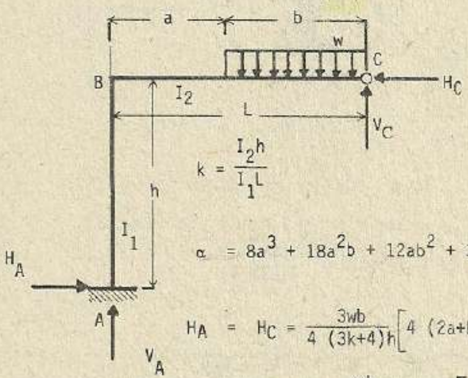
Esfuerzos por doblado o flexión en la viga :

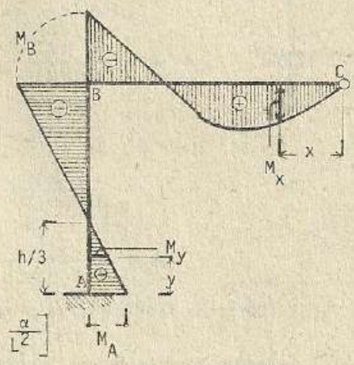
$$M_x = V_C x \qquad x < c$$

$$M_x = V_C x - \frac{w(x-c)^2}{2} \qquad x \geqslant c$$

Esfuerzos por flexión en la columna : $M_y = V_C L - \frac{wb^2}{2} + H_C y$

(57)





$$k = \frac{I_2 h}{I_1 L}$$

$$\alpha = 8a^3 + 18a^2 b + 12ab^2 + 3b^3$$

$$H_A = H_C = \frac{3wb}{4(3k+4)h}\left[4(2a+b) - \frac{\alpha}{L^2}\right]$$

$$V_C = \frac{wb}{2(3k+4)L}\left[\frac{\alpha}{L^2} + (6a+3b)k\right] \qquad V_A = wb - V_C$$

$$M_B = V_C L - \frac{wb}{2}(2a+b) \qquad M_A = V_C L - \frac{wb}{2}(2a+b) + H_C h$$

Esfuerzo de flexión en la viga :

$$M_x = V_C x - \frac{wx^2}{2} \qquad x < b$$

$$M_x = V_C x - wb\left(x - \frac{b}{2}\right) \qquad x > b$$

Esfuerzo de flexión en la columna : $M_y = V_C L - \frac{wb}{2}(2a+b) + H_C y$

En caso que $a = b = \frac{L}{2}$; $\alpha = \frac{41}{8} L^3$; $H_A = H_C = \frac{21wL^2}{64(3k+4)h}$;

$$V_C = \frac{36k+41}{32(3k+4)} \cdot wL \quad ; \qquad V_A = \frac{12k+23}{32(3k+4)} wL$$

$$M_B = -\frac{7}{32(3k+4)} wL^2 \quad ; \qquad M_A = \frac{7}{64(3k+4)} wL^2$$

Esfuerzos de flexión en la viga :

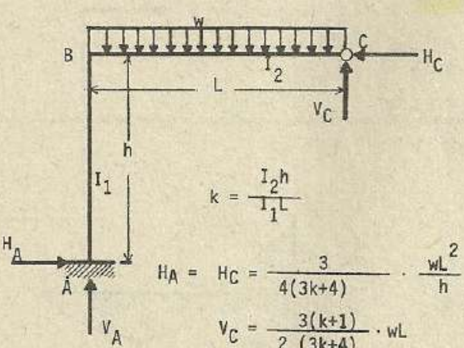
$$M_x = \frac{36k+41}{32(3k+4)} wLx - \frac{wx^2}{2} \qquad x < \frac{L}{2}$$

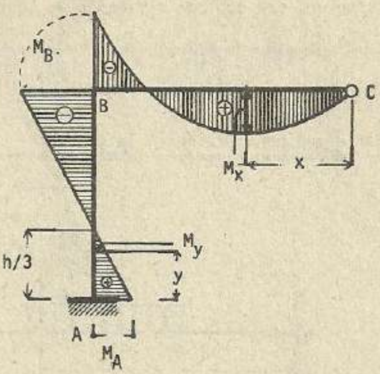
$$M_x = \frac{36k+41}{32(3k+4)} wLx - \frac{wL}{2}\left(x - \frac{L}{4}\right) \qquad x > \frac{L}{2}$$

Esfuerzos de flexión en la columna :

$$M_x = \frac{36k+41}{32(3k+4)} wL^2 - \frac{3wL^2}{8} + \frac{21\, wL^2 y}{64(3k+4)h}$$

(58)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{3}{4(3k+4)} \cdot \frac{wL^2}{h}$$

$$V_C = \frac{3(k+1)}{2\,(3k+4)} \cdot wL$$

$$V_A = \frac{(3k+5)}{3\,(3k+4)} \cdot wL$$

$$M_B = -\frac{wL^2}{2\,(3k+4)} \quad ; \quad M_A = \frac{wL^2}{4\,(3k+4)}$$

Esfuerzo de flexión en la viga : $M_x = V_C x - \frac{wx^2}{2}$

Esfuerzo de flexión en la columna : $M_y = V_C L - \frac{wL^2}{2} + H_C y$

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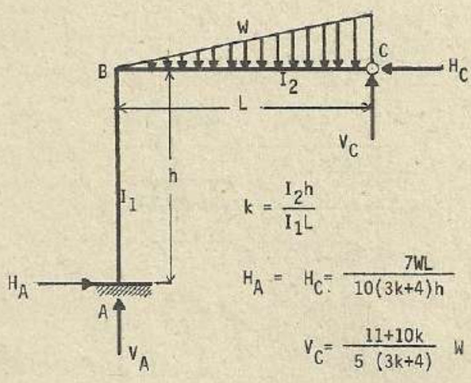
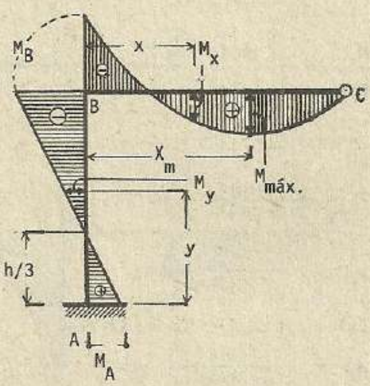
(59)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{7WL}{10(3k+4)h}$$

$$V_C = \frac{11+10k}{5\,(3k+4)}\,W$$

$$V_A = \frac{5k+9}{5\,(3k+4)}\,W \quad ; \quad M_B = -\frac{7WL}{15\,(3k+4)} \quad ; \quad M_A = \frac{7WL}{30\,(3k+4)}$$

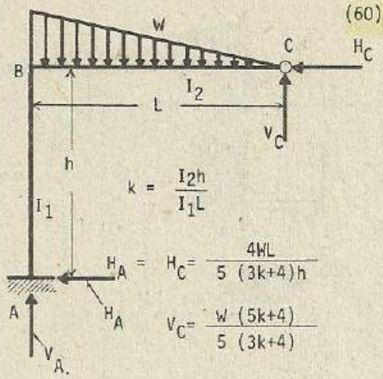
Esfuerzo por flexión en la viga :

$$M_x = M_B + V_A x - \frac{Wx^3}{3L^2}$$

Esfuerzo por flexión en la columna :

$$M_y = M_A - H_A y$$

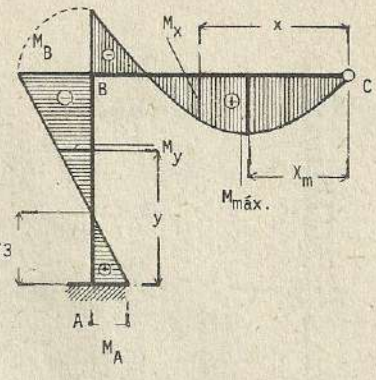
(60)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{4WL}{5\,(3k+4)h}$$

$$V_C = \frac{W\,(5k+4)}{5\,(3k+4)}$$



$$V_A = \frac{2W\,(5k+8)}{5(3k+4)} \qquad M_B = -\frac{3WL}{15\,(3k+4)} \qquad M_A = \frac{4WL}{15\,(3k+4)}$$

Esfuerzo de flexión en la columna : $M_y = M_A - H_A y$

Esfuerzo de flexión en la viga : $M_x = V_C x - \dfrac{Wx^3}{3L^2}$ $\qquad x_m = L\sqrt{\dfrac{5k+4}{5(3k+4)}}$

$$M_{máx.} = \frac{2WL(5k+4)}{15(3k+4)}\sqrt{\frac{5k+4}{5(3k+4)}}$$

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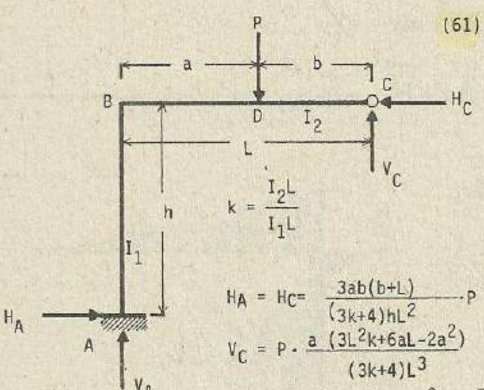
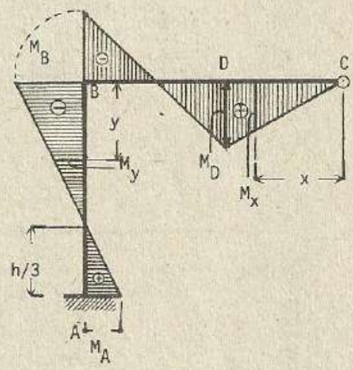
(61)



$$k = \frac{I_2 L}{I_1 L}$$

$$H_A = H_C = \frac{3ab(b+L)}{(3k+4)hL^2}\cdot P$$

$$V_C = P \cdot \frac{a\,(3L^2k+6aL-2a^2)}{(3k+4)L^3}$$



$$V_A = P \cdot \frac{3kL^2b+4L^3 - 6a^2L+2a^3}{(3k+4)L^3} \qquad M_B = -\frac{2ab\,(b+L)}{(3k+4)L^2}\,P$$

$$M_A = \frac{ab\,(b+L)}{(3k+4)L^2}\cdot P \qquad M_D = \frac{Pab\,(3L^2k+6aL - 2a^2)}{(3k+4)L^2}$$

Esfuerzo de flexión en la viga : $M_x = V_C x \qquad x \leq b$

$M_x = V_C x - P\,(x-b) \qquad x \geq b$

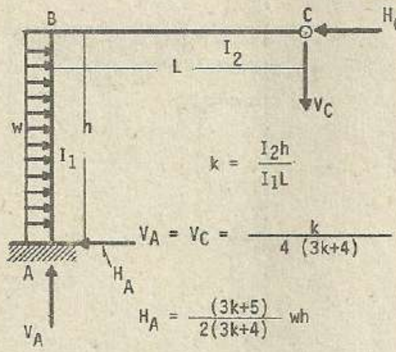
Esfuerzo de flexión en la columna : $M_y = V_C L - Pa + H_C y$

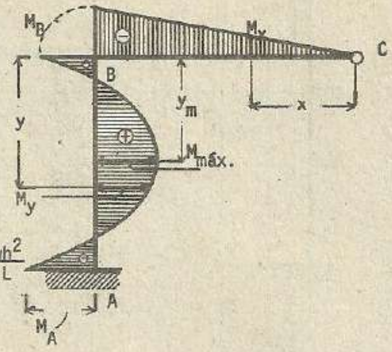
Sí $a = b = \dfrac{L}{2}$; $H_A = H_C = \dfrac{9}{8\,(3k+4)}\cdot\dfrac{PL}{h}$; $V_C = \dfrac{6k+5}{4(3k+4)}\cdot P$

$$V_A = \frac{6k+11}{4\,(3k+4)}\cdot P \qquad M_B = -\frac{3}{4\,(3k+4)}\cdot PL$$

$$M_A = \frac{3}{8\,(3k+4)}\cdot PL \qquad M_D = \frac{6k+5}{8\,(3k+4)}\cdot PL$$

(62)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{k}{4\ (3k+4)} \cdot \frac{wh^2}{L}$$

$$H_A = \frac{(3k+5)}{2(3k+4)}\ wh$$

$$H_C = \frac{3\ (k+1)}{2(3k\ +\ 4)}\ wh \qquad\qquad M_A = -\frac{k+2}{4(3k+4)}\ wh^2$$

$$M_B = -\frac{k}{4\ (3k+4)}\ wh^2$$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{k}{4\ (3k+4)} \cdot \dfrac{wh^2}{L} \cdot x$

Esfuerzo de flexión en la columna :

$$M_y = -\frac{k}{4(3k+4)}\ wh^2 + \frac{3(k+1)}{2(3k+4)}\ why - \frac{wy^2}{2}$$

$$y_m = \frac{3(k+1)}{2(3k+4)} \qquad M_{máx} = \frac{(9+10k+3k^2)}{8\ (3k+4)^2}\ wh^2$$

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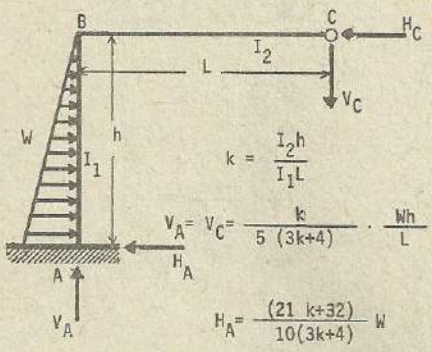
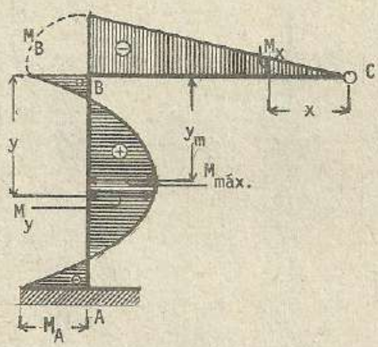
(63)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{k}{5\ (3k+4)} \cdot \frac{Wh}{L}$$

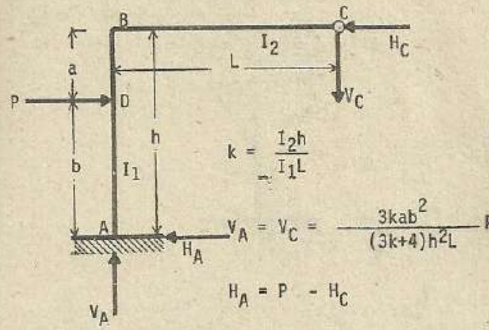
$$H_A = \frac{(21\ k+32)}{10(3k+4)}\ W$$

$$H_C = \frac{(9k+8)}{10\ (3k+4)}\ W \qquad M_B = -\frac{k}{5\ (3k+4)}\ Wh \ ; \qquad M_A = -\frac{(9k+16)}{30\ (3k+4)}\ Wh$$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{k}{5(3k+4)} \cdot \dfrac{Wh}{L}\ x$

Esfuerzo de flexión en la columna :

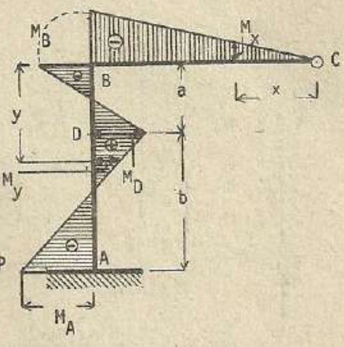
$$M_y = -\frac{k}{5\ (3k+4)}\ Wh + \frac{(9k+8)}{10\ (3k+4)}\ Wy - \frac{Py^3}{3h^2}$$



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{3kab^2}{(3k+4)h^2 L}\,P$$

$$H_A = P - H_C$$



$$H_C = \frac{\{3k\,(2a+h) + 2\,(a+2h)\}}{(3k+4)\,h^3}\,Pb^2 \; ; \quad M_A = -\frac{3ka + 2\,(a+h)}{(3k+4)\,h^2}\,Pab$$

$$M_B = -\frac{3kab^2}{(3k+4)h^2}\cdot P \quad ; \quad M_D = \frac{\{6ka + 2(a+2h)\}}{(3k+4)h^3}\,Pab^2$$

Esfuerzo de flexión en la viga :

$$M_x = -\frac{3kab^2}{(3k+4)h^2 L}\,Px$$

Esfuerzo de flexión en la columna :

$$M_y = -\frac{3kab^2}{(3k+4)h^2}\cdot P + \frac{\{3k\,(2a+b) + 2\,(a+2h)\}}{(3k+4)h^3}\,Pb^2 y \qquad y \leq a$$

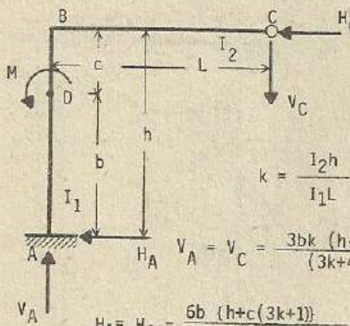
$$M_y = -\frac{3kab^2}{(3k+4)h^2}\cdot P + \frac{\{3k\,(2a+b) + 2\,(a+2h)\}}{(3k+4)h^3}\,Pb^2 y - P\,(y-a); \; y \geq a$$

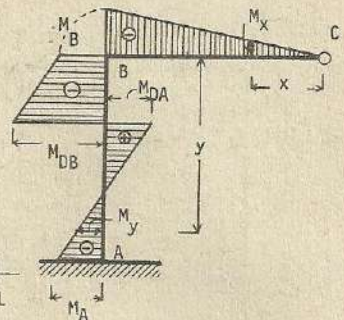
Si $a = b = \frac{h}{2}$ $\qquad V_A = V_C = \frac{3kh}{8(3k+4)L}\,P \quad ; \quad H_A = \frac{6k+11}{4(3k+4)}\,P$

$$H_C = \frac{(6k+5)}{4(3k+4)}\,P$$

$$M_A = \frac{(3k+6)}{8\,(3k+4)}\cdot Ph \quad ; \quad M_B = -\frac{3k}{8(3k+4)}\cdot Ph \quad ; \quad M_D = \frac{(3k+5)}{8(3k+4)}\,Ph$$

(65)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{3bk\ (h-3c)}{(3k+4)} \cdot \frac{M}{h^2 L}$$

$$H_A = H_C = \frac{6b\ \{h+c(3k+1)\}}{(3k+4)} \cdot \frac{M}{h^3} \ ; \qquad M_B = -\frac{3bk\ (h-3c)}{(3k+4)} \cdot \frac{M}{h^2}$$

$$M_{DB} = -\frac{3bM}{(3k+4)h^2}\left[k\ (h-3c) + \frac{2c\{h+c\ (3k+1)\}}{h}\right] \ ; \qquad M_{DA} = M - M_{DB}$$

$$M_A = M - V_c L - H_c h$$

Esfuerzo de flexión en la viga : $M_x = -V_c x$

Esfuerzo de flexión en la columna:

$$M_y = -V_c L - H_c y \qquad y \leq a$$

$$M_y = M - V_c L - H_c y \qquad y \geq a$$

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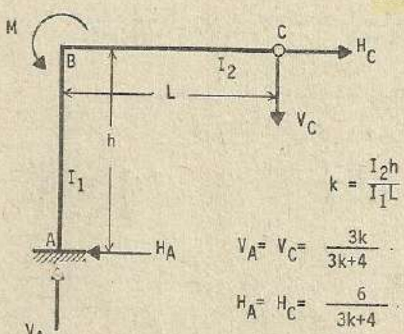
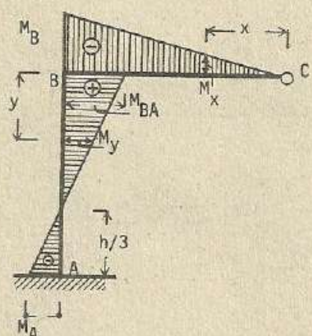
(66)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{3k}{3k+4} \cdot \frac{M}{L}$$

$$H_A = H_C = \frac{6}{3k+4} \cdot \frac{M}{h}$$

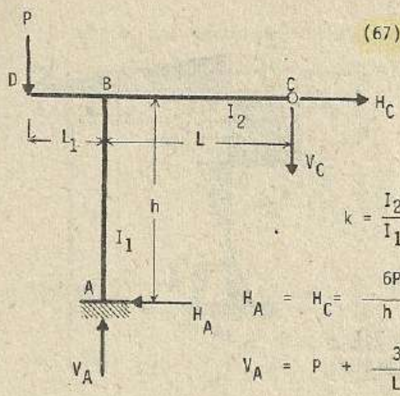
$$M_{BC} = -\frac{3k}{3k+4} M \ ; \qquad M_{BA} = \frac{4}{3k+4} M \ : \qquad M_A = -\frac{2}{3k+4} M$$

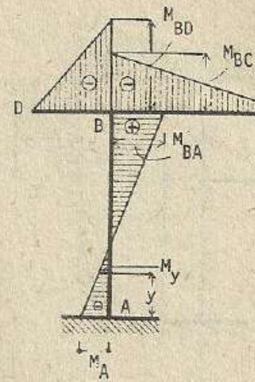
Esfuerzo de flexión en la viga : $M_x = -\frac{3k}{3k+4} \cdot \frac{x}{L} \cdot M$

Esfuerzo de flexión en la columna :

$$M_y = -\frac{3k}{3k+4} \cdot M - \frac{6y}{3k+4} \cdot \frac{M}{h} + M$$

(67)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{6PL_1}{h\ (3k+4)}$$

$$V_A = P + \frac{3PL_1 k}{L\ (3k+4)}$$

$V_C = \dfrac{3PL_1 k}{L(3k+4)}$; $M_{BD} = -\ PL_1$; $M_{BA} = \dfrac{4PL_1}{3k+4}$; $M_{BC} = -\dfrac{3PL_1 k}{(3k+4)}$

$M_A = -\dfrac{2PL_1}{3k+4}$; Esfuerzo de flexión en la viga : $M_x = -\dfrac{PL_1 kx}{(3k+4)L}$

Esfuerzo de flexión en la columna :

$$M_y = -\frac{2PL_1}{(3k+4)} + \frac{6PL_1 y}{(3k+4)\ h}$$

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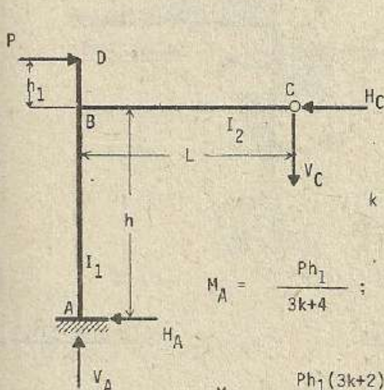
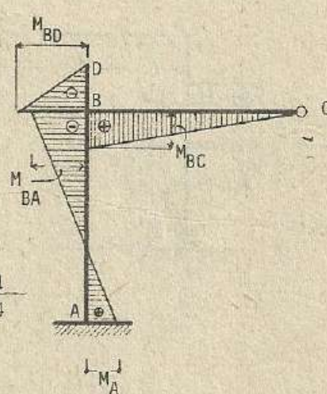
(68)



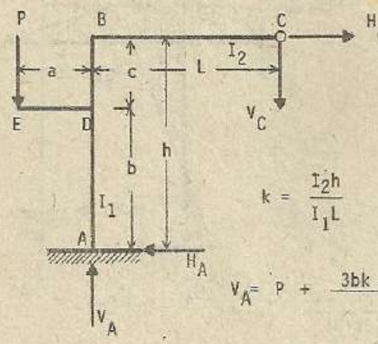


$$k = \frac{I_2 h}{I_1 L}$$

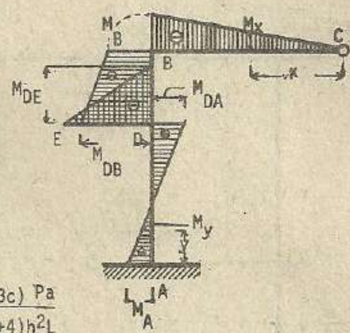
$M_A = \dfrac{Ph_1}{3k+4}$; $M_{BA} = -\dfrac{2Ph_1}{3k+4}$

$M_{BC} = \dfrac{Ph_1(3k+2)}{3k+4}$; $M_{BD} = -\ Ph_1$

(69)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = P + \frac{3bk\,(h-3c)\,Pa}{(3k+4)h^2 L}$$

$$V_C = \frac{3bk\,(h-3c)Pa}{2\,(3k+4)h^2 L}\quad ;\quad H_A = H_C = \frac{6b\left[h+c\,(3k+1)\right]Pa}{(3k+4)h^3}$$

$$M_{DB} = -\frac{3bPa}{(3k+4)\,h^2}\left[k(h-3c) + \frac{2c(h+c\,(3k+1))}{h}\right]; \quad M_B = -\frac{3bk\,(h-3c)\,Pa}{(3k+4)\,h^2}$$

$$M_{DA} = Pa - M_{DB} \quad ; \quad M_A = Pa - V_C L - H_C h$$

Esfuerzo de flexión en la columna : $M_y = -V_C L - H_C y \qquad y \leqslant c$

$$M_y = Pa - V_C L - H_C y \qquad y \geqslant c$$

Esfuerzo de flexión en la viga : $M_x = -V_C x$

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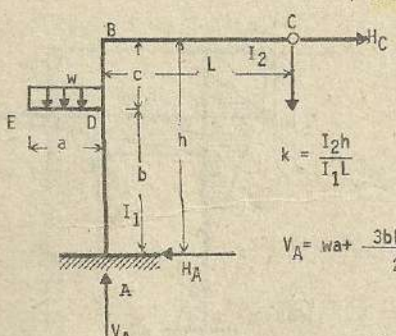
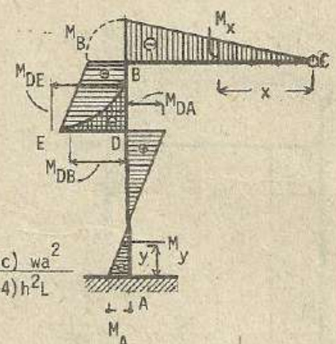
(70)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = wa + \frac{3bk\,(h-3c)\,wa^2}{2\,(3k+4)h^2 L}$$

$$V_C = \frac{3bk\,(h-3c)wa^2}{2\,(3k+4)h^2 L} \qquad H_A = H_C = \frac{3b\left[h+c(3k+1)\right]wa^2}{(3k+4)h^3}$$

$$M_{DB} = -\frac{3bwa^2}{2(3k+4)h^2}\cdot\left[k\,(h-3c) + \frac{2c\,(h+c\,(3k+1))}{h}\right]$$

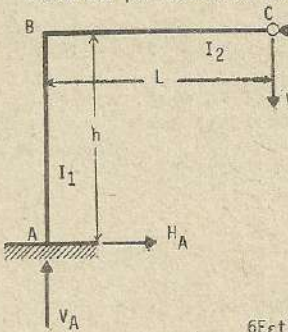
$$M_B = -\frac{3bk\,(h-3c)\,wa^2}{2\,(3k+4)\,h^2}\quad ; \quad M_{DA} = \frac{wa^2}{2} + M_{DB}; \qquad M_A = \frac{wa^2}{2} - V_C L - H_C h$$

Esfuerzo de flexión en la viga : $-V_C x$

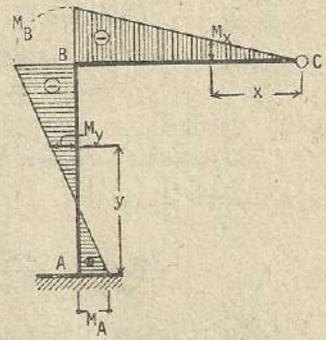
Esfuerzo de flexión en la columna : $M_y = -V_C L - H_C y \qquad y \leqslant c$

$$M_y = \frac{wa^2}{2} - V_C L - H_C y \qquad y \geqslant c$$

Esfuerzos por cambio de temperatura .



$$K = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{6E\varepsilon t I_1 \left[2L^2 (3k+1) + 3kh^2\right]}{(3k+4)h^3 L}$$

$$V_A = V_C = \frac{6E\varepsilon t I_1 (2h^2+3L^2)\ k}{(3k+4)h^2L^2} \quad ; \quad M_B = -\frac{6E\varepsilon t I_1 (2h^2 + 3L^2)\ k}{(3k+4)h^2 L}$$

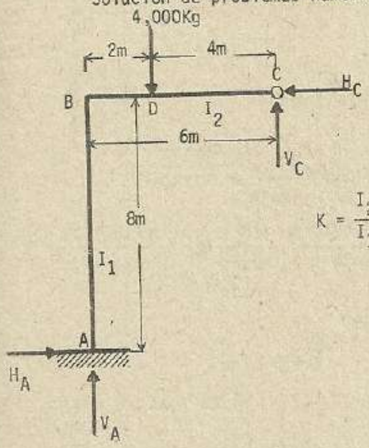
$$M_A = \frac{6E\varepsilon t I_1 \left[(3k+2) + kh^2\right]}{(3k+4)h^2 L}$$; Esfuerzo de flexión en la viga :

$$M_x = -V_C x$$

Momento de flexión en la columna : $M_y = M_B + H_C y$

Nota : En caso de descenso de la temperatura, los esfuerzos se anotan con los signos contrarios.

Solución de problemas numéricos:

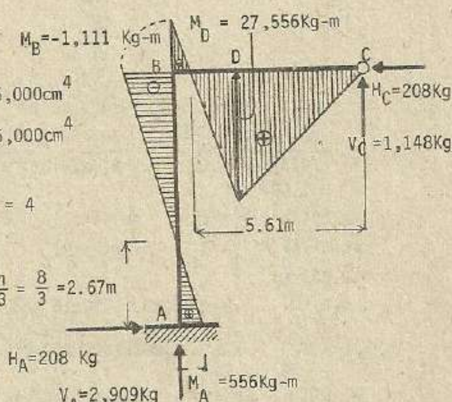


I_2=45,000cm^4

I_1=15,000cm^4

$$K = \frac{I_2 h}{I_1 L} = \frac{45,000 \times 8}{15,000 \times 6} = 4$$

$$\frac{h}{3} = \frac{8}{3} = 2.67\text{m}$$



$$H_A = H_C = \frac{3ab(b+L)}{(3k+4)hL^2}\,P = \frac{3\times2\times4\times(4+6)}{(3\times4+4)\times8\times6\times6} \times 4,000 = 208 \text{ Kg.}$$

$$V_C = P\,\frac{a(3L^2k+6aL-2a^2)}{(3k+4)L^3} = 4,000\times\frac{2\ (3\times6\times6\times4+6\times2\times6-2\times2\times2)}{(3\times4+4)\times6\times6\times6} = 1,148 \text{ Kg.}$$

$$V_A = P\,\frac{3kL^2b + 4L^3 - 6a^2L + 2a^3}{(3k+4)L^3}$$

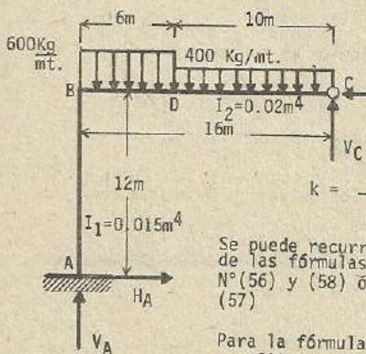
$$= 4{,}000 \times \frac{3x4x6x6x4 + 4x6x6x6 - 6x2x2x6 + 2x2x2x2}{(3x4+4)\,x6x6x6} \doteq 2{,}852 \text{ Kg.}$$

$$M_B = -\frac{2ab(b+L)}{(3k+4)L^2}\cdot P = -\frac{2x2x4\,(4+6)}{(3x4+4)x6x6} \times 4{,}000 \doteq -1{,}111 \text{ Kg-m}$$

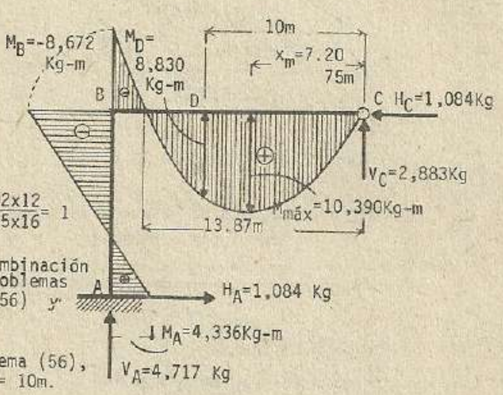
$$M_A = \frac{ab(b+L)}{(3k+4)L^2}\,xP = \frac{2x4x(4+6)}{(3x4+4)x6x6} \times 4{,}000 \doteq 556 \text{ Kg-m}$$

$$M_D = M_{máx.} = \frac{Pab\,(3L^2k + 6aL - 2a^2)}{(3k+4)L^2}$$

$$= \frac{4{,}000x2x4\,(3x6x6x4 + 6x2x6 - 2x2x2)}{(3x4+4)\,x6\,x6} \doteq 27{,}556 \text{ Kg-m}$$



$$k = \frac{I_2h}{I_1L} = \frac{0.02x12}{0.015x16} = 1$$



Se puede recurrir a la combinación de las fórmulas de los problemas N°(56) y (58) ó también (56) y (57)

Para la fórmula del problema (56), w_1 = 200 Kg/m; b =6m, y c = 10m.

Para (58), w_2=400 Kg/m.

$$H_A = H_C = \frac{3w_1b^2(4Lc + b^2)}{4(3k+4)L^2h} + \frac{3w_2L^2}{4(3k+4)h} = \frac{3x200x6^2x(4x16x10 + 6^2)}{4x(3x1+4)x16^2x12} + \frac{3x400x16^2}{4(3x1+4)x12}$$

$\doteq$ 1,084 Kg.

$$V_C = \frac{w_1b^2(3b^2+4bc+3kL^2)}{2(3k+4)L^3} + \frac{3w_2L(k+1)}{2(3k+4)} = \frac{200x6^2x(3x6^2+4x6x10+3x1x16^2)}{2(3x1+4)x16^3} + \frac{3x400x16(1+1)}{2(3x1+4)} =$$

$\doteq$ 2,883 Kg.

$V_A = w_1b + w_2L - V_C$ = 200x6 + 400x16 - 2,883 = 4,717 Kg.

$$M_B = V_CL - w_1\frac{b^2}{2} - w_2\frac{L^2}{2} = 2{,}883x16 - 200x\frac{6x6}{2} - 400x\frac{16x16}{2} = -8{,}672 \text{ Kg-m}$$

$M_A = M_B + H_Ch$ = -8,672 + 1,084x12 = 4,336 Kg-m

Fuerza cortante en el miembro (BC) ; $V = V_C - wx$

Si hacemos como (x_m) la distancia desde (C) al punto donde la cortante es cero (0).

$V_C - wx_m = 0 \longrightarrow 2{,}883 - 400x_m = 0$

x_m = 7,2075 m

Luego, $M_{máx.} = V_C x_m - w \dfrac{x_m^2}{2} = 2{,}883 \times 7.2075 - 400 \cdot \dfrac{7.2075^2}{2} \doteq 10{,}390$ Kg -m

$M_D = V_C \times 10 - w_2 \dfrac{10^2}{2} = 2{,}883 \times 10 - 400 \times \dfrac{10^2}{2} \doteq 8{,}830$ Kg -m

La distancia aproximada (x_0) desde (C) hasta el punto donde el esfuerzo de momento es cero (0) :

$$M_x = V_C x - w_1 \frac{(x-10)^2}{2} - w_2 \cdot \frac{x^2}{2}$$

Si $x_0 > 10$m.

$$V_C x_0 - w_1 \frac{(x_0 - 10)^2}{2} - w_2 \cdot \frac{x_0^2}{2} = 0$$

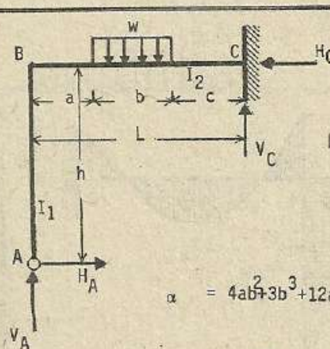
$$2{,}883 x_0 - 200 \times \frac{(x_0 - 10)^2}{2} - 400 \cdot \frac{x_0^2}{2} = 0$$

$$300 x_0^2 - 4{,}883 x_0 + 10{,}000 = 0$$

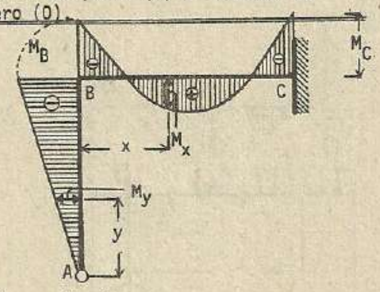
donde : $x_0 = 2.6$.

$x_0 = 13.87$ Luego, a 13.87 m del punto (C) el momento de flexión es cero (0).

(72)



$$k = \frac{I_2 h}{I_1 L}$$



$$\alpha = 4ab^3 + 3b^3 + 12abc + 12b^2c + 12a^2c + 18bc^2 + 8c^3$$

$$H_A = H_C = \frac{wb}{4(4k+3)L^2h}\left[3\alpha - 8L\,(b^2 + 3c^2 + 3bc)\right]$$

$$V_A = \frac{wb}{2(4k+3)L^3}\left[(k+3)\;\alpha - 6L\,(b^2 + 3c^2 + 3bc)\right] \quad ; \quad M_C = -M_B + V_A L - \frac{wb}{2}(b+2c)$$

$$V_C = wb - V_A \quad ; \quad M_B = -\frac{wb}{4\,(4k+3)L^2h}\left[3\,\alpha - 8L\,(b^2 + 3c^2 + 3bc)\right]$$

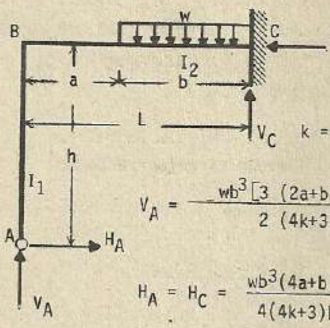
Esfuerzo de flexión en la columna : $M_y = -H_A y$

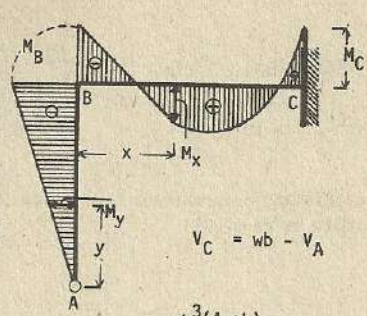
Esfuerzo de flexión en la viga : $M_x = -H_A h + V_A x$ $\quad x \leq a$

$M_x = -H_A h + V_A x - \dfrac{w\,(x-a)^2}{2}$ $\quad a \leq x \leq a+b$

$M_x = -H_A h + V_A x - wb\,(x - a - \dfrac{b}{2})$ $\quad x \geq a + b$

(73)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = \frac{wb^3[3\ (2a+b) + (4a+3b)\ k]}{2\ (4k+3)\ L^3}$$

$$V_C = wb - V_A$$

$$H_A = H_C = \frac{wb^3(4a+b)}{4(4k+3)L^2h}$$

$$M_B = -\frac{wb^3(4a+b)}{4\ (4K+3)\ L^2}$$

$$M_C = -H_Ch + V_CL - \frac{wb^2}{2}$$

Esfuerzo de flexión en la viga : $M_x = -H_Ch + V_Cx \qquad x \leq a$

$$M_x = -H_Ch + V_Cx - \frac{w\ (x-a)^2}{2} \qquad x \geq a$$

Esfuerzo de flexión en la columna : $M_y = -H_Cy$

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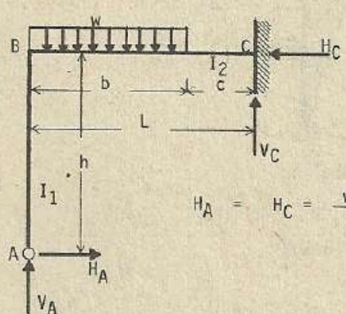
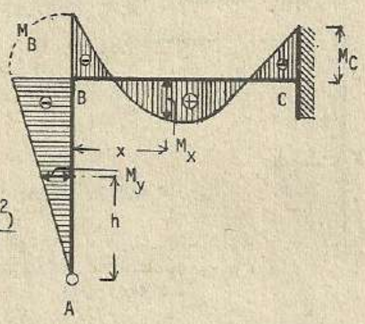
(74)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wb^2(b^2+4bc+6c^2)}{4\ (4k+3)L^2h}$$

$$V_A = \frac{wb}{2\ (4k+3)\ L^3}\left[3L(b^2+3bc+2c^2) + Lk(3b^2+9bc+8c^2) + bc^2k\right]$$

$V_C = wb - V_A$; $M_B = -\dfrac{wb^2(b^2+4bc+6c^2)}{4\ (4k+3)\ L^2}$; $M_C = -H_Ch + V_CL - \dfrac{wb^2}{2}$

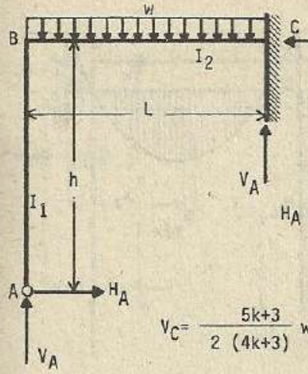
Esfuerzo de flexión en la columna : $M_x = -H_Ch + V_Cx - \dfrac{wx^2}{2} \qquad x \leq b$

$$M_x = -H_Ch + V_Cx - \frac{wb}{2}\ (2x - b) \qquad x \geq b$$

Esfuerzo de flexión en la columna :

$$M_y = -H_Cy$$

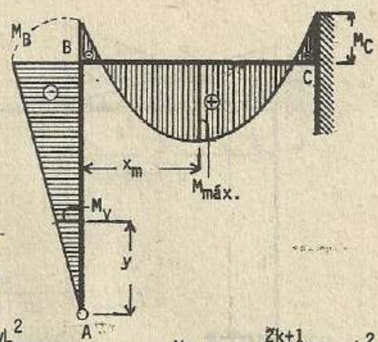
(75)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{wL^2}{4\,(4k+3)\,h}$$

$$V_A = \frac{3\,(k+1)}{2(4k+2)} wL$$



$$V_C = \frac{5k+3}{2\,(4k+3)}\, wL \;; \qquad M_B = -\frac{wL^2}{4\,(4k+3)} \qquad M_C = -\frac{2k+1}{4\,(4k+3)}\, wL^2$$

Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga :

$$M_x = -H_A h + V_A x - \frac{wx^2}{2} \;; \quad x_m = \frac{3(k+1)L}{2(4k+3)} \;; \quad M_{máx.} = \frac{(3+10k+9k^2)}{8(4k+3)^2}\, wL^2$$

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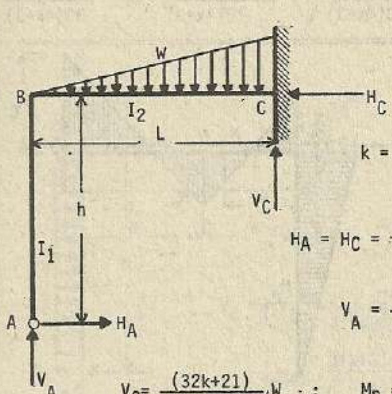
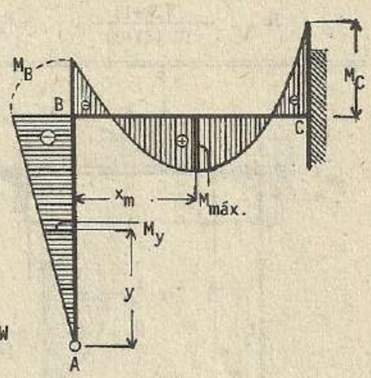
(76)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{WL}{5(4k+3)h}$$

$$V_A = \frac{(8k+9)}{10(4k+3)} \cdot W$$



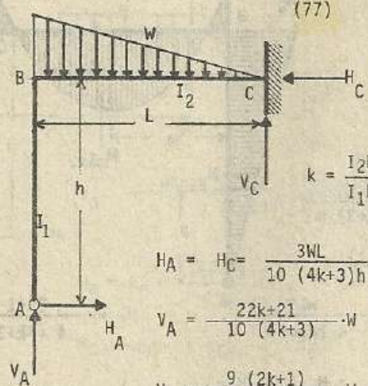
$$V_C = \frac{(32k+21)}{10(4k+3)} \cdot W \;; \qquad M_B = -\frac{WL}{5\,(4k+3)} \;; \qquad M_C = -\frac{(16k+9)}{30\,(4k+3)} \cdot WL$$

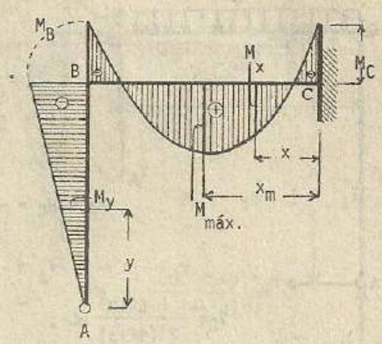
Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga : $M_x = -H_A h + V_A x - \dfrac{Wx^3}{3L^2}$

$$x_m = L \cdot \sqrt{\frac{8k+9}{10(4k+3)}} \;; \qquad M_{máx} = \frac{WL}{15}\left[\frac{(8k+9)}{(4k+3)} \cdot \sqrt{\frac{(8k+9)}{10(4k+3)}} - \frac{3}{(4k+3)}\right]$$

(77)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{3WL}{10\ (4k+3)h}$$

$$V_A = \frac{22k+21}{10\ (4k+3)} \cdot W$$

$$V_C = \frac{9\ (2k+1)}{10\ (4k+3)} \cdot W \qquad M_B = -\frac{3WL}{10\ (4k+3)}$$

$$M_C = -\frac{(7k+3)}{15\ (4k+3)}\ WL$$

Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga : $M_x = -M_C + V_C x - \frac{Wx^3}{3L^2}$

$$x_m = 3L\sqrt{\frac{(2k+1)}{10\ (4k+3)}}\ ; \qquad M_{máx.} = 2WL\left[\frac{9(2k+1)}{10(4k+3)}\sqrt{\frac{(2k+1)}{10(4k+3)}} - \frac{7k+3}{30(4k+3)}\right]$$

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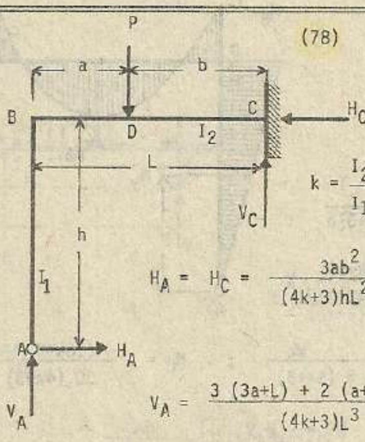
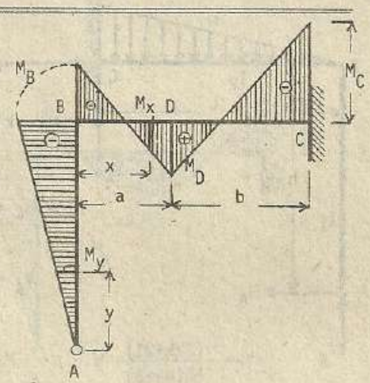
(78)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{3ab^2}{(4k+3)hL^2} \cdot P$$

$$V_A = \frac{3\ (3a+L) + 2\ (a+2L)k}{(4k+3)L^3}\ Pb^2 \qquad V_C = P - V_A$$

$$M_B = -\frac{3ab^2}{(4k+3)L^2}\ P\ ; \qquad M_C = -\frac{3a + 2\ (a+L)k}{(4k+3)L^2}\ Pab$$

$$M_D = \frac{6a + 2\ (a+2L)\ k}{(4k+3)L^3} \cdot Pab^2$$

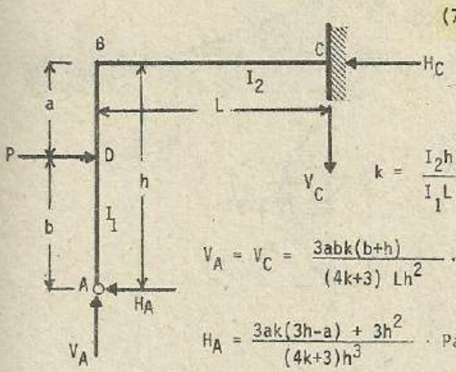
Esfuerzo de flexión en la columna : $M_y = -H_A y$

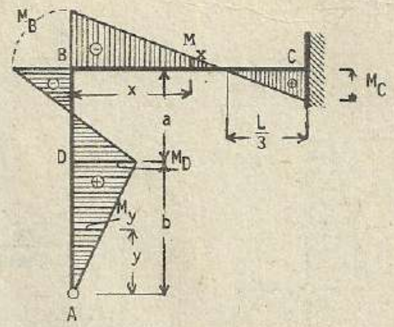
Esfuerzo de flexión en la viga :

$$M_x = -H_A h + V_A x \qquad x \leq a$$

$$M_x = -H_A h + V_A x - P\ (x-a) \qquad x \geq a$$

(79)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{3abk(b+h)}{(4k+3)\ Lh^2} \cdot P$$

$$H_A = \frac{3ak(3h-a) + 3h^2}{(4k+3)h^3} \cdot Pa$$

$$H_C = P - H_A$$

$$M_C = \frac{abk\ (b+h)}{(4k+3)h^2} \cdot P \quad ; \quad M_B = \frac{2abk\ (b+h)}{(4k+3)\ h^2} \cdot P \ ; \quad M_D = \frac{2ak(3h-a) + 3h^2}{(4k+3)h^3} \cdot Pab$$

Esfuerzo de flexión en la columna : $M_y = H_A y \qquad y \leqslant b$

$M_y = H_A y - P\ (y-b) \qquad y \geqslant b$

Esfuerzo de flexión en la viga : $M_x = H_A h - Pa + V_A x$

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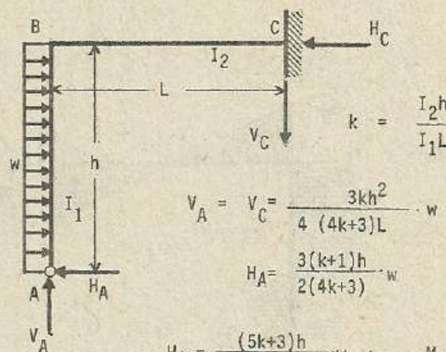
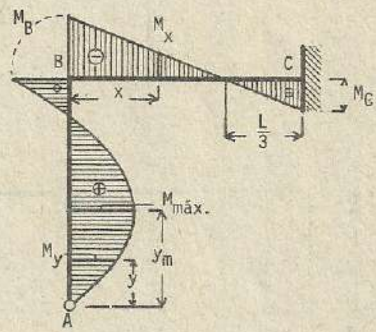
(80)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{3kh^2}{4\ (4k+3)L} \cdot w$$

$$H_A = \frac{3(k+1)h}{2(4k+3)} \cdot w$$

$$H_C = \frac{(5k+3)h}{2\ (4k+3)} \cdot w \quad ; \quad M_B = -\frac{k}{2\ (4k+3)} \cdot wh^2 \ ; \quad M_C = \frac{k}{4\ (4k+3)} \cdot wh^2$$

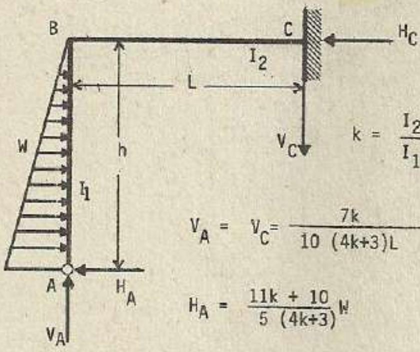
Esfuerzo de flexión en la columna :

$$M_y = H_A y - \frac{wy^2}{2} \quad ; \quad y_m = \frac{3\ (k+1)}{2\ (4k+3)} \cdot h \ ; \quad M_{máx.} = \frac{9\ (k+1)^2}{8\ (4k+3)^2} \cdot wh^2$$

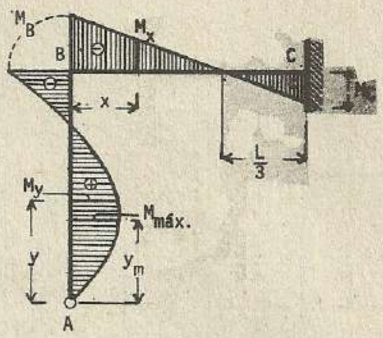
Esfuerzo de flexión en la viga :

$$M_x = H_A h - \frac{wh^2}{2} + V_A x$$

(81)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = V_C = \frac{7k}{10\ (4k+3)L}\ Wh$$

$$H_A = \frac{11k + 10}{5\ (4k+3)}\ W$$

$$H_C = \frac{9k+3}{5\ (4k+3)}\ W \qquad ; \qquad M_B = -\frac{7k}{15\ (4k+3)}\ Wh$$

$$M_C = \frac{7k}{30\ (4k+3)}\ Wh$$

Esfuerzo de flexión en la columna : $M_y = H_A y - \frac{Wy^2}{3h^2} \cdot (3h-y)$

Esfuerzo de flexión en la viga : $M_x = V_A x + H_A h - \frac{2Wh}{3}$

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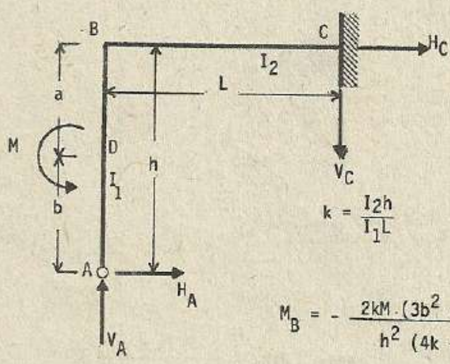
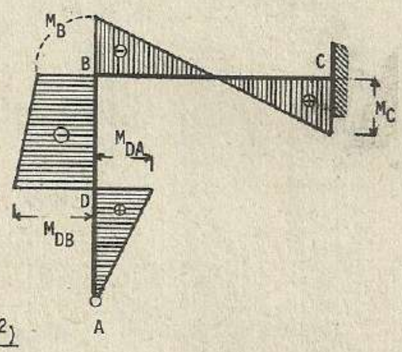
(82)



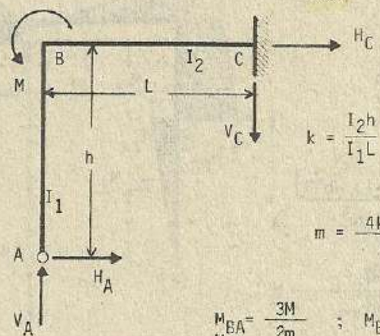
$$k = \frac{I_2 h}{I_1 L}$$



$$M_B = -\frac{2kM \cdot (3b^2 - h^2)}{h^2\ (4k + 3)}$$

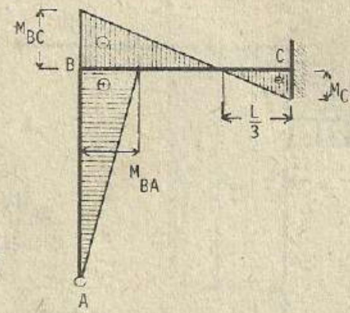
$$M_C = \frac{kM\ (3b^2 - h^2)}{h^2\ (4k+3)}$$

(83)



$$k = \frac{I_2 h}{I_1 L}$$

$$m = \frac{4k+3}{2}$$



$$M_{BA} = \frac{3M}{2m} \quad ; \quad M_{BC} = -\frac{2kM}{m}$$

$$M_C = -\frac{M_{BC}}{2} \quad ; \quad H_A = H_C = \frac{M_{BA}}{h} \quad ; \quad V_A = V_C = \frac{3M_C}{L}$$

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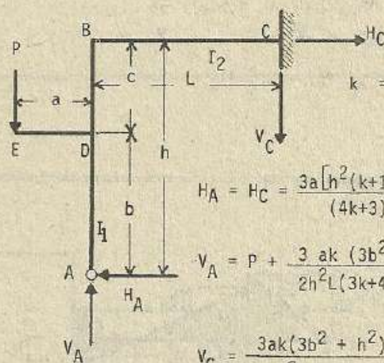
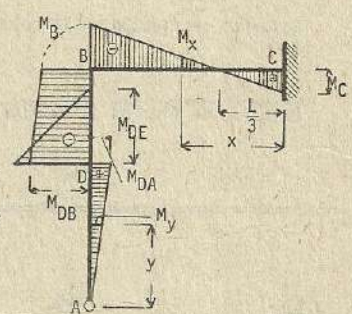
(84)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_C = \frac{3a\left[h^2(k+1) - b^2 k\right]}{(4k+3)\,h^3} \cdot P$$

$$V_A = P + \frac{3\,ak\,(3b^2+h^2)}{2h^2L(3k+4)} \cdot P$$

$$V_C = \frac{3ak(3b^2+h^2)}{2h^2L\,(4k+3)} \cdot P \quad ; \quad M_B = -\frac{ak\,(3b^2+h^2)}{(4k+3)\,h^2} \cdot P$$

$$M_C = \frac{ak\,(3b^2+h^2)}{2(4k+3)h^2} \cdot P \quad ; \quad M_{DB} = \frac{3b\left[h^2(k+1)-b^2k\right]-(4k+3)h^3}{(4k+3)h^3}\,Pa; \quad M_{DE} = -\,Pa$$

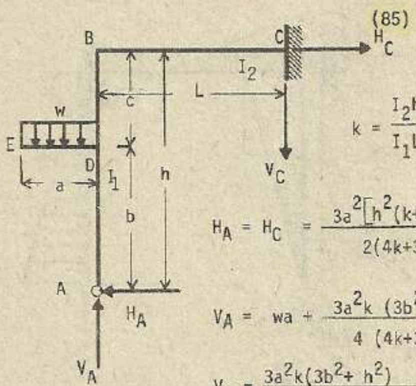
$$M_{DA} = \frac{3ab\left[h^2(k+1) - b^2k\right]}{(4k+3)h^3} \cdot P$$

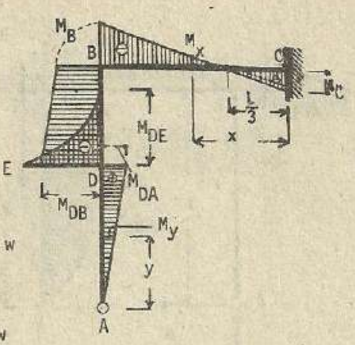
Esfuerzo de flexión en la columna : $M_y = H_A y$ $\quad y \leq b$

$M_y = H_A y - Pa$ $\quad y \geq b$

Esfuerzo de flexión en la viga : $M_x = -V_C\left(x - \frac{L}{3}\right)$

(85)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{3a^2\left[h^2(k+1) - b^2k\right]}{2(4k+3)h^3}\cdot w$$

$$V_A = wa + \frac{3a^2k\,(3b^2+h^2)}{4\,(4k+3)h^2L}\,w$$

$$V_C = \frac{3a^2k(3b^2+h^2)}{4\,(4k+3)h^2L}\cdot w \quad ; \quad M_{DE} = -\frac{wa^2}{2}$$

$$M_{DB} = \frac{3b\left[h^2(k+1) - b^2k\right] - (4k+3)h^3}{2\,(4k+3)h^3}\,wa^2 \quad : \quad M_{DA} = \frac{3a^2b\left[h^2(k+1) - b^2k\right]}{4\,(4k+3)h^3}\cdot w$$

$$M_B = -\frac{a^2k\,(3b^2+h^2)}{2\,(4k+3)h^2}\cdot w \quad ; \quad M_C = -\frac{a^2k\,(3b^2+h^2)}{4\,(4k+3)h^2}\,w$$

Esfuerzo de flexión en la columna : $M_y = H_A y$ $\quad y \leq b$

$M_y = H_A y - \frac{wa^2}{2}$ $\quad y \geq b$

Esfuerzo de flexión en la viga : $M_x = -V_C\,(x - \frac{L}{3})$

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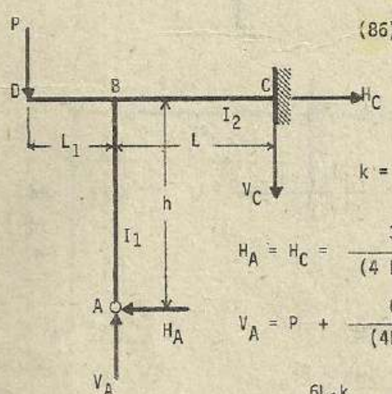
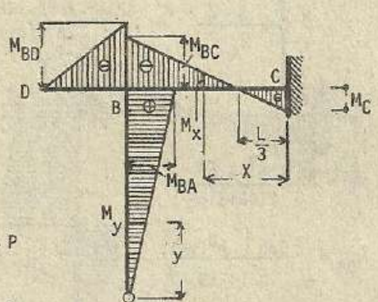
(86)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{3L_1}{(4k+3)\,h}\cdot P$$

$$V_A = P + \frac{6L_1k}{(4k+3)\,L}\,P$$

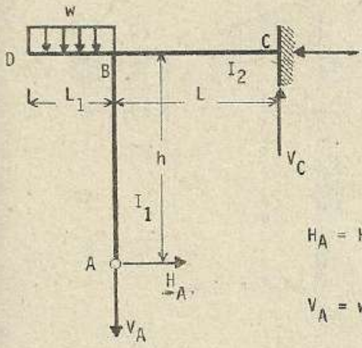
$$V_C = \frac{6L_1k}{(4k+3)L}\cdot P \quad ; \quad M_{BD} = -PL_1 \quad ; \quad M_{BA} = \frac{3L_1}{(4k+3)}\cdot P$$

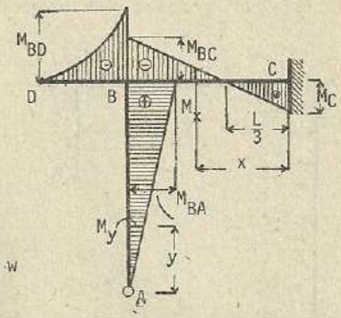
$$M_{BC} = -\frac{4L_1k}{(4k+3)}\cdot P \quad ; \quad M_C = \frac{2L_1k}{(4k+3)}\cdot P$$

Esfuerzo de flexión en la columna : $M_y = H_A y$

Esfuerzo de flexión en la viga : $M_x = -V_C\,(x - \frac{L}{3})$

(87)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{3L_1^2}{2\,(4k+3)\,h}\,w$$

$$V_A = wL_1 + \frac{3L_1^2 k}{(4k+3)L}\,w$$

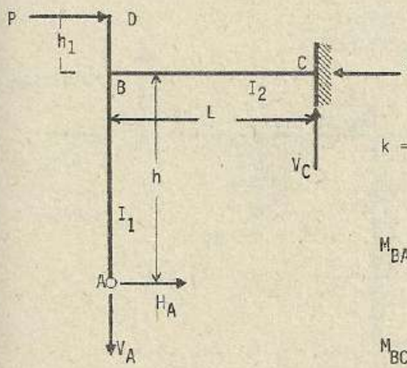
$$V_C = \frac{3L_1^2 k}{(4k+3)L}\,w \;; \qquad M_{BD} = \frac{wL_1^2}{2} \;; \qquad M_{BA} = \frac{3L_1^2}{2\,(4k+3)}\,w$$

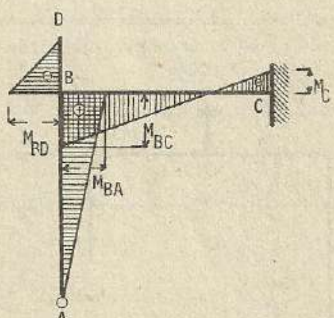
$$M_{BC} = -\frac{2L_1^2 k}{(4k+3)}\,w \;; \qquad M_C = \frac{L_1^2 k}{(4k+3)}\,w$$

Esfuerzo de flexión en la columna : $M_y = H_A y$

Esfuerzo de flexión en la viga : $M_x = -V_C\,(x - \frac{L}{3})$

(88)





$$k = \frac{I_2 h}{I_1 L}$$

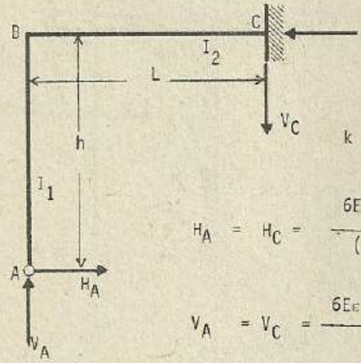
$$M_{BA} = -\frac{Ph_1}{4k+3}$$

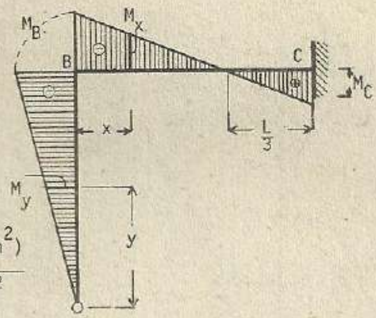
$$M_{BC} = \frac{Ph_1(2k+1)}{4\,k+3}$$

$$M_{BD} = -Ph_1 \;; \qquad M_C = -\frac{Ph_1(2k+1)}{4k+3}$$

Esfuerzos por cambio de temperatura

(89)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_C = \frac{6E\varepsilon t I_2\,(2L^2 + 3h^2)}{(4k+3)\,h^2\,L^2}$$

$$V_A = V_C = \frac{6E\varepsilon t I_2\left[2\,(k+3)\,h^2 + 3L^2\right]}{(4k+3)\,hL^3}$$

$$M_B = -\frac{6E\varepsilon t I_2\,(2L^2 + 3h^2)}{(4k+3)\,hL^2} \quad ; \quad M_C = \frac{6E\varepsilon t I_2\left[(2k+3)\,h^2 + L^2\right]}{(4k+3)hL^2}$$

Esfuerzo de flexión en la columna : $M_y = -H_A y$

Esfuerzo de flexión en la viga : $M_x = M_B + V_A x$

Nota : En caso de descenso de la temperatura, los esfuerzos se anotan con los signos contrarios

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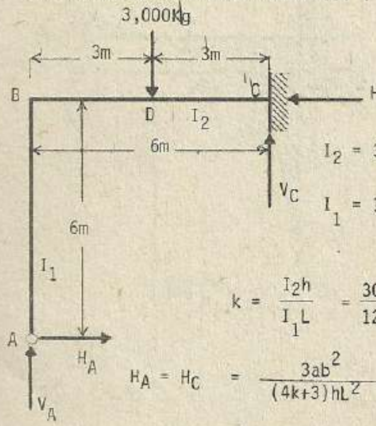
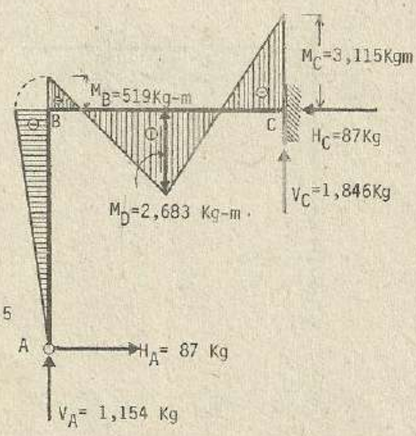
Solución de problemas numéricos :



$I_2 = 30,000\ cm^4$

$I_1 = 12,000\ cm^4$



$$k = \frac{I_2 h}{I_1 L} = \frac{30,000 \times 6}{12,000 \times 6} = 2.5$$

$$H_A = H_C = \frac{3ab^2}{(4k+3)hL^2}\,P = \frac{3 \times 3 \times 3 \times 3}{(4 \times 2.5+3)\ \times 6 \times 6 \times 6} \times 3,000$$

$$= 87\ Kg.$$

$$V_A = \frac{3\,(3a+L) + 2\,(a+2L)\,k}{(4k+3)\,L^3}\,.\,Pb^2 = \frac{3\,(3 \times 3+6) + 2\,(3+2 \times 6) \times 2.5}{(4 \times 2.5+3) \times 6 \times 6 \times 6} \times 3,000 \times 3 \times 3$$

$$= 1,154\ Kg \quad ; \quad V_C = P - V_A = 3,000 - 1,154 = 1,846\ Kg.$$

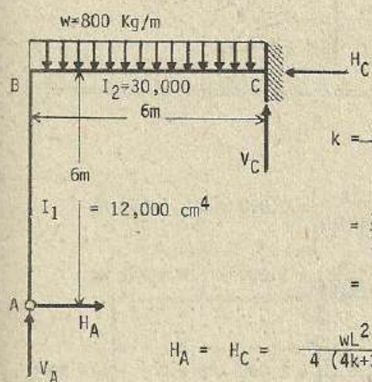
$$M_B = -\frac{3ab^2}{(4k+3)L^2}\cdot P = -\frac{3x3x3x3}{(4x2.5+3)x6x6} \times 3{,}000 = -519 \ \text{Kg-m}$$

$$M_C = -\frac{3a+2(a+L)k}{(4k+3)L^2} \times Pab = -\frac{3x3+2(3+6) \times 2.5}{(4x2.5+3)\ x6\ x6} \times 3{,}000 \times 3 \times 3$$

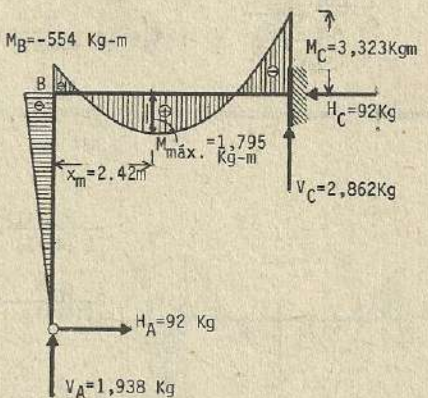
$$= -3{,}115 \ \text{Kg-m}$$

$$M_D = M_{máx.} = \frac{6a+2(a+2L)k}{(4k+3)L^3}\cdot Pab^2 = \frac{6 \times 3 + 2(3+2x6)\ 2.5}{(4x2.5+3)x6x6x6} \times 3{,}000x3x3x3$$

$$\doteqdot 2{,}683 \ \text{Kg-m}$$



$$k = \frac{I_2h}{I_1L} = \frac{30{,}000x6}{12{,}000x6} = 2.5$$



$$H_A = H_C = \frac{wL^2}{4\ (4k+3)\ k} = \frac{800x6x6}{4\ (4x2.5+3)x6} \doteqdot 92 \ \text{Kg}$$

$$V_A = \frac{3\ (k+1)}{2\ (4k+3)}\cdot wL = \frac{3\ (2.5+1)}{2\ (4x2.5+3)} \times 800 \times 6 \doteqdot 1{,}938 \ \text{Kg}$$

$$V_C = \frac{5k+3}{2\ (4k+3)}\ wL = \frac{5 \times 2.5 + 3}{2(4x2{,}5+3)} \times 800 \times 6 = 2{,}862 \ \text{Kg}$$

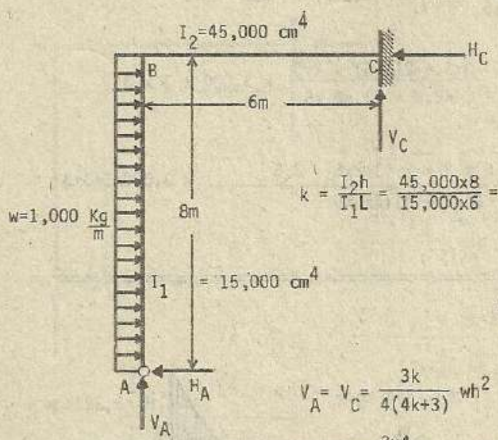
$$M_B = -\frac{wL^2}{4\ (4k+3)} = -\frac{800\ x6 \times 6}{4\ (4x2.5+3)} \doteqdot -554 \ \text{Kg-m} \ ; \quad M_C = -\frac{(2k+1)}{4(4k+3)}\ wL^2$$

$$= -\frac{2x2.5+1}{4(4x2.5+3)} x800x6x6$$

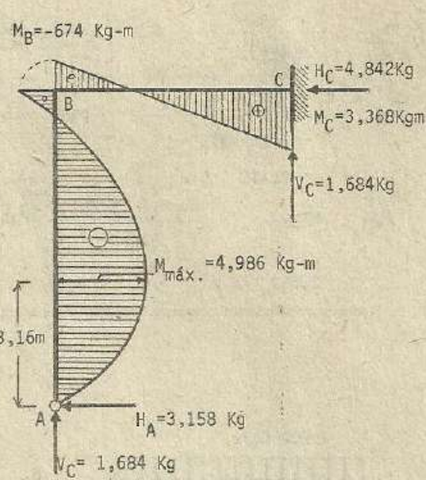
$$\doteqdot -3{,}323 \ \text{Kg-m}$$

$$M_{máx.} = \frac{3+10k+9K^2}{8(4k+3)^2}\cdot wL^2 = \frac{3 + 10 \times 2.5 + 9x2.5x2.5}{8\ (4x2.5+3)^2} \times 800 \times 6 \times 6 = 1{,}795 \ \text{Kg-m}$$

$$x_m = \frac{3(k+1)L}{2(4k+3)} = \frac{3\ (2.5+1)x6}{2\ (4x2.5+3)} = 2.42 \ \text{m.}$$



$$k = \frac{I_2 h}{I_1 L} = \frac{45{,}000 \times 8}{15{,}000 \times 6} = 4$$



$$V_A = V_C = \frac{3k}{4(4k+3)}\, wh^2$$

$$= \frac{3 \times 4}{4(4 \times 4+3) \times 6} \times 1000 \times 8 \times 8$$

$$= 1{,}684 \text{ Kg}$$

$$H_A = \frac{3\,(k+1)}{2\,(4k+3)}\, wh = \frac{3\,(4+1)}{2\,(4 \times 4+3)} \times 1{,}000 \times 8 = 3{,}158 \text{ Kg.}$$

$$H_C = \frac{5k+3}{2(4k+3)}\, wh = \frac{5 \times 4+3}{2\,(4 \times 4+3)} \times 1{,}000 \times 8 = 4{,}842 \text{ Kg.}$$

$$M_B = -\frac{k}{2\,(4k+3)}\, wh^2 = \frac{4}{2\,(4 \times 4+3)} \times 1{,}000 \times 8 \times 8 = -674 \text{ Kg-m}$$

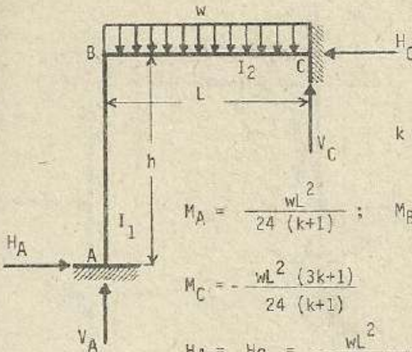
$$M_C = \frac{k}{4\,(4k+3)}\, wh^2 = \frac{4}{4(4 \times 4+3)} \times 1{,}000 \times 8 \times 8 = 3{,}368 \text{ Kg-m}$$

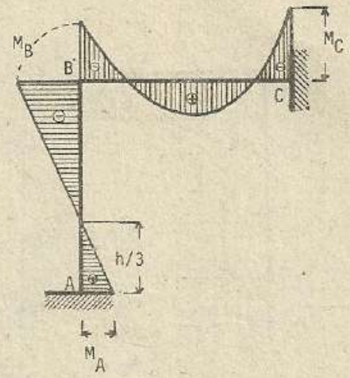
$$M_{máx.} = \frac{9\,(k+1)^2}{8\,(4k+3)^2}\, wh^2 = \frac{9(4+1)^2}{8(4 \times 4+3)^2} \times 1{,}000 \times 8 \times 8 =$$

$$= 4{,}986 \text{ Kg-m}$$

$$y_m = \frac{3\,(k+1)}{2(4k+3)}\, h = \frac{3\,(4+1)}{2\,(3 \times 4+3)} \times 8 = 3.16 \text{ m}$$

(90)





$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = \frac{wL^2}{24\ (k+1)} \quad ; \quad M_B = -\frac{wL^2}{12\ (k+1)}$$

$$M_C = -\frac{wL^2\ (3k+1)}{24\ (k+1)}$$

$$H_A = H_C = \frac{wL^2}{8\ (k+1)\ h}$$

$$V_A = \frac{wL}{2} - \frac{kwL}{8\ (k+1)} \quad ; \quad V_C = \frac{wL}{2} + \frac{kwL}{8\ (k+1)}$$

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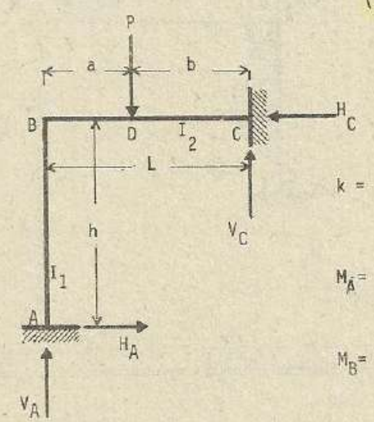
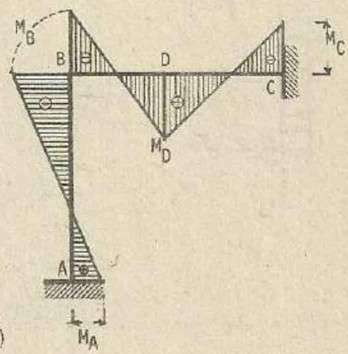
(91)





$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = \frac{Pab^2}{2L^2\ (k+1)}$$

$$M_B = -\frac{Pab^2}{L^2\ (k+1)}$$

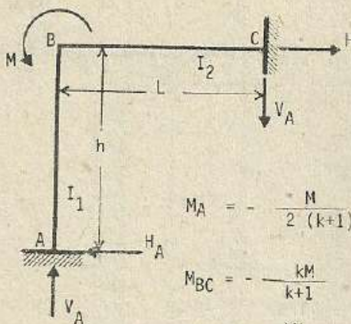
$$M_C = -\frac{Pab\ [\ kL + (k+2)\ a\ ]}{2L^2\ (k+1)}$$

$$V_A = \frac{Pb - M_B + M_C}{L} \quad ; \quad V_C = P - V_A \quad ; \quad H_A = H_C = \frac{3M_A}{h}$$

Si $a = b = \frac{L}{2}$

$$M_A = \frac{PL}{16\ (k+1)} \quad ; \quad M_B = -\frac{PL}{8\ (k+1)} \quad ; \quad M_C = -\frac{PL\ (3k+2)}{16\ (k+1)}$$

(92)

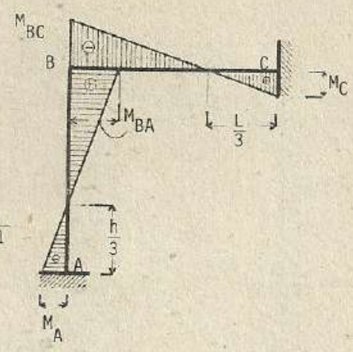


$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{M}{2\,(k+1)} \;;\qquad M_{BA} = \frac{M}{k+1}$$

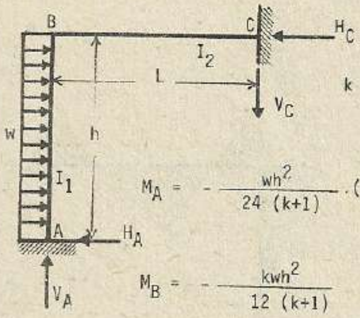
$$M_{BC} = -\frac{kM}{k+1}$$

$$M_C = \frac{kM}{2\,(k+1)}$$



$$V_A = V_C = \frac{3M_C}{L} \;;\qquad H_A = H_C = -\frac{3M_A}{h}$$

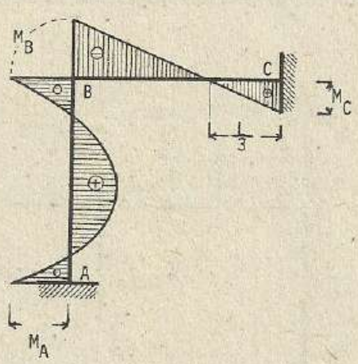
(93)



$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{wh^2}{24\,(k+1)}\cdot(2k+3)$$

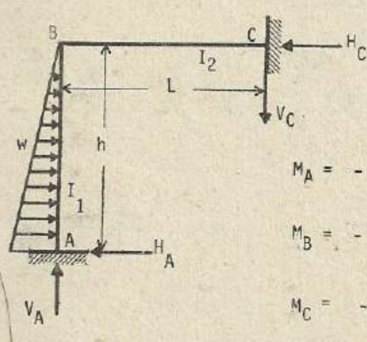
$$M_B = -\frac{kwh^2}{12\,(k+1)}$$



$$M_C = -\frac{M_B}{2} = +\frac{kwh^2}{24\,(k+1)}$$

$$V_A = V_C = \frac{kwh^2}{8L\,(k+1)} \;;\qquad H_A = \frac{wh}{2} + \frac{wh}{8(k+1)} \;;\qquad H_C = \frac{wh}{2} - \frac{wh}{8(k+1)}$$

(94)

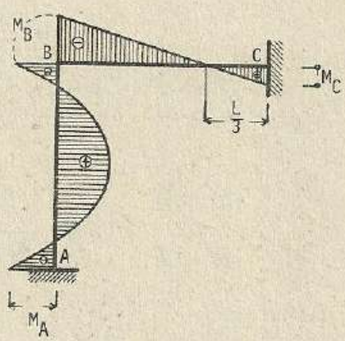


$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{Wh\,(3k+4)}{30\,(k+1)}$$

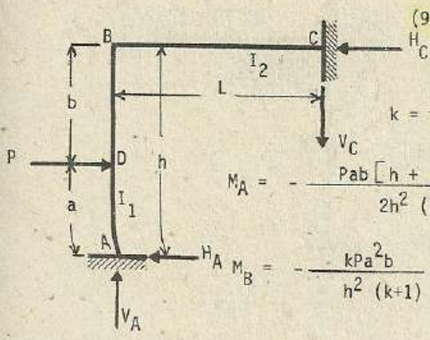
$$M_B = -\frac{kWh}{15\,(k+1)}$$

$$M_C = \frac{kWh}{30\,(k+1)}$$



$$H_A = \frac{1}{h}\cdot\left(W \times \frac{2}{3}\,h + M_B - M_A\right) \;;\qquad H_C = W - H_A \;;\qquad V_A = V_C = \frac{3M_C}{L}$$

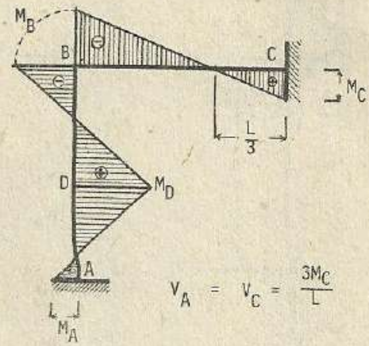
(95)



$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{Pab\left[h + (2k+1)\,b\right]}{2h^2\,(k+1)}$$

$$M_B = -\frac{kPa^2 b}{h^2\,(k+1)}$$



$$V_A = V_C = \frac{3M_C}{L}$$

$$M_C = \frac{kPa^2 b}{2h^2(k+1)} \quad ; \quad H_A = \frac{Pb + M_B - M_A}{h} \quad ; \quad H_C = P - H_A$$

Si $a = b = \frac{h}{2}$ $\quad M_A = -\frac{Ph\,(2k+3)}{16\,(k+1)} \quad ; \quad M_B = -\frac{kPh}{8\,(k+1)} \quad ; \quad M_C = \frac{kPh}{16\,(k+1)}$

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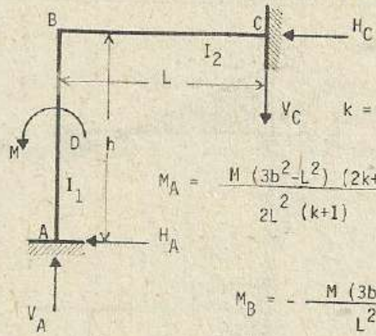
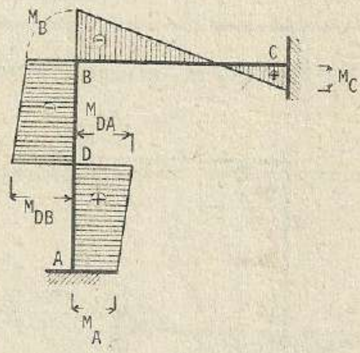
(96)



$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = \frac{M\,(3b^2 - L^2)\,(2k+1)}{2L^2\,(k+1)}$$



$$M_B = -\frac{M\,(3b^2 - L^2)\,k}{L^2\,(k+1)}$$

$$M_C = \frac{M\,(3b^2 - L^2)\,(2k+1)}{6L^2\,(k+1)}$$

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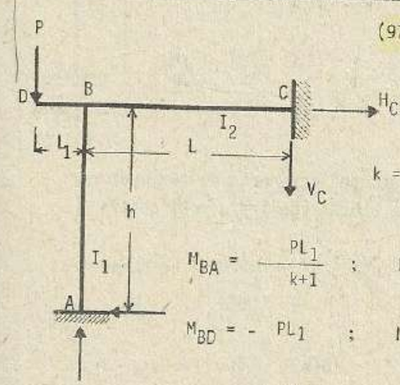
(97)

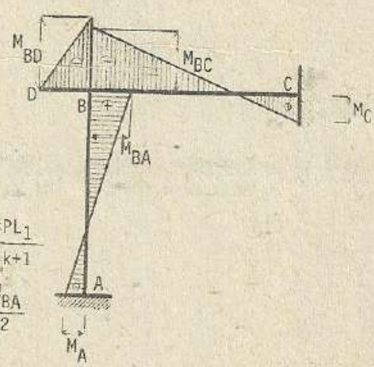


$$k = \frac{I_2 h}{I_1 L}$$

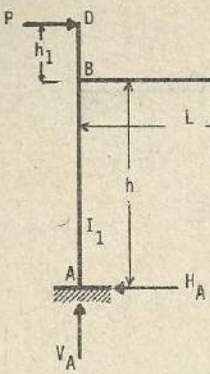
$$M_{BA} = \frac{PL_1}{k+1} \quad ; \quad M_{BC} = -\frac{kPL_1}{k+1}$$

$$M_{BD} = -\,PL_1 \quad ; \quad M_A = -\frac{M_{BA}}{2}$$

$$M_C = -\frac{M_{BC}}{2}$$



(98)



$$k = \frac{I_2 h}{I_1 L}$$

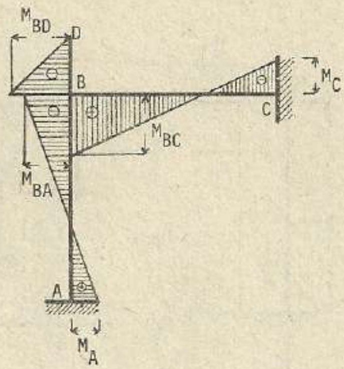
$$M_{BA} = -\frac{Ph_1}{k+1}$$

$$M_{BC} = \frac{kPh_1}{k+1}$$

$$M_{BD} = Ph_1$$

$$M_A = -\frac{M_{BA}}{2}$$

$$M_C = -\frac{M_{BC}}{2}$$



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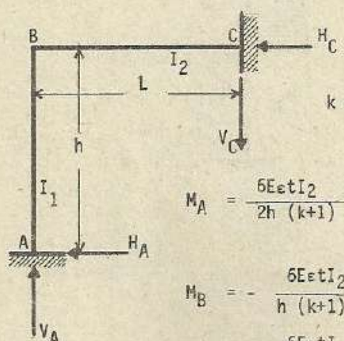
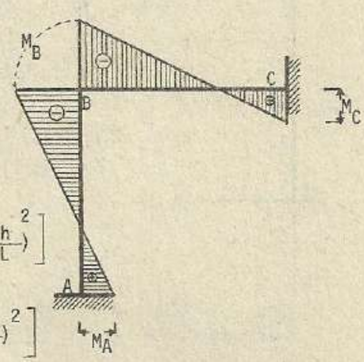
(99) : Esfuerzos por cambio de temperatura



$$k = \frac{I_2 h}{I_1 L}$$



$$M_A = \frac{6E\varepsilon t I_2}{2h\,(k+1)}\left[2 + \frac{1}{k} + \left(\frac{h}{L}\right)^2\right]$$

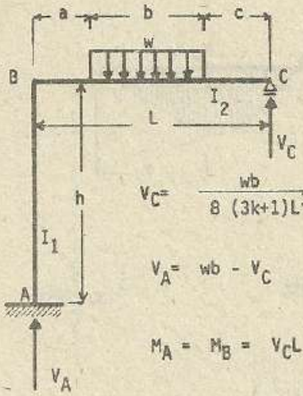
$$M_B = -\frac{6E\varepsilon t I_2}{h\,(k+1)}\left[1 + \left(\frac{h}{L}\right)^2\right]$$

$$M_C = \frac{6E\varepsilon t I_2}{2h(k+1)}\left[1 + (k+2)\cdot\left(\frac{h}{L}\right)^2\right]$$

$$V_A = V_C = \frac{(M_C - M_B)}{L} \quad ; \qquad H_A = H_C = \frac{(M_A - M_B)}{h}$$

Nota : El presente problema corresponde al caso de incremento de temperatura. En caso de descenso, los esfuerzos se anotan con los signos contrarios.

(100)

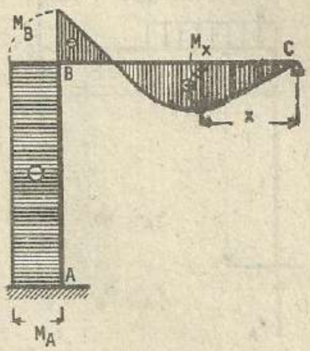


$$k = \frac{I_2 h}{I_1 L}$$

$$V_C = \frac{wb}{8\,(3k+1)L^3}\left[\alpha + 12L^2 k(2a+b)\right]$$

$$V_A = wb - V_C$$

$$M_A = M_B = V_C L - \frac{wb}{2}(2a+b)$$



$$\alpha = 4b^2c + 3b^3 + 12abc + 12ab^2 + 12a^2c + 18a^2b + 8a^3$$

Esfuerzo de flexión en la viga : $M_x = V_C x$ $\qquad x \leq c$

$$M_x = V_C x - \frac{w\,(x-c)^2}{2} \qquad c \leq x \leq b+c$$

$$M_x = V_C x - \frac{wb}{2}(2x-b-2c) \qquad x \geq b+c$$

Esfuerzo de flexión en la columna :

$$M_y = V_C L - \frac{wb}{2}(2a+b)$$

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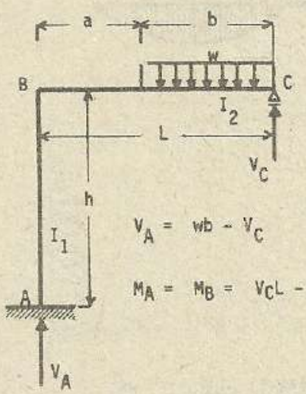
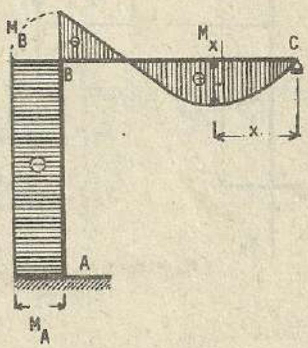
(101)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = wb - V_C$$

$$M_A = M_B = V_C L - \frac{wb}{2}(2a+b)$$



$$V_C = \frac{wb}{8\,(3k+1)L^2}\left[3b^3 + 12ab^2 + 18a^2b + 8a^3 + 12kL^2(2a+b)\right]$$

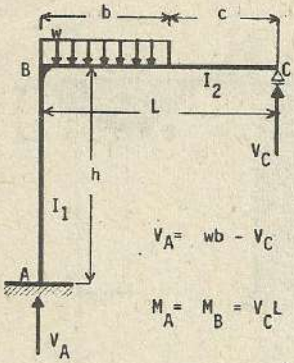
Esfuerzo de flexión en la viga :

$$M_x = V_C x - \frac{wx^2}{2} \qquad x \leq b$$

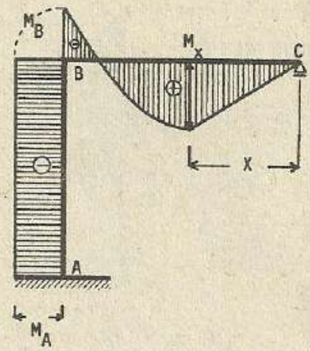
$$M_x = V_C x - \frac{wb}{2}(2x-b) \qquad x \geq b$$

Esfuerzo de flexión en la columna : $M_y = V_C L - \frac{wb}{2}(2a+b)$

(102)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = wb - V_C$$

$$M_A = M_B = V_C L - \frac{wb^2}{2}$$

$$V_C = \frac{wb^2}{8(3k+1)L^2}\left[4\,bc + 3b^2 + 12kL^2\right]$$

Esfuerzo de flexión en la viga : $M_x = V_C x \qquad x \leq c$

$$M_x = V_C x - \frac{w\,(x-c)^2}{2} \qquad x \geq c$$

Esfuerzo de flexión en la columna : $M_y = V_C L - \dfrac{wb^2}{2}$

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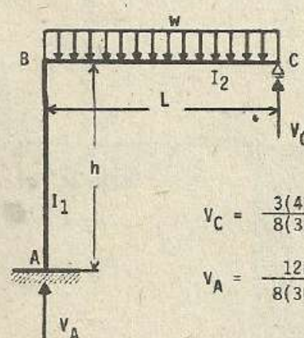
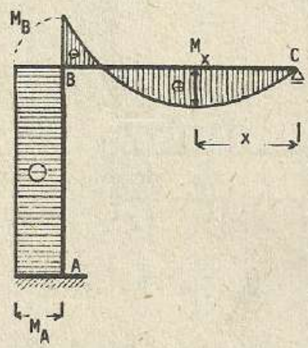
(103)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_C = \frac{3(4k+1)}{8(3k+1)} \cdot wL$$

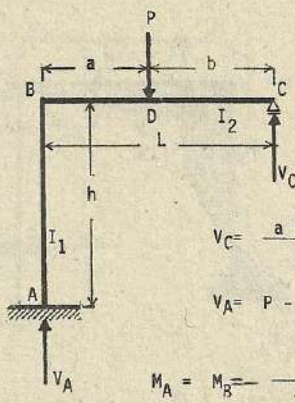
$$V_A = \frac{12k+5}{8(3k+1)} \cdot wL$$

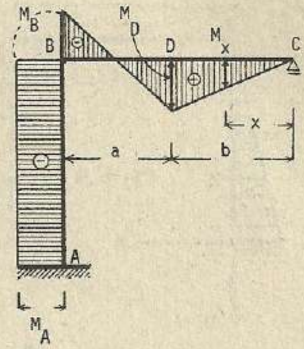
$$M_A = M_B = -\frac{1}{8\,(3k+1)} \cdot wL^2$$

Esfuerzo de flexión en la viga : $M_x = \dfrac{3\,(4k+1)}{8\,(3k+1)} \cdot wLx - \dfrac{wx^2}{2}$

Esfuerzo de flexión en la columna : $M_y = -\dfrac{1}{8\,(3k+1)} \cdot wL^2$

(104)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_C = \frac{a\,(3L-a) + 6kL^2}{2\,(3k+1)L^3} \cdot Pa$$

$$V_A = P - V_C$$

$$M_A = M_B = -\frac{L+b}{2\,(3k+1)\,L^2} \cdot Pab$$

$$M_D = \frac{a\,(3L-a) + 6kL^2}{2\,(3k+1)L^3} \cdot Pab$$

Esfuerzo de flexión en la viga : $M_x = V_C x \qquad x \leq b$

$M_x = V_C x - P\,(x-b) \qquad x > b$

Esfuerzo de flexión en la columna

$$= -\frac{L+b}{2\,(3k+1)L^2} \cdot Pab$$

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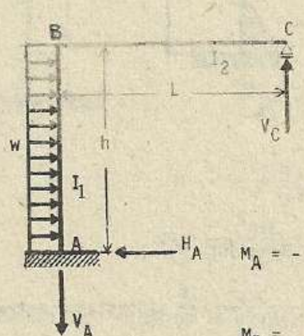
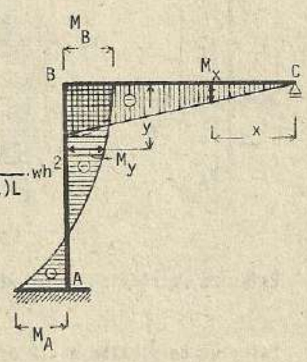
(105)





$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{k}{2(3k+1)L} \cdot wh^2$$

$$H_A = wh$$

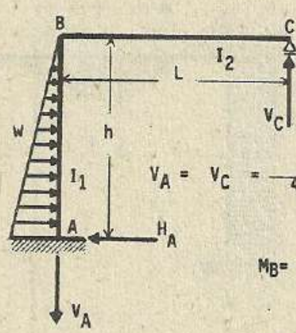
$$M_A = -\frac{2k+1}{2\,(3k+1)} \cdot wh^2$$

$$M_B = \frac{k}{2\,(3k+1)} \cdot wh^2$$

Esfuerzo de flexión en la viga : $M_x = \frac{k}{(3k+1)L} \cdot wh^2 x$

Esfuerzo de flexión en la columna $\frac{\ldots}{2\,(3k+1)} \cdot wh^2 - \frac{wy^2}{2}$

(106)

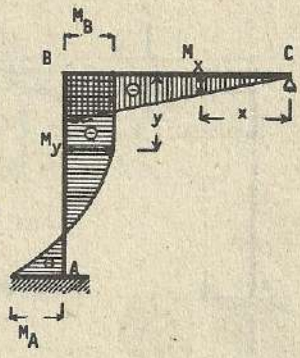


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{k}{4\,(3k+1)L}\,Wh \;;\; H_A = W$$

$$M_B = \frac{k}{4\,(3k+1)}\,Wh$$

$$M_A = -\frac{9k+4}{12\,(3k+1)}\,Wh$$



Esfuerzo de flexión en la viga : $M_x = \frac{k}{4\,(3k+1)L}\,Whx$

Esfuerzo de flexión en la columna : $M_y = \frac{k}{4\,(3k+1)}\,Wh - \frac{Wy^3}{3h^2}$

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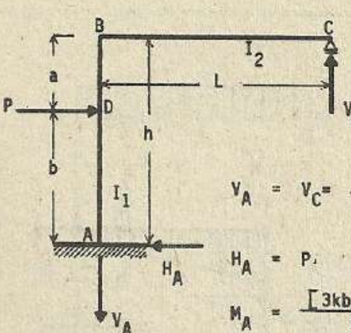
(107)



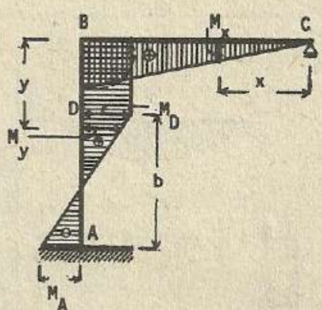
$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_C = \frac{3kb^2}{2\,(3k+1)\,hL}\,P$$

$$H_A = P$$

$$M_A = \frac{[3kb - 2\,(3k+1)\,h]}{2\,(3k+1)\,h}\,Pb$$

$$M_B = M_D = \frac{3kb^2}{2\,(3k+1)} \cdot \frac{P}{h}$$

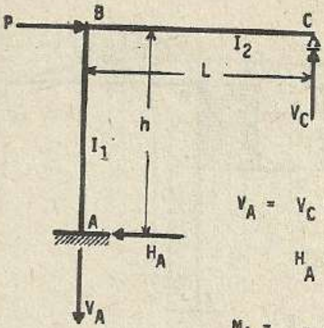


Esfuerzo de flexión en la viga : $M_x = \frac{3kb^2}{2(3k+1)hL} \cdot Px$

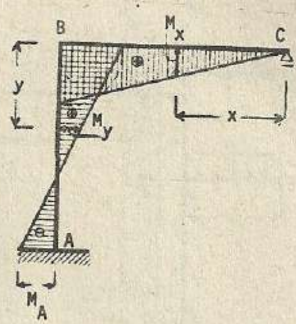
Esfuerzo de flexión en la columna : $M_y = \frac{3kb^2}{2(3k+1)} \cdot \frac{P}{h}$; (Constante entre B-D)

$$M_y = \frac{3kb^2}{2(3k+1)} \cdot \frac{P}{h} - P\,(y-a)\;;\quad y \geq a$$

(108)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = V_C = \frac{3k}{2\,(3k+1)\,L} \cdot Ph$$

$$H_A = P$$

$$M_A = -\frac{3k+2}{2\,(3k+1)} \cdot Ph$$

$$M_B = \frac{3k}{2\,(3k+1)} \cdot Ph$$

Esfuerzo de flexión en la viga : $M_x = \frac{3k}{2(3k+1)L} \cdot Phx$

Esfuerzo de flexión en la columna : $M_y = \frac{3k}{2(3k+1)} \cdot Ph - Py$

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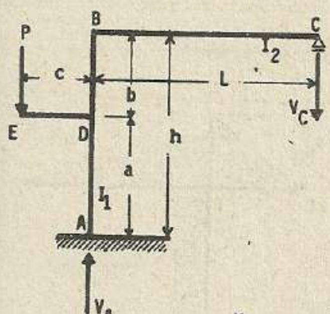
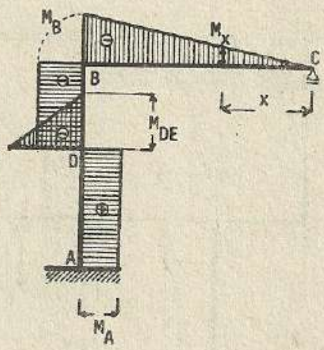
(109)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = P + \frac{3kacP}{hL\,(3k+1)}$$

$$V_C = \frac{3kacP}{hL\,(3k+1)}$$

$$M_A = -\frac{3kacP}{h\,(3k+1)} + Pc$$

$$M_B = -\frac{3kacP}{h\,(3k+1)}$$

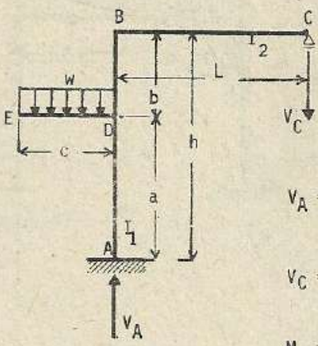
Esfuerzo de flexión en la viga: $M_x = -V_C x$

Esfuerzo de flexión en la columna :

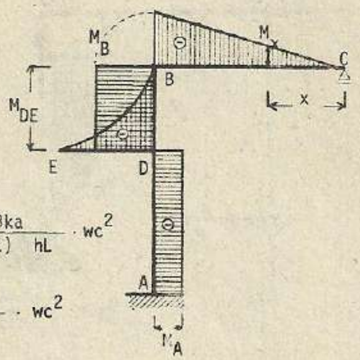
$$M_y = -\frac{3kacP}{h\,(3k+1)} + Pc \qquad (A - D)$$

$$M_y = -\frac{3kacP}{h\,(3k+1)} \qquad (D - B)$$

(110)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = wc + \frac{3ka}{2\,(3k+1)\;\;hL} \cdot wc^2$$

$$V_C = \frac{3ka}{2\,(3k+1)\,hL} \cdot wc^2$$

$$M_A = \frac{h + 3k\,(h-a)}{2\,(3k+1)\,h} \cdot wc^2$$

$$M_{DE} = -\frac{wc^2}{2} \quad ; \quad M_{DB} = -\frac{3ka}{2\,(3k+1)\,h} \cdot wc^2 \quad ; \quad M_{DA} = \frac{h+3k(h-a)}{2\,(3k+1)\,h}\,wc^2$$

$$M_B = -\frac{3ka}{2\,(3k+1)\,h} \cdot wc^2$$

Esfuerzo de flexión en la viga : $M_x = -\frac{3ka}{2\,(3k+1)\,hL} \cdot wc^2 x$

Esfuerzo de flexión en la columna: $M_y = \frac{h + 3k\,(h-a)}{2\,(3k+1)\,h}\,wc^2$ porción (A-D)

$M_y = -\frac{3ka}{2\,(3k+1)\;\;h}\,wc^2$ porción (D-B)

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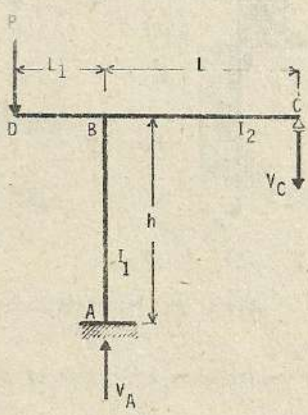
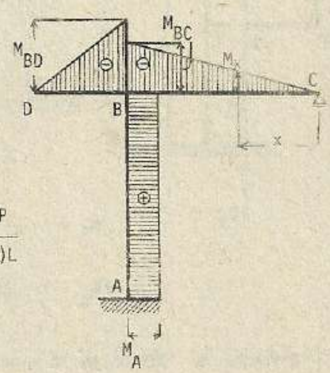
(11)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = P + \frac{3L_1 kP}{(3k+1)L}$$

$$V_C = \frac{3L_1 kP}{(3k+1)\,L}$$

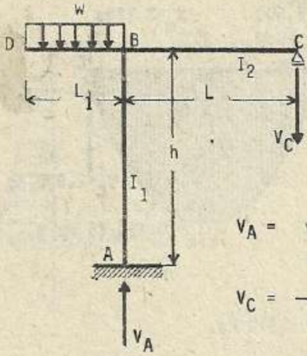
$$M_A = \frac{PL_1}{(3k+1)}$$

$$M_{BD} = -PL_1 \quad ; \quad M_{BA} = \frac{PL_1}{(3k+1)} \quad ; \quad M_{BC} = -\frac{3PL_1 k}{(3k+1)}$$

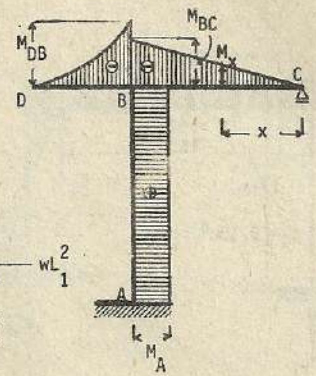
Esfuerzo de flexión en la viga : $M_x = -$ ——— . x

Esfuerzo de flexión en la columna: $M_y =$ (

(112)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = wL_1 + \frac{3k}{2\,(3k+1)\;L}\,wL_1^2$$

$$V_C = \frac{3k}{2\,(3k+1)\;L}\,wL_1^2$$

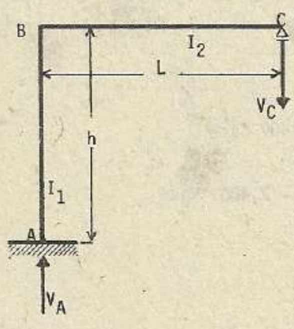
$$M_A = M_{BA} = \frac{wL_1^2}{2\,(3k+1)}$$

$$M_{BD} = -\frac{wL_1^2}{2} \quad ; \quad M_{BC} = -\frac{3k}{2\,(3k+1)}\,wL_1^2$$

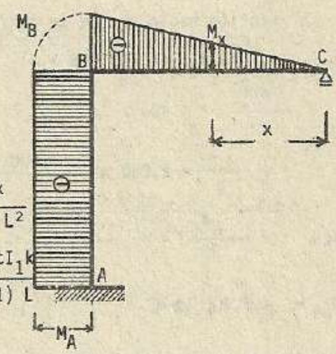
Esfuerzo de flexión en la viga : $M_x = -\dfrac{3k}{2\,(3k+1)\;L}\,wL_1^2 x$

Esfuerzo de flexión en la columna : $M_y = \dfrac{wL_1^2}{2(3k+1)}$

Esfuerzo por cambio de temperatura (113)



$$k = \frac{I_2 h}{I_1 L}$$



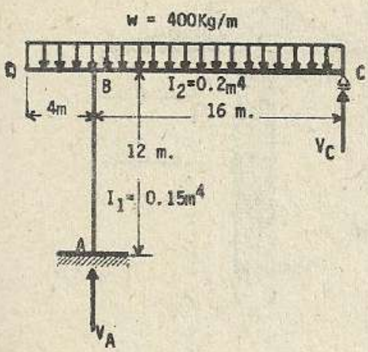
$$V_A = V_C = \frac{3E\varepsilon t I_1 k}{(3k+1)\;L^2}$$

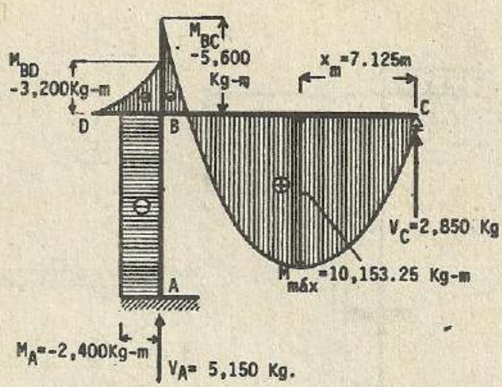
$$M_A = M_B = -\frac{3E\varepsilon t I_1 k}{(3k+1)\;L}$$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{3E\varepsilon t I_1 k}{(3k+1)\;L^2}\cdot x$

Esfuerzo de flexión en la columna : $M_y = -\dfrac{3E\varepsilon t I_1 k}{(3k+1)\;L}$ Constante

Nota : El presente pórtico trata del problema de aumento de temperatura, en caso contrario, los esfuerzos se anotan con los signos contrarios.





$$k = \frac{I_2 h}{I_1 L} = \frac{0.2 \times 12}{0.15 \times 16} = 1$$

Reacción debido a la carga en la viga BC :

$$V'_C = \frac{3(4k+1)}{8(3k+1)} \cdot wL = \frac{3 \times 5}{8 \times 4} \times 400 \times 16 = 3{,}000 \text{ Kg.}$$

Debido al volado :

$$V''_C = \frac{3k}{2(3k+1)L} \cdot wL^2 = \frac{3 \times 400 \times 4 \times 4}{2(3 \times 1+1) \times 16} = 150 \text{ Kg.} \quad \therefore \; V_C = 3{,}000 - 150 = 2{,}850 \text{ Kg.}$$

Nota: La reacción hacia arriba se considera como signo positivo

$$V_A = w(L + L_1) - V_C = 400(16 + 4) - 2{,}850 = 5{,}150 \text{ Kg.}$$

$$M_{BD} = -\frac{wL_1^2}{2} = -400 \times \frac{4 \times 4}{2} = -3{,}200 \text{ Kg-m}$$

$$M_{BC} = V_C L - w \cdot \frac{L^2}{2} = 2{,}850 \times 16 - \frac{400 \times 16 \times 16}{2} = -5{,}600 \text{ Kg-m}$$

$$M_{BA} = V_C L - w\frac{L^2}{2} + w \cdot \frac{L_1^2}{2} = -5{,}600 + 3{,}200 = -2{,}400 \text{ Kg-m}$$

$$M_A = M_{BA} = -2{,}400 \text{ Kg-m}$$

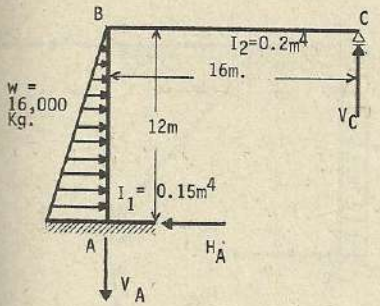
Para encontrar el punto de máximo esfuerzo de momento

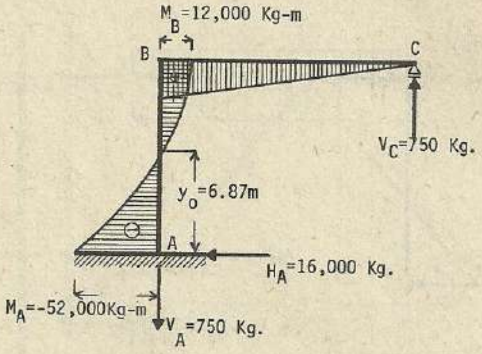
$$V = V_C - wx = 0$$

$$2{,}850 - 400x = 0 \qquad \therefore \quad x = \frac{2{,}850}{400} = 7.125 \text{ m}$$

$$M_{máx} = V_C x - w \cdot \frac{x^2}{2} = 2{,}850 \times 7.125 - 400 \times \frac{7.125^2}{2} =$$

$$= 10{,}153.125 \text{ Kg-m.}$$





$$k = \frac{I_2 h}{I_1 L} = \frac{0.2 \times 12}{0.15 \times 16} = 1$$

$$V_A = V_C = \frac{k}{4\,(3k+1)\,L}\,wh = \frac{1}{4 \times (3+1)\,16} \times 16{,}000 \times 12 = 750 \text{ Kg.}$$

$$H_A = W = 16{,}000 \text{ Kg.}$$

$$M_B = \frac{k}{4\,(3k+1)} \cdot wh = \frac{1}{4 \times (3+1)} \times 16{,}000 \times 12 = 12{,}000 \text{ Kg-m}$$

$$M_A = -\frac{9k+4}{12\,(3k+1)} \cdot wh = -\frac{9+4}{12\,(3+1)} \times 16{,}000 \times 12 = -52{,}0000 \text{ Kg-m}$$

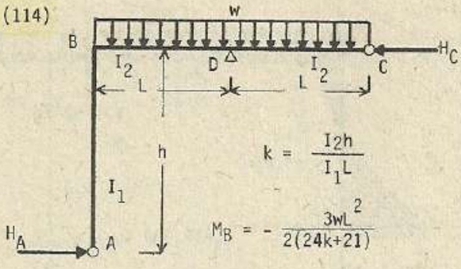
Considerando como (y_o) la distancia que media entre el punto (A) y el punto donde el esfuerzo de momento es cero, tenemos :

$$M_y = V_C\,L - \frac{W y_o^3}{3h^2} = 0$$

$$750 \times 16 - \frac{16{,}000\, y_o^3}{3 \times 12 \times 12} = 0 \qquad y_o^3 = \frac{750 \times 16 \times 3 \times 12 \times 12}{16{,}000} = 324$$

$$\therefore \quad y_o = \sqrt[3]{324} = 6.87 \text{ m.}$$

(114)



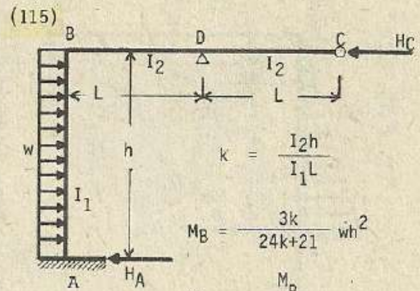
$$k = \frac{I_2 h}{I_1 L}$$

$$M_B = -\frac{3wL^2}{2(24k+21)}$$

$$M_D = -\frac{3\,(4k+3)}{4\,(24k+21)} \cdot wL^2$$

$$H_A = H_C = \frac{M_B}{h}$$

(115)



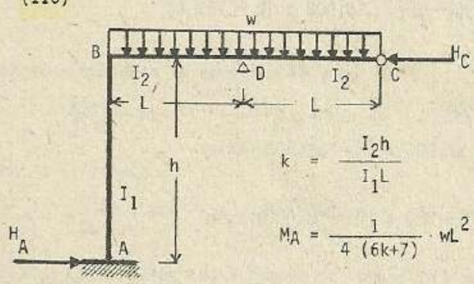
$$k = \frac{I_2 h}{I_1 L}$$

$$M_B = \frac{3k}{24k+21}\, wh^2$$

$$M_D = -\frac{M_B}{4}$$

$$H_A = \frac{wh}{2} + \frac{M_B}{h} \quad ; \quad H_C = \frac{wh}{2} - \frac{M_B}{h}$$

(116)



$$k = \frac{I_2 h}{I_1 L}$$

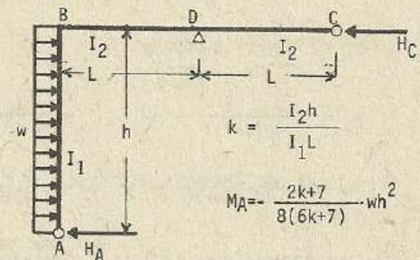
$$M_A = \frac{1}{4\,(6k+7)} \cdot wL^2$$

$$M_B = -\frac{1}{2\,(6k+7)} \cdot wL^2$$

$$H_A = H_C = -\frac{3M_B}{2h}$$

$$M_D = -\frac{3\,(k+1)}{4\,(6k+7)} \cdot wL^2$$

(117)



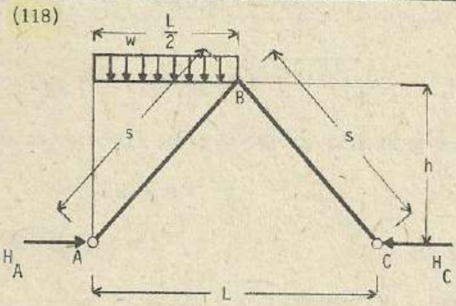
$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = -\frac{2k+7}{8(6k+7)} \cdot wh^2$$

$$M_B = -\frac{k}{2\,(6k+7)} \cdot wh^2$$

$$M_D = -\frac{M_B}{4}$$

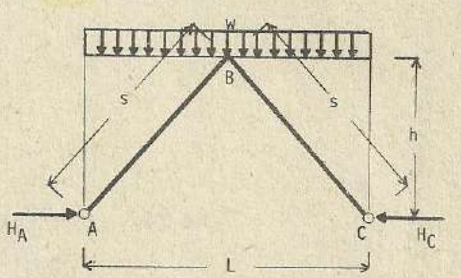
(118)



$$W = \frac{wL}{2} \quad ; \quad M_B = -\frac{WL}{64}$$

$$H_A = H_C = \frac{5wL^2}{64h} = 0.07813\ w \cdot \frac{L^2}{h}$$

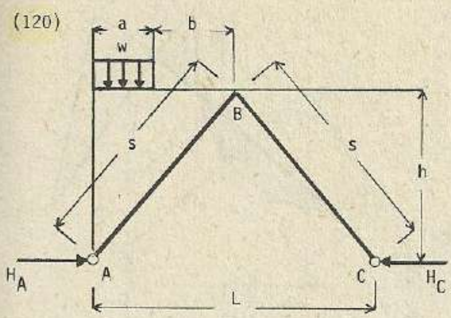
(119)



$$M_B = -\frac{wL^2}{32}$$

$$H_A = H_C = \frac{5wL^2}{32h} = 0.15625\ w \cdot \frac{L^2}{h}$$

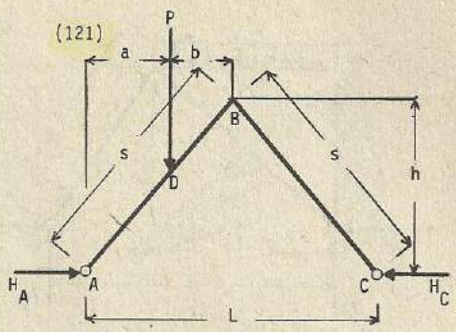
(120)



$$H_A = H_C = \frac{wL}{8} \cdot \frac{L}{h}\, \alpha^2 \quad (3 - 2\,\alpha^2)$$

$$\alpha = \frac{a}{h}$$

(121)



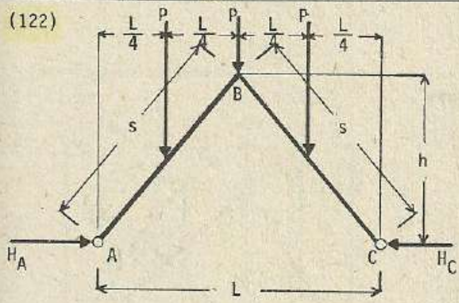
$$\alpha = \frac{a}{L} \quad ; \qquad M_B = - \frac{Pab\,(L + 2b)}{2L^2}$$

$$H_A = H_C = \frac{P}{4} \cdot \frac{L}{h}\, \alpha\, (3 - 4\,\alpha^2)$$

Si $a = b = \frac{L}{2}$ → $H_A = H_C = \frac{PL}{4h}$

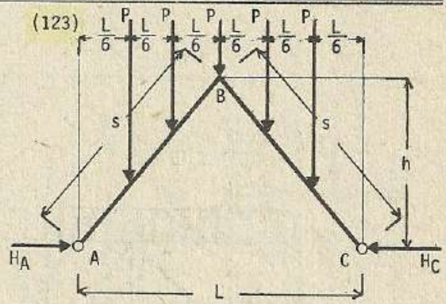
Si $a = b = \frac{L}{4}$ → $M_B = - \frac{3PL}{64}$

(122)



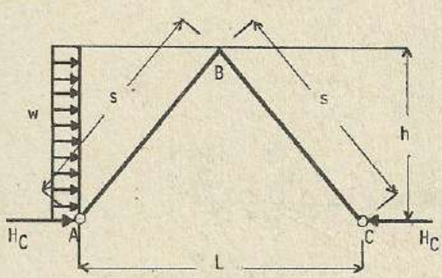
$$M_B = - \frac{3PL}{32}$$

(123)



$$M_B = - \frac{PL}{6}$$

(124)

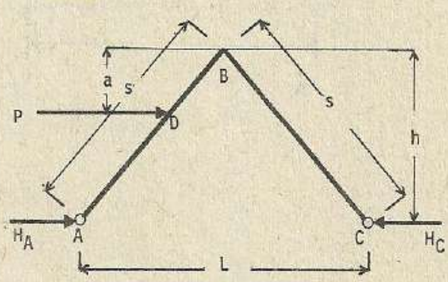


$$H_A = - \frac{11}{16}\, wh = - 0.6875\, wh$$

$$H_C = \frac{5}{16}\, wh = - 0.3125\, wh$$

$$M_B = - \frac{wL^2}{16}$$

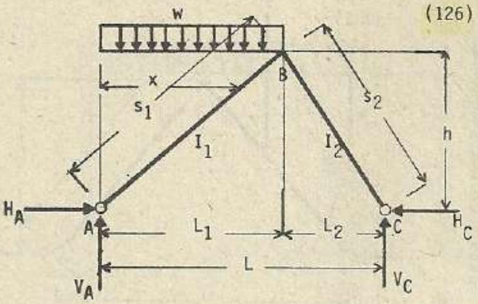
(125)

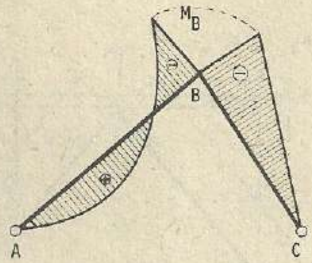


$$H_A = \frac{P}{4}\left[2 + 3 \cdot \left(\frac{a}{h}\right)^2 - \left(\frac{a}{h}\right)^3\right]$$

$$H_C = \frac{P}{4}\left[- 2 + 3 \cdot \left(\frac{a}{h}\right)^2 - \left(\frac{a}{h}\right)^3\right]$$

(126)





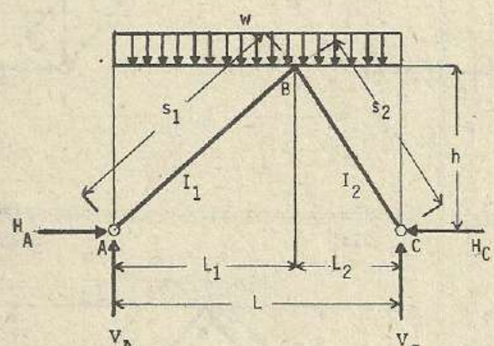
$$k = \frac{I_2 s_1}{I_1 s_2} \quad ; \quad W = \frac{wL_1}{2}$$

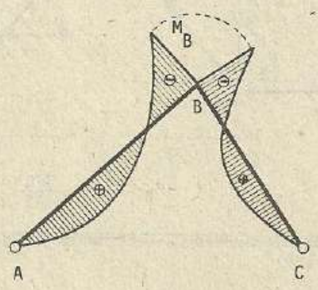
$$M_B = -\frac{kWL_1}{8\,(k+1)} \quad ; \quad V_A = \frac{wL_1}{2L}\,.\,(L + L_2) \quad ; \quad V_C = \frac{wL_1}{2L}$$

$$H_A = H_C = \frac{wL_1^2}{8hL}\;\frac{\left[4L_2 + k\,(4L_2 + L)\right]}{(k+1)} \quad ; \quad M_x = V_A x - H_A\,\frac{hx}{L_1} - \frac{wx^2}{2}$$

$(M_B = V_C L_2 - H_C h)$

(127)

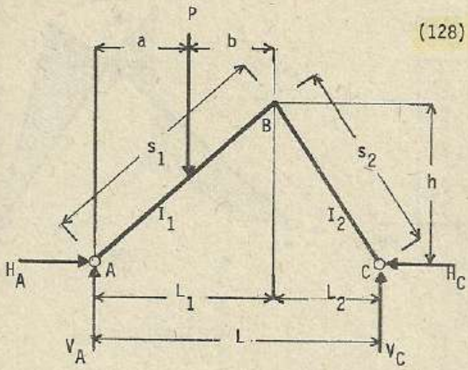


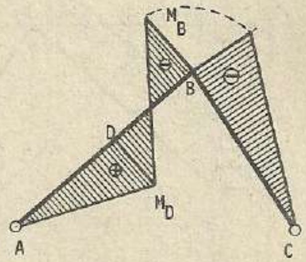


$$k = \frac{I_2\, s_1}{I_1\, s_2}$$

$$M_B = -\frac{w\,(kL_1^2 + L_2^2)}{8\,(k + 1)}$$

(128)





$$k = \frac{I_2 s_1}{I_1 s_2} \quad ; \quad M_B = -\frac{KPab(L_1+a)}{2L_1^2\,(k+1)}$$

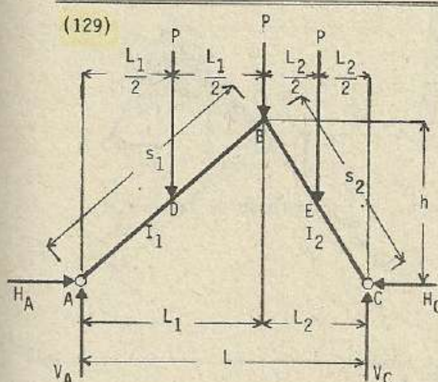
Si $a = b = \frac{L}{2} \rightarrow M_B = -\frac{3kPL_1}{16\,(k+1)}$

Considerando $\alpha = \frac{a}{L_1}$ $\qquad V_A = \frac{P\,(L-a)}{L} \; ; \; V_C = \frac{Pa}{L}$

$$H_A = H_C = \frac{Pa}{2hL} \cdot \frac{\left[2L_2 + (3L - L\,\alpha^2 - 2L_1\,k)\right]}{k+1}$$

$$M_D = V_A a - H_A\,\frac{ha}{L_1} \qquad M_B = V_C L_2 - H_C h$$

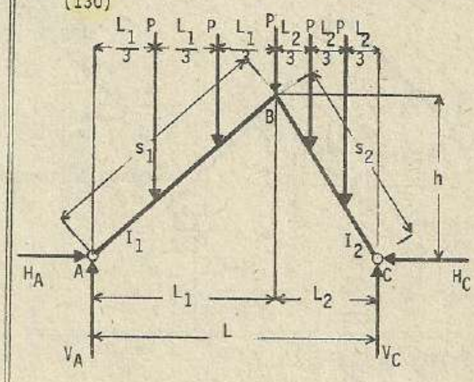
(129)



$$k = \frac{I_2 s_1}{I_1 s_2}$$

$$M_B = -\frac{3P\,(kL_1 + L_2)}{16\,(k+1)}$$

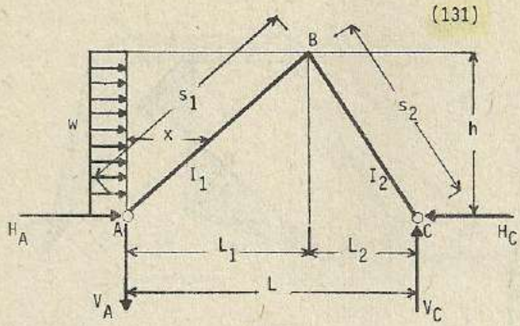
(130)



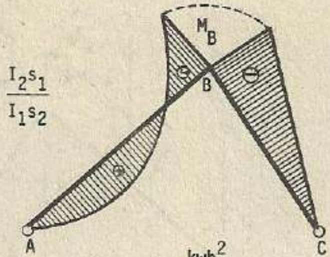
$$k = \frac{I_2 s_1}{I_1 s_2}$$

$$M_B = -\frac{P\,(kL_1 + L_2)}{3\,(k+1)}$$

(131)



$$k = \frac{I_2 s_1}{I_1 s_2}$$



$$M_B = -\frac{kwh^2}{8(k+1)}$$

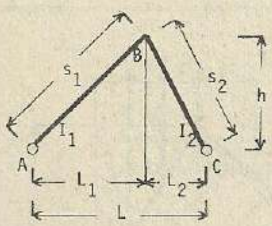
$$M_B = V_C L_2 - H_C h$$

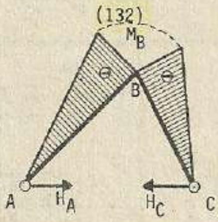
$$-V_A = V_C = \frac{wh^2}{2L} \quad ; \quad H_C = \frac{wh}{8L} \cdot \frac{\left[4L_2 + k\,(L + 4L_2)\right]}{k+1}$$

$$H_A = wh - H_C$$

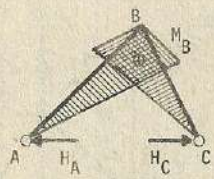
$$H_x = H_A \frac{hx}{L_1} - V_A x - \frac{wh^2 x^2}{2L_1^2}$$

(132)





Aumento de temperatura

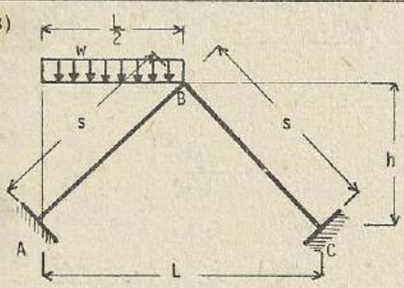


Descenso de temperatura

$$k = \frac{I_2 s_1}{I_1 s_2}$$

$$H_A = H_C = H = \frac{2E\varepsilon t I_2 L}{h^2 s_2\,(k+1)} \quad ; \quad M_B = Hh$$

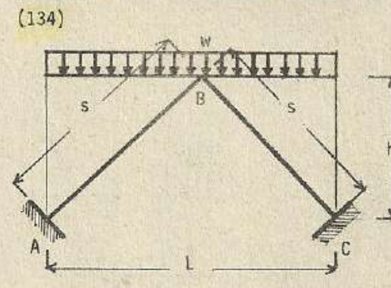
(133)



$$W = w \cdot \frac{L}{2} \quad ; \quad M_A = -\frac{5}{96}\,WL$$

$$M_B = -\frac{1}{48}\,WL \quad ; \quad M_C = -\frac{1}{96}\,WL$$

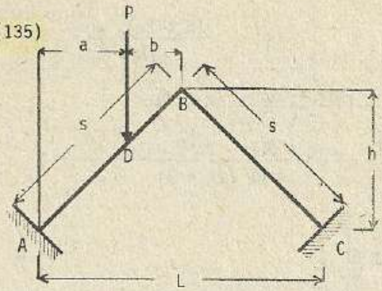
(134)



$$W = w \cdot \frac{L}{2} \quad ; \quad M_A = -\frac{1}{24}\,WL$$

$$M_B = -\frac{1}{24}\,WL; \quad M_C = -\frac{1}{24}\,WL$$

(135)



$$M_A = -\frac{Pab\,(L + 6b)}{2L^2}$$

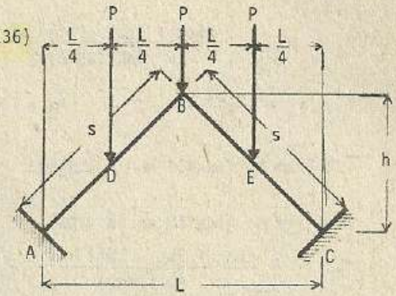
$$M_B = -\frac{2Pa^2b}{L^2} \quad ; \quad M_C = \frac{Pa^2b}{L^2}$$

Si $a = b = \frac{L}{4}$

$$M_A = -\frac{5}{64}\,PL \; ; \quad M_B = -\frac{1}{32}\,PL$$

$$M_C = \frac{1}{64}\,PL$$

(136)

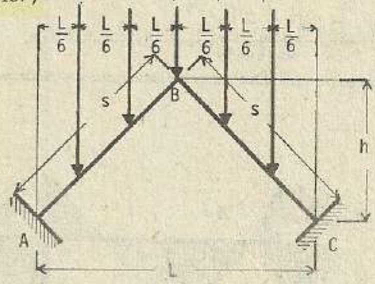


$$M_A = -\frac{1}{16}\,P$$

$$M_B = -\frac{1}{16}\,P$$

$$M_C = -\frac{1}{16}\,P$$

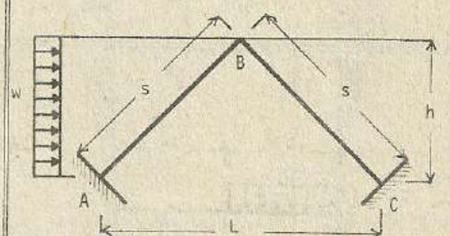
(137)



$$M_A = -\frac{1}{9}\,PL \; ; \quad M_B = -\frac{1}{9}\,PL$$

$$M_C = -\frac{1}{9}\,PL$$

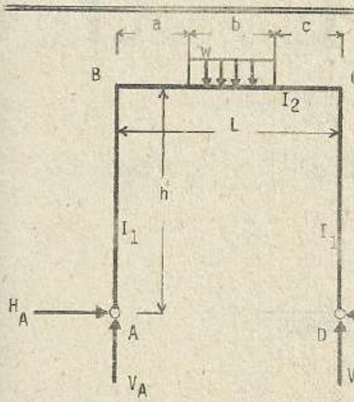
(138)



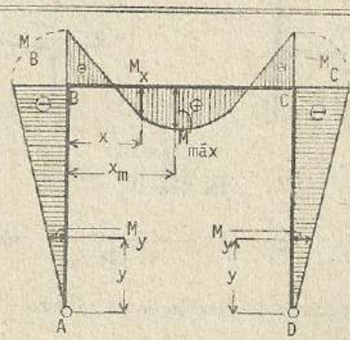
$$M_A = -\frac{5}{96}\,wh^2 \; ; \quad M_B = -\frac{1}{48}\,wh^2$$

$$M_C = \frac{1}{96}\,wh^2$$

(139)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{(6ac + 3bL - 2b^2)\cdot wb}{4hL\ (2k+3)} \qquad V_A = \frac{(b + 2c)\cdot wb}{2L}$$

$$V_D = \frac{(2a + b)\ wb}{2L} \quad ; \quad M_B = M_C = -\frac{(6ac + 3bL - 2b^2)\cdot wb}{4\ L\ (2k+3)}$$

Esfuerzo de flexión en la columna : $M_y = -\dfrac{(6ac + 3bL - 2b^2)\ .\ wb}{4\ hL\ (2k + 3)}\ y$

Esfuerzo de flexión en la viga :

$$M_x = -\frac{(6ac + 3bL - 2b^2)\ .\ wb}{4L\ (2k+3)} + \frac{(b + 2c)\cdot wb}{2L}\ x \qquad x \leq a$$

$$M_x = -\frac{(6ac + 3bL - 2b^2).wb}{4L\ (2k + 3)} + \frac{(b + 2c)\ wb}{2L}x - \frac{w(x - a)^2}{2} \qquad a \leq x \leq a + b$$

$$M_x = -\frac{(6ac + 3bL - 2b^2)\ .\ wb}{4L\ (2k + 3)} + \frac{(b+2c)wb}{2L}x - \frac{wb\ (2x - 2a - b)}{2} \qquad x \geq a + b$$

$$x_m = a + \frac{b\ (b + 2c)}{2L}$$

$$M_{máx.} = \frac{\left[(2k+3)\ (b+2c)\ (4aL+b^2+ 2bc) - (12\ acL+6bL^2 - 4b^2 L)\right]}{8L^2\ (2k+3)}\ .\ wb$$

Si $a = c$:

$$H_A = H_D = \frac{(3L^2-b^2)}{8hL(2k+3)}\ wb \ ; \quad V_A = V_D = \frac{wb}{2}$$

$$M_B = M_C = \frac{(3L^2 - b^2)}{8L\ (2k+3)}\cdot wb$$

$$x_m = \frac{L}{2} \ ; \quad M_{máx} = \frac{(2k+3)\ (4a+b)\ L - \left[\ 6\ aL + 2b\ (L+a)\right]}{8L\ (2k+3)}\ wb$$

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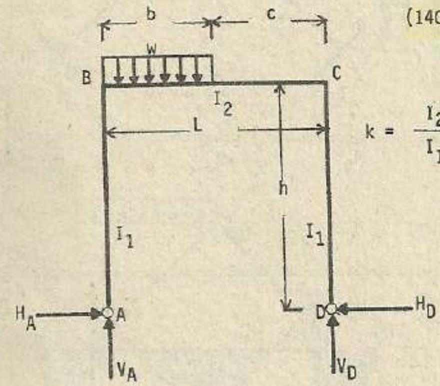
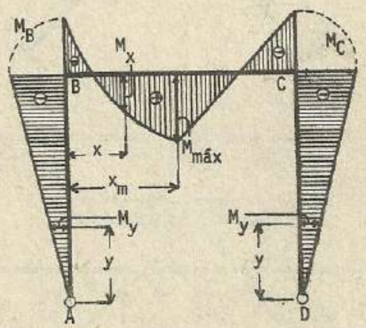
(140)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{(L+2c)}{4hL\ (2k+3)}\cdot wb^2 \ ; \quad V_A = \frac{(L+c)}{2L}\cdot wb \ ; \quad V_D = \frac{wb^2}{2L}$$

$$M_B = M_C = -\frac{(L+2c)}{4L\ (2k+3)}\cdot wb^2$$

Esfuerzo de flexión en la columna : $M_y = -\dfrac{(L+2c)}{4hL\ (2k+3)}\cdot wb^2 y$

Esfuerzo de flexión en la viga :

$$M_x = -\frac{wb^2(L+2c)}{4L\ (2k+3)} + \frac{wb\ (L+c)}{2L}x - \frac{wx^2}{2} \qquad x \leq b$$

$$M_x = -\frac{wb^2\ (L+2c)}{4L\ (2k+3\ )} + \frac{wb\ (L+c)}{2L}x - \frac{wb}{2}(2x-b) \qquad x \geq b$$

$$x_m = \frac{b\ (L+c)}{2L} \qquad M_{máx.} = \frac{wb^2\left[\ (L+c)^2\ (2k+1) + 2c^2\right]}{8L^2\ (2k+3)}$$

Si $b = c = \frac{L}{2}$

$$H_A = H_D = \frac{wL^2}{8h\ (2k+3)} \quad ; \quad V_A = \frac{3wL}{8} \quad ; \quad V_D = \frac{wL}{8}$$

$$M_B = M_C = -\frac{wL^2}{8\ (2k+3)}$$

$$x_m = \frac{3L}{8} \qquad M_{máx.} = \frac{(18\ k+11)}{128\ (2k+3)}wL^2$$

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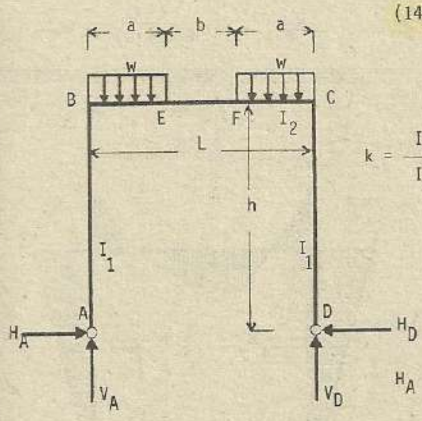
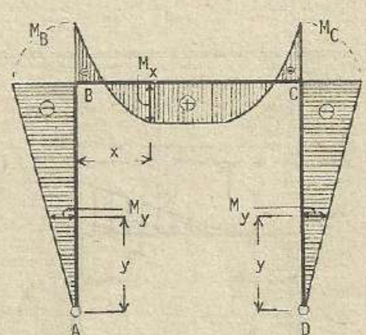
(141)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{(2\ L+b)}{2hL\ (2k+3)}wa^2 \qquad V_A = V_D = wa$$

$$M_B = M_C = -\frac{(2L+b)}{2L\ (2k+3)}wa^2$$

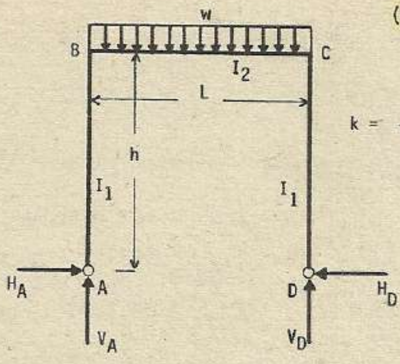
Esfuerzos de flexión en la columna : $M_y = -\frac{(2L+b)}{2hL\ (2k+3)}wa^2y$

Esfuerzo de flexión en la viga : $M_x = -\frac{(2\ L+b)}{2L\ (2k+3)}wa^2 + wax - \frac{wx^2}{2} \qquad x \leq a$

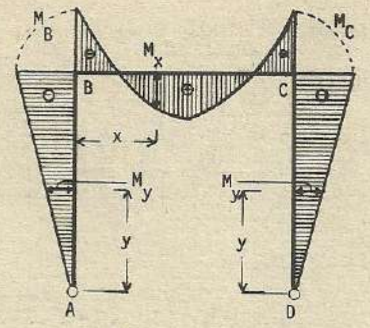
$$M_E = M_F = \frac{(a+Lk)}{L\ (2k+3)} \cdot wa^2$$

Entre los puntos (E) y (F) el momento de doblado o flexión es constante.

(142)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{wL^2}{4h\ (2k+3)} \qquad V_A = V_D = \frac{wL}{2}$$

$$M_B = M_C = -\frac{wL^2}{4\ (2k+3)}$$

Esfuerzo de flexión en la columna : $M_y = \dfrac{wL^2 y}{4h\ (2k+3)}$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{wL^2}{4\ (2k+3)} + \dfrac{wL}{2} x - \dfrac{wx^2}{2}$

$$x_m = \frac{L}{2} \qquad M_{máx.} = \frac{2k+1}{8\ (2k+3)} \cdot wL^2$$

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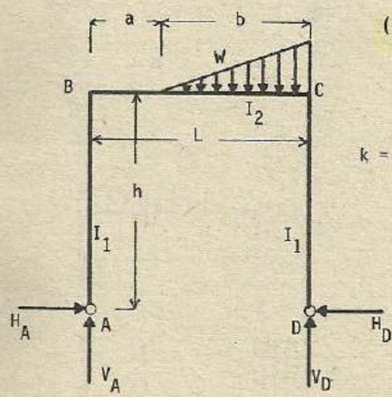
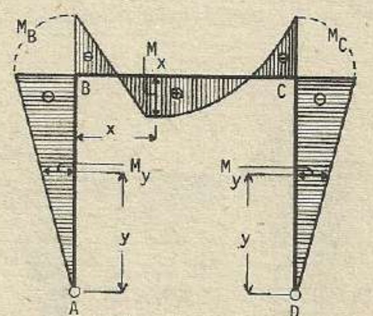
(143)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D - \frac{Wb\ (L + a)}{4\ hL\ (2k+3)} \ ; \ V_A = \frac{Wb}{3L}$$

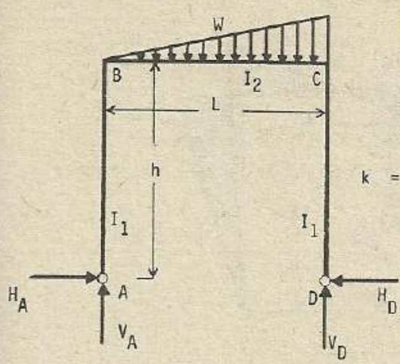
$$V_D = \frac{W\ (2L+a)}{3L} \ ; \qquad M_B = M_C = -\frac{Wb\ (L+a)}{4L\ (2k+3)}$$

Esfuerzo de flexión en la columna : $M_y = -\dfrac{Wb\ (L+a)}{4hL\ (2k+3)} \cdot y$

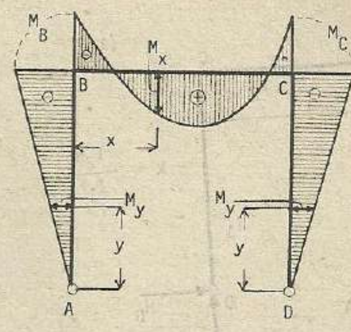
Esfuerzo de flexión en la viga : $M_x = -\dfrac{Wb\ (L+a)}{4L\ (2k+3)} + \dfrac{Wb}{3L} x$ —— $x \leq a$

$$M_x = -\frac{Wb\ (L+a)}{4L\ (2k+3)} + \frac{Wb}{3L} x - \frac{W\ (x-a)^3}{3b^2} \quad x \geq a$$

(144)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{WL}{4h\,(2k+3)} \quad ; \quad V_A = \frac{W}{3}$$

$$V_D = \frac{2}{3}\,W \qquad M_B = M_C = -\frac{WL}{4\,(2k+3)}$$

Esfuerzo de flexión en la columna : $M_y = -\dfrac{WLy}{4h\,(2k+3)}$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{WL}{4\,(2k+3)} - \dfrac{Wx^3}{3L^2} + \dfrac{W}{3}\,x$

$$x_m = \frac{L}{\sqrt{3}} \quad ; \quad M_{máx.} = \frac{8\sqrt{3}\,(2k+3) - 27}{108\,(2k+3)}\,WL$$

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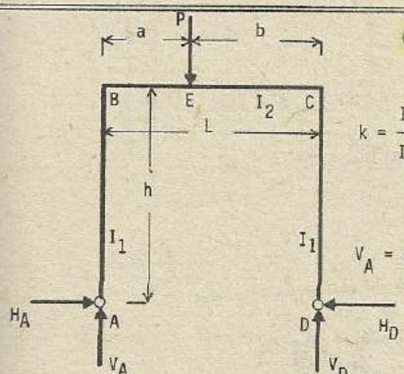
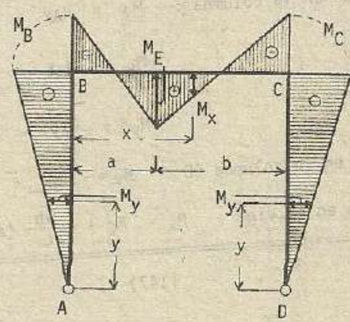
(145)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = \frac{Pb}{L}$$



$$V_D = \frac{Pa}{L}$$

$$H_A = H_D = \frac{3ab}{2hL\,(2k+3)}\,.P$$

$$M_B = M_C = -\frac{3ab}{2\,L\,(2k+3)}\,.P \qquad M_E = \frac{ab\,(4k+3)}{2L\,(2k+3)}\,.P$$

Esfuerzo de flexión en la columna : $M_y = -\dfrac{3ab}{2hL\,(2k+3)}\,Py$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{3ab}{2L\,(2k+3)}\,.P + \dfrac{Pb}{L}\,.x \longrightarrow x \leq a$

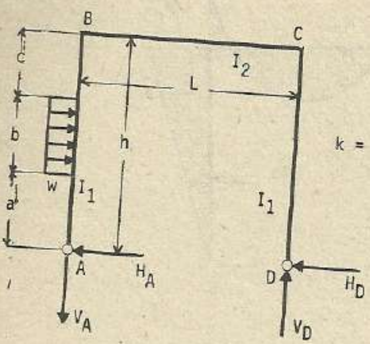
$$M_x = -\frac{3ab}{2L\,(2k+3)}\,P + \frac{Pb}{L}\,.x - P\,(x-a) \rightarrow x \geq a$$

Si $a = b = \dfrac{L}{2}$

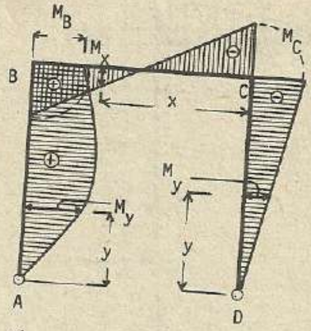
$$H_A = H_D = \frac{3L}{8h\,(2k+3)}\,.P \qquad V_A = V_D = \frac{P}{2}$$

$$M_B = M_C = -\frac{3}{8\,(2k+3)}\,.PL \qquad M_E = \frac{(4k+3)}{8\,(2k+3)}\,.PL$$

(146)



$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = V_D = \frac{wb}{L} \cdot \left(a + \frac{b}{2}\right)$$

$$H_A = wb - \frac{(2a+b)\left[6h^2(k+1) - k(2a^2 + 2ab + b^2)\right]}{8h^3(2k+3)} \cdot wb$$

$$H_D = \frac{(2a+b)\left[6h^2(k+1) - k(2a^2 + 2ab + b^2)\right]}{8h^3(2k+3)} \cdot wb \quad ; \quad M_B = H_A h - \frac{wb}{2}(b+2c)$$

$$M_C = -\frac{(2a+b)\left[6h^2(k+1) - k(2a^2 + 2ab + b^2)\right]}{8h^2(2k+3)} wb$$

Esfuerzo de flexión en la columna : $M_y = H_A y$ ⟶ $y \leq a$

$M_y = H_A y - \frac{w(y-a)^2}{2}$ ⟶ $a \leq y \leq a+b$

$M_y = H_A y - \frac{wb}{2}(2y-2a-b)$ ⟶ $y \geq a+b$

Momento de flexión en la columna CD : $M_y = - H_D y$

Esfuerzo de momento en la viga : $M_x = M_C + \frac{wb}{L}\left(a + \frac{b}{2}\right) x$

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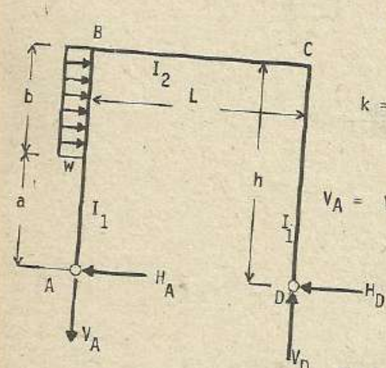
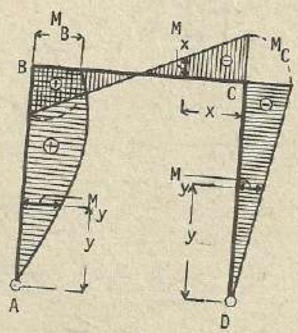
(147)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{wb}{2L}(a+h)$$



$$H_A = wb - \frac{(h+a)\left[6h^2(k+1) - k(h^2 + a^2)\right]}{8h^3(2k+3)} \cdot wb$$

$$H_D = \frac{(h+a)\left[6h^2(k+1) - k(h^2 + a^2)\right]}{8h^3(2k+3)} wb \quad ; \quad M_B = H_A h - \frac{wb^2}{2}$$

$$M_C = -\frac{(h+a)\left[6h^2(k+1) - k(h^2 + a^2)\right]}{8h^3(2k+3)} wb$$

Esfuerzo de momento por flexión en la columna (AB) :

$$M_y = H_A y \qquad y \leq a$$

$$M_y = H_A y - \frac{w(y-a)^2}{2} \qquad y \geq a$$

Esfuerzo de momento por flexión en la columna (CD) : $M_y = -H_D y$

Esfuerzo de flexión en la viga :

$$M_x = M_C + \frac{wb}{2L}(a+h)\,x$$

Si $a = b = \frac{h}{2}$; $V_A = V_D = \frac{3h^2 w}{8L}$; $H_A = \frac{71k+120}{128(2k+3)} wh$; $H_D = \frac{3\,(19k+24)}{128\,(2k+3)} wh$

$$M_B = \frac{39k+72}{128\,(2k+3)} wh^2 \;;\; M_C = -\frac{3\,(19k+24)}{128\,(2k+3)} wh^2$$

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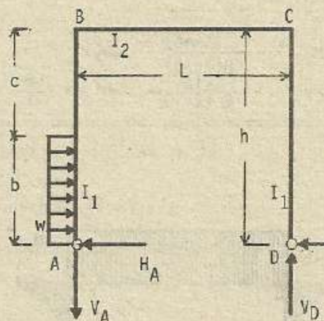
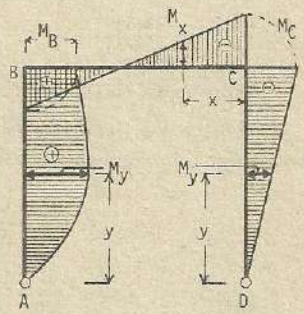
(148)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{wb^2}{2L}$$



$$H_A = wb - \frac{6h^2(k+1) - kb^2}{8h^3\,(2k+3)} wb^2$$

$H_D = \frac{6h^2(k+1) - kh^2}{8h^3\,(2k+3)} wb^2$; $M_B = H_A h - \frac{wb}{2}(b+2c)$; $M_C = -\frac{[6h^2\,(k+1)-(kb^2)]}{8h^2\,(2k+3)} wb^2$

Esfuerzo de flexión en la columna AB : $M_y = H_A y - \frac{wy^2}{2} \longrightarrow y \leq b$

$M_y = H_A y - \frac{wb}{2}(2y-b) \longrightarrow y \geq b$

Esfuerzo de fleción en la columna CD : $M_y = -H_D y$

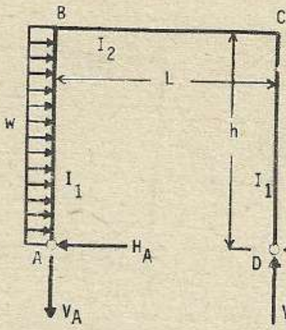
Esfuerzo de flexión en la viga : $M_x = M_C + \frac{wb}{2L} x$

Si $b = c = \frac{h}{2}$: $V_A = V_D = \frac{wh^2}{8L}$

$$H_A = \frac{105k+168}{128\,(2k+3)} wh \;;\; H_D = \frac{23k+24}{128\,(2k+3)} wh$$

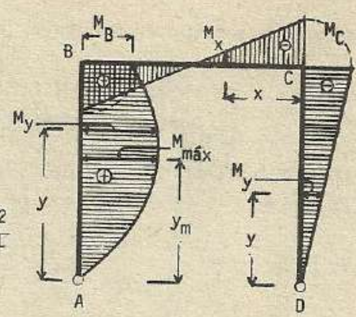
$$M_B = \frac{9k+24}{128\,(2k+3)} wh^2 \;;\; M_C = -\frac{(23k+24)}{128\,(2k+3)} wh^2$$

(149)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{wh^2}{2L}$$



$$H_A = \frac{11k+18}{8\ (2k+3)} \cdot wh \qquad H_D = \frac{5k+6}{8\ (2k+3)} \cdot wh$$

$$M_B = \frac{3(k+2)}{8\ (2k+3)} \cdot wh^2 \qquad M_C = -\frac{(5k+6)}{8\ (2k+3)} \cdot wh^2$$

Esfuerzo de flexión en la columna AB : $M_y = \dfrac{11k+18}{8\ (2k+3)}\ why - \dfrac{wy^2}{2}$

$y_m = \dfrac{11k+18}{8\ (2k+3)}\ h$; $M_{máx.} = \dfrac{(11k+18)^2}{128\ (2k+3)^2}\ wh^2$

Esfuerzo de flexión en la columna CD : $M_y = -\dfrac{(5k+6)}{8(2k+3)} \cdot why$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{5k+6}{8\ (2k+3)} \cdot wh^2 + \dfrac{wh^2}{2L}\ x$

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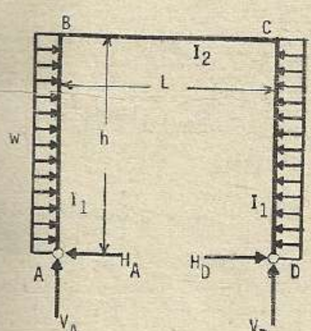
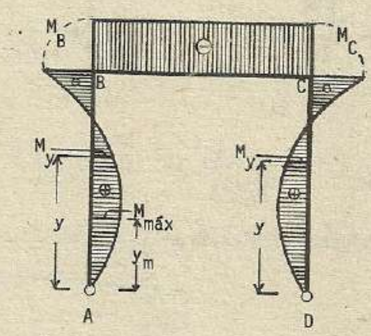
(150)



$$k = \frac{I_2 h}{I_1 L}$$

$V_A = V_D = 0$



$$H_A = H_D = \frac{3(k+2)}{4(2k+3)} wh \ ; \qquad M_B = M_C = -\frac{k}{4(2k+3)} \cdot wh^2$$

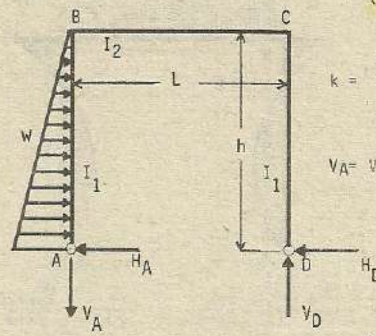
Esfuerzo de flexión en la columna :

$$M_y = \frac{3(k+2)}{4\ (2k+3)} \cdot why - \frac{wy^2}{2}$$

$y_m = \dfrac{3\ (k+2)}{4\ (2k+3)} \cdot h \longrightarrow M_{máx.} = \dfrac{9\ (k+2)^2}{32\ (2k+3)^2}\ wh^2$

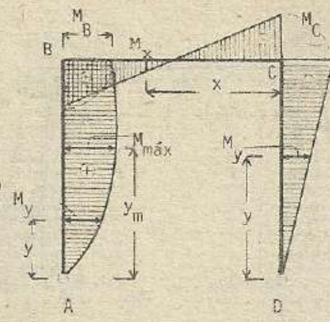
Esfuerzo de flexión en la viga : $M_x = -\dfrac{k}{4\ (2k+3)} \cdot wh^2$ (constante)

(151)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = -\frac{h}{3L}\cdot W$$



$$H_A = \frac{31k+50}{20\ (2k+3)}\cdot W \ ; \qquad H_D = \frac{9k+10}{20\ (2k+3)}\cdot W$$

$$M_B = \frac{13k+30}{60\ (2k+3)}\cdot Wh \ ; \qquad M_C = -\frac{(9k+10)}{20\ (2k+3)}\cdot Wh$$

Esfuerzo de flexión en la columna AB : $M_y = \frac{(31k+50)}{20\ (2k+3)}\, Wy - \frac{Wy^2}{3h^2}\,(3h-y)$

$$y_m = h\left[1 - \sqrt{\frac{9k+10}{20\ (2k+3)}}\right] ; \quad M_{máx.} = 2Wh\left[1 - \sqrt{\frac{9k+10}{20\ (2k+3)}}\right] \times \left[\frac{(31k+50)}{40\ (2k+3)}\right.$$
$$\left. - \frac{1}{6}\left[2 + \sqrt{\frac{9k+10}{20\ (2k+3)}}\right]\cdot\left[1 - \sqrt{\frac{9k+10}{20(2k+3)}}\right]\right]$$

Esfuerzo de flexión en la columna DC :

$$M_y = -\frac{(9k+10)}{20\ (2k+3)}\cdot Wy$$

Esfuerzo de flexión en la viga : $M_x = -\frac{(9k+10)}{20\ (2k+3)}\cdot Wh + \frac{h}{3L}\cdot Wx$

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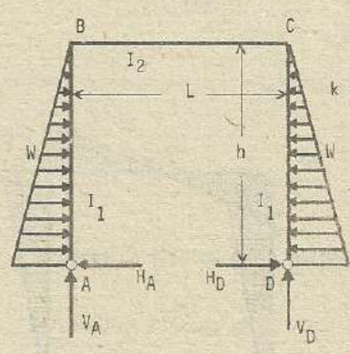
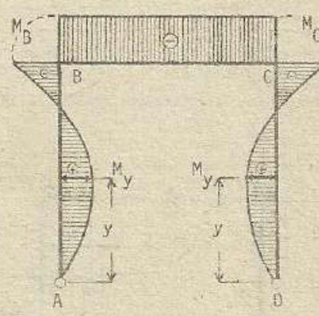
(152)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = 0$$



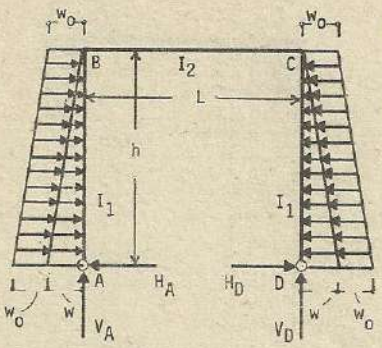
$$H_A = H_D = \frac{11k+20}{10\ (2k+3)}\cdot W$$

$$M_B = M_C = -\frac{7k}{30\ (2k+3)}\cdot Wh$$

Esfuerzo de flexión en la columna : $M_y = \frac{11k+20}{10\ (2k+3)}\cdot Wy - \frac{Wy^2}{3h^2}(3h-y)$

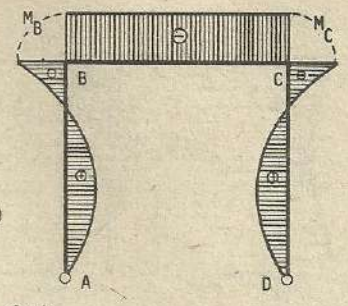
Esfuerzo de flexión en la viga : $M_x = -\frac{7k}{30\ (2k+3)}\cdot Wh$ (constante)

(153)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = 0$$



$$H_A = H_D = \frac{3w_o h}{4} \cdot \frac{k+2}{2k+3} + \frac{wh}{20} \cdot \frac{11k+20}{2k+3}$$

$$M_B = M_C = -\frac{w_o h^2}{4} \cdot \frac{k}{2k+3} - \frac{wh^2}{60} \cdot \frac{7k}{2k+3}$$

Nota : para hallar($M_{máx}$) analizar el problema (12)

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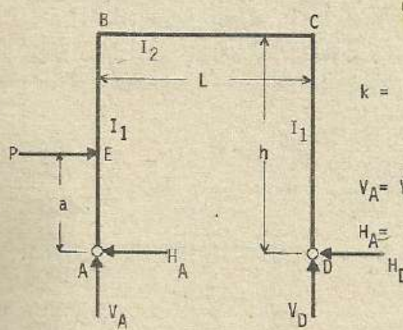
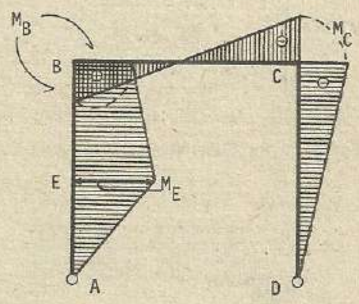
(154)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{Pa}{L}$$

$$H_A = P - H_D$$



$$H_D = \frac{Pa}{2} \cdot \frac{3h^2 + k\,(3h^2 - a^2)}{h^3(2k+3)}$$

$$M_B = Pa - H_D h$$

$$M_C = -H_D h$$

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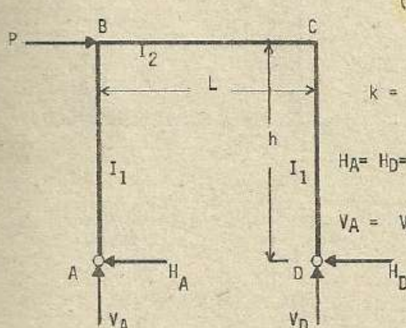
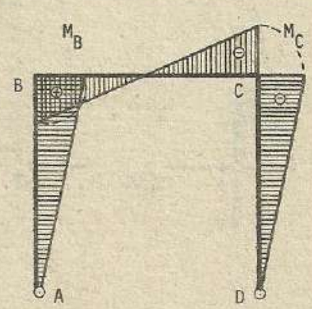
(155)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_D = H = \frac{P}{2}$$

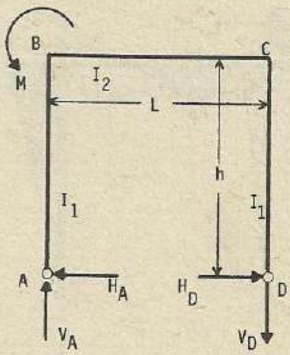
$$V_A = V_D = \frac{Ph}{L}$$



$$M_B = \frac{1}{2} \cdot Ph$$

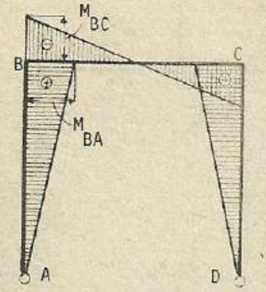
$$M_C = -\frac{1}{2} \cdot Ph$$

(156)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{M}{L}$$



$$H_A = H_D = H = \frac{3M}{2\,(2k+3) + h}$$

$$M_{BA} = M_C = Hh \qquad M_{BC} = M_{BA} - M$$

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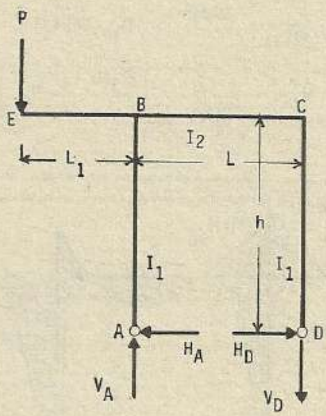
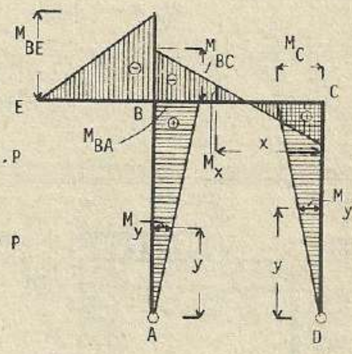
(157)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = \frac{L_1 + L}{L} \cdot P$$

$$V_D = \frac{L_1}{L} \cdot P$$



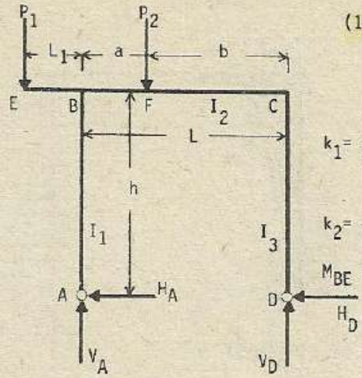
$$H_A = H_D = \frac{3PL_1}{2\,(2k+3)h} \qquad M_{BE} = -PL_1$$

$$M_{BA} = \frac{3PL_1}{2\,(2k+3)} \quad ; \qquad M_{BC} = -\frac{4k+3}{2(2k+3)} \cdot PL_1 \quad ; \qquad M_C = \frac{3PL_1}{2\,(2k+3)}$$

Esfuerzo de flexión en la columna : $M_y = \dfrac{3PL_1 y}{2\,(2k+3)\,h}$

Esfuerzo de flexión en la viga : $M_x = \dfrac{3PL_1}{2\,(2k+3)} - P \cdot \dfrac{L_1}{L} \cdot x$

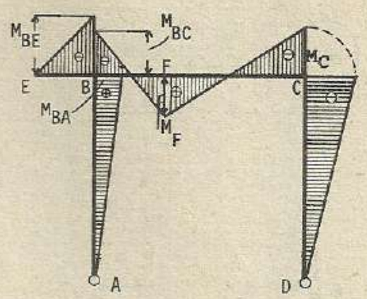
(158)



$k_1 = \dfrac{I_2 h}{I_1 L}$

$k_2 = \dfrac{I_3}{I_1}$

$M_{BE} = -P_1 L_1$



$$M_{BC} = -\frac{1}{2(3k_2+k_1+k_1k_2)}\left[\frac{2k_2P_2ab}{L} + P_1L_1(3k_2+2k_1k_2+2k_1)\right]$$

$$M_{BA} = -\frac{3k_2}{2(3k_2+k_1+k_1k_2)}\left[\frac{P_2ab}{L} - P_1L_1\right] \; ; \quad M_C = -\frac{3k_2}{2(3k_2+k_1+k_1k_2)}\left[\frac{P_2ab}{L} - P_1L_1\right]$$

Si $I_1 = I_3$

$$k = \frac{I_2 h}{I_1 L} \qquad M_{BE} = -P_1L_1 \; ; \quad M_{BC} = -\frac{1}{2(2k+3)}\left[\frac{3P_2ab}{L} + P_1L_1(4k+3)\right]$$

$$M_{BA} = -\frac{3}{2(2k+3)}\left[\frac{P_2ab}{L} - P_1L_1\right] \; ; \quad M_C = -\frac{3}{2(2k+3)}\left[\frac{P_2ab}{L} - P_1L_1\right]$$

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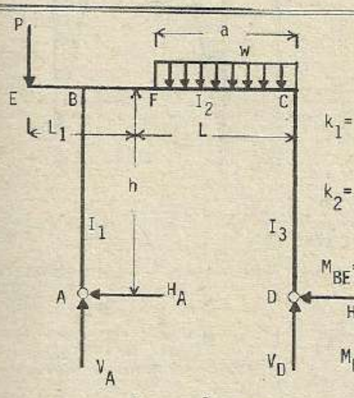
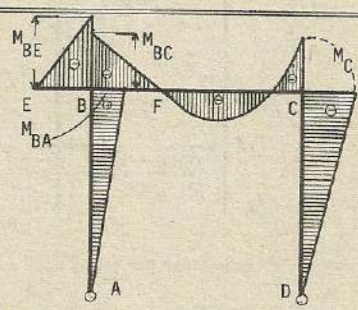
(159)



$k_1 = \dfrac{I_2 h}{I_1 L}$

$k_2 = \dfrac{I_3}{I_1}$

$M_{BE} = -PL_1$



$$M_{BC} = -\left[\frac{k_2wa^2(3L-2a)}{4L(3k_2+k_1+k_1k_1)} + \frac{PL_1(3k_2+2k_1k_2+2k_1)}{2(3k_2+k_1+k_1k_2)}\right]$$

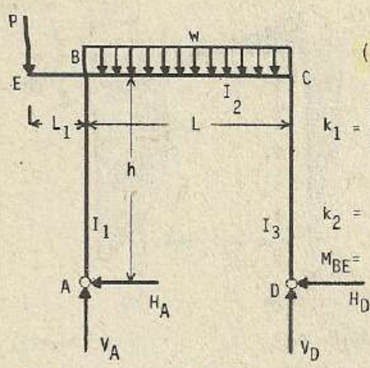
$$M_{BA} = -\left[\frac{k_2wa^2(3L-2a)}{4L(3k_2+k_1+k_1k_2)} - \frac{3k_2PL_1}{2(3k_2+k_1+k_1k_2)}\right]$$

$$M_C = -\left[\frac{k_2wa^2(3L-2a)}{4L(3k_1+k_1+k_1k_2)} - \frac{3k_2PL_1}{2(3k_2+k_1+k_1k_2)}\right]$$

Si $k_3 = k_1 \qquad k = \dfrac{I_2 h}{I_1 L}$

$$M_{BE} = -PL_1 \qquad M_{BC} = \left[\frac{wa^2(3L-2a)}{4L(2k+3)} + \frac{PL_1(4k+3)}{2(2k+3)}\right]$$

$$M_{BA} = -\left[\frac{wa^2(3L-2a)}{4L(2k+3)} - \frac{3PL_1}{2(2k+3)}\right]; \quad M_C = -\left[\frac{wa^2(3L-2a)}{4L(2k+3)} - \frac{3PL_1}{2(2k+3)}\right]$$

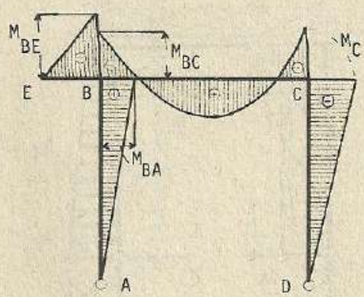


(160)

$$k_1 = \frac{I_2 h}{I_1 L}$$

$$k_2 = \frac{I_3}{I_1}$$

$$M_{BE} = -PL_1$$



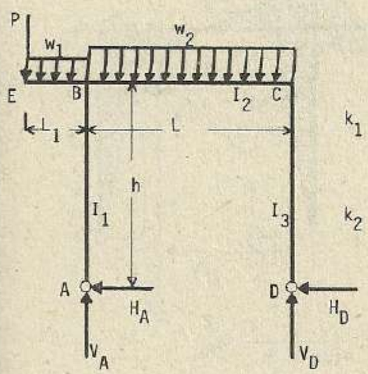
$$M_{BC} = -\frac{1}{4\,(3k_2+k_1+k_1+k_2)}\left[k_2 wL^2 + 2PL_1(3k_2+2k_1k_2+2k_1)\right]$$

$$M_{BA} = +\frac{3k_2}{4\,(3k_2+k_1+k_1\,k_2)}\left[\frac{wL^2}{3} - 2PL_1\right] \;;\quad M_C = -\frac{3k_2}{4\,(3k_2+k_1+k_1k_2)}\left[\frac{wL^2}{3} - 2PL_1\right]$$

Si $I_1 = I_3$ $\quad k = \frac{I_2 h}{I_1 L}$ $\quad M_{BE} = -PL_1$

$$M_{BC} = -\frac{1}{4\,(2k+3)}\left[wL^2 + 2PL_1(4k+3)\right] \;;\quad M_{BA} = -\frac{3}{4\,(2k+3)}\left[\frac{wL^2}{3} - 2PL_1\right]$$

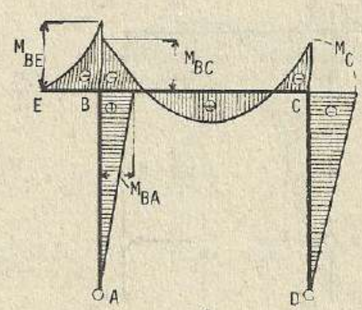
$$M_C = -\frac{3}{4\,(2k+3)}\left[\frac{wL^2}{3} - 2PL_1\right]$$



(161)

$$k_1 = \frac{I_2 h}{I_1 L}$$

$$k_2 = \frac{I_3}{I_1}$$



$$M_{BE} = -\left(PL_1 + \frac{w_1 L_1^2}{2}\right)$$

$$M_{BC} = -\frac{1}{4(3k_2+k_1+k_1k_2)}\left[k_2w_2L^2 + (2PL_1 + w_1L_1^2)\cdot(3k_2+2k_1k_2+2k_1)\right]$$

$$M_{BA} = -\frac{3k_2}{4(3k_2+k_1+k_1k_2)}\left[\frac{w_2L^2}{3} - 2PL_1 - w_1L_1^2\right]$$

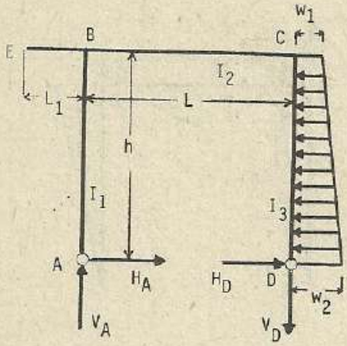
$$M_C = -\frac{3k_2}{4\,(3k_2+k_1+k_1k_2)}\left[\frac{w_2L^2}{3} - 2PL_1 + w_1L_1^2\right]$$

Si $I_3 = I_1$ $\quad k = \frac{I_2 h}{I_1 L}$ $\quad M_{BE} = -\left(PL_1 + \frac{w_1L_1^2}{2}\right)$

$$M_{BC} = -\frac{1}{4\,(2k+3)}\left[w_2L^2 + (2PL_1 + w_1L_1^2)\,(4k+3)\right]$$

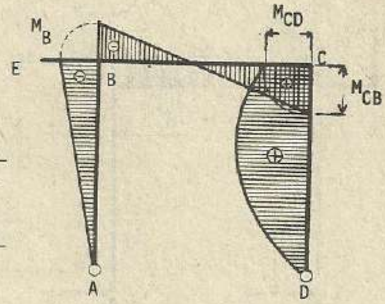
$$M_{BA} = +\frac{3}{4\,(2k+3)}\left[\frac{w_2L^2}{3} - 2PL_1 - w_1L_1^2\right] = -M_C$$

(162)



$$k_1 = \frac{I_2 h}{I_1 L}$$

$$k_2 = \frac{I_3}{I_1}$$



$$m = \frac{h^2}{180}\,(8w_1 + 7w_2) \quad ; \quad n = \frac{h^2}{6}\,(2w_1+w_2)$$

$$M_B = -\frac{2k_1 m + n\,(3k_2 + 2k_1)}{2\,(3k_2 + k_1 + k_1 k_2)} \qquad M_C = \frac{(3k_2 + 2k_1 k_2)\,n - 2k_1 m}{2\,(3k_2 + k_1 + k_1 k_2)}$$

Si $I_3 = I_1$ $\quad k = \frac{I_2 h}{I_1 L}$

$$M_B = -\frac{\{2\,km + n\,(2k+3)\}}{2\,(2k+3)} \qquad M_C = \frac{(2k+3)\,n - 2\,km}{2\,(2k+3)}$$

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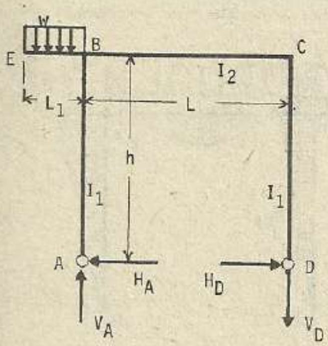
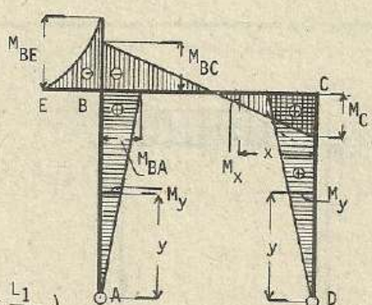


$$k = \frac{I_2 h}{I_1 L}$$

$$V_D = \frac{wL_1^2}{L}$$

$$V_A = \frac{wL_1\,(L + \frac{L_1}{2})}{L}$$



$$H_A = H_D = \frac{3}{4(2k+3)\,h}\,wL_1^2$$

$$M_C = \frac{3}{4\,(2k+3)}\cdot wL_1^2 \;;\; M_{BE} = -\frac{wL_1^2}{2} \;;\; M_{BC} = -\frac{4k+3}{4\,(2k+3)}\,wL_1^2$$

$$M_{BA} = \frac{3}{4\,(2k+3)}\,wL_1^2$$

Esfuerzo de flexión en la columna : $M_y = \frac{3}{4\,(2k+3)\,h}\cdot wL_1^2\,y$

Esfuerzo de flexión en la viga : $M_x = \frac{3}{4\,(2k+3)}\cdot wL_1^2 - \frac{wL_1^2}{L}\,x$

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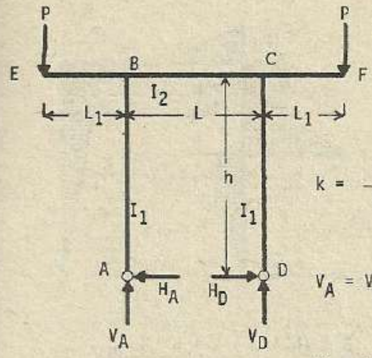
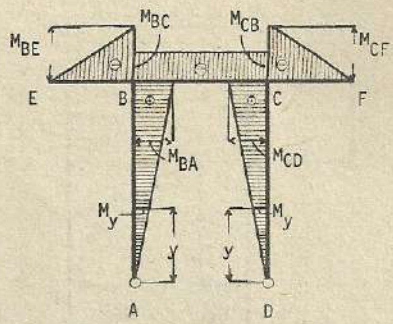


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = P$$

$$H_A = H_D = \frac{3PL_1}{(2k + 3)\,h}$$



$M_{BE} = M_{CF} = -PL_1$; $M_{BA} = M_{CD} = \dfrac{3PL_1}{(2k+3)}$; $M_{BC} = M_{CB} = -\dfrac{2k}{(2k+3)} \cdot PL_1$

Esfuerzo de flexión en la columna : $M_y = \dfrac{3PL_1 y}{(2k+3)\,h}$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{2k}{(2k+3)} \cdot PL_1$ (constante)

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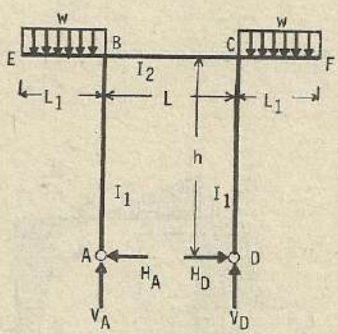
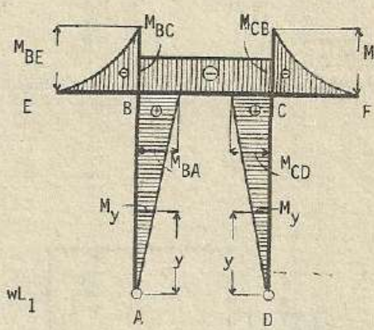


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = wL_1$$



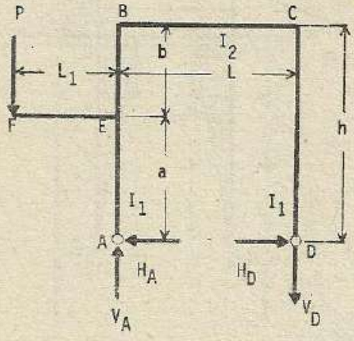
$$H_A = H_D = \frac{3}{2\,(2k+3)\,h} \cdot wL_1^2$$

$M_{BA} = M_{CD} = \dfrac{3}{2\,(2k+3)} \cdot wL_1^2$; $M_{BE} = M_{CF} = -\dfrac{wL_1^2}{2}$; $M_{BC} = M_{CB} = -\dfrac{k}{(2k+3)} \cdot wL_1^2$

Esfuerzo de flexión en la columna : $M_y = \dfrac{3}{2\,(2k+3)h} \cdot wL_1^2 y$

Esfuerzo de flexión en viga : $M_x = -\dfrac{k}{(2k+3)} \cdot wL_1^2$ (constante)

(166)

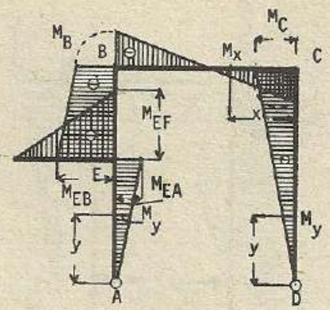


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = \frac{L+L_1}{L} \cdot P$$

$$V_D = \frac{L_1}{L} \cdot P$$

$$H_A = H_D = \frac{3\left[(h^2-a^2)\,k + h^2\right]}{2\,(2k+3)\,h^3}\,PL_1$$



$$M_C = \frac{3\left[(h^2-a^2)\,k + h^2\right]}{2\,(2k+3)\,h^2}\,PL_1 \quad ; \quad M_B = -\frac{(h^2+3a^2)\,k + 3h^2}{2\,(2k+3)\,h^2}\,PL_1 \quad ; \quad M_{EF} = -PL_1$$

$$M_{EB} = \frac{\left[3h^3(2k+3) - 3\left\{(h^2-a^2)\,k + h^2\right\}a\right]}{2\,(2k+3)\,h^3}\,.PL_1 \quad ; \quad M_{EA} = \frac{3\left[(h^2-a^2)\,k + h^2\right]}{2\,(2k+3)\,h^3}\,PL_1 a$$

Esfuerzo de flexión en la columna AB : $M_y = H_A y \qquad y \leqslant a$

$M_y = H_A y - PL_1 \qquad y \geqslant a$

en la columna DC : $M_y = H_D y$

Esfuerzo de flexión en la viga : $M_x = \frac{3\left[(h^2-a^2)\,k + h^2\right]}{2\,(2k+3)\,h^2}\,.PL_1 - P\,\frac{L_1}{L}\,.x$

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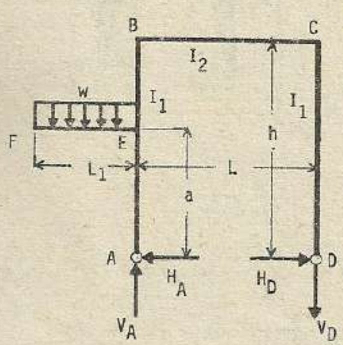
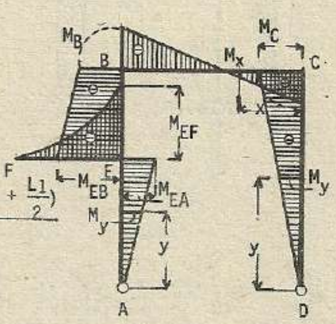


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = \frac{wL_1\left(L + \frac{L_1}{2}\right)}{L}$$

$$V_D = \frac{wL_1^2}{2L}$$



$$H_A = H_D = \frac{3\left[k\,(h^2-a^2) + h^2\right]}{4\,(2k+3)\,h^3}\,wL_1^2$$

$$M_C = \frac{3\left[k\,(h^2-a^2) + h^2\right]}{2\,(2k+3)\,h^2}\,wL_1^2 \quad ; \quad M_B = -\frac{k\cdot(h^2+3a^2) + 3h^2}{4\,(2k+3)h^2}\,wL_1^2 \quad ; \quad M_{EF} = -\frac{wL_1^2}{2}$$

$$M_{EB} = -\frac{\left[2\,h^3(2k+3) - 3\left\{(h^2-a^2)\cdot k + h^2\right\}a\right]}{4\,(2k+3)\,h^3}\,wL_1^2 \qquad M_{EA} = \frac{3\left[k\,(h^2-a^2) + h^2\right]a}{4\,(2k+3)\,h^3}\,wL_1^2$$

Esfuerzo de flexión en la columna AB :

$$M_y = \frac{3\left[k\,(h^2 - a^2) + h^2\right]}{4\,(2k+3)\,h^3}\, wL_1^2\, y \qquad y \leq a$$

$$M_y = \frac{3\left[k\,(h^2 - a^2) + h^2\right]}{4\,(2k+3)\,h^3}\, wL_1^2\, y - \frac{wL_1^2}{2} \qquad y \geq a$$

Esfuerzo de flexión en la columna CD :

$$M_y = \frac{3\left[k\,(h^2 - a^2) + h^2\right]}{4\,(2k+3)\,h^3}\, wL_1^2\, y$$

Esfuerzo de flexión en la viga : $M_x = \frac{3\left[(h^2 - a^2)\,k + h^2\right]}{4\,(2k+3)\,h^2} . wL_1^2 - \frac{wL_1^2}{2L}\, x$

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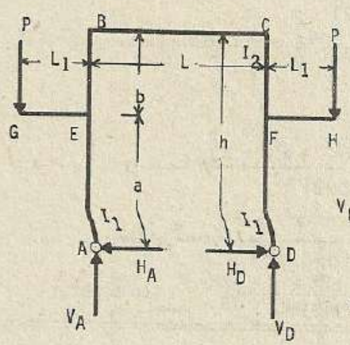
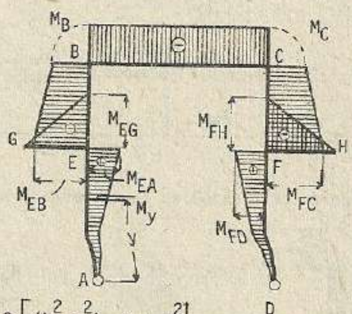


$k = \frac{I_2 h}{I_1 L}$



$V_A = V_D = P$

$$H_A = H_D = \frac{3\left[(h^2 - a^2)\;k + h^2\right]}{(2k+3)\,h^3} PL_1$$

$M_B = M_C = -\frac{(3a^2 - h^2)\,k}{(2k+3)\,h^2} \cdot PL_1$; $M_{EG} = M_{FH} = -PL_1$

$$M_{EB} = M_{FC} = -\frac{(2k+3)\,h^3 - 3a\left[(h^2 - a^2)\,k + h^2)\right]}{(2k+3)\,h^3} \cdot PL_1$$

$$M_{EA} = M_{ED} = \frac{3\left[(h^2 - a^2)\,k + h^2\right]}{(2k+3)\,h^3} \cdot PL_1 a$$

Momento de flexión en la columna $M_y = H_A y \qquad y \leq a$

$M_y = H_A y - PL_1 \qquad y \geq a$

Esfuerzo de flexión en la viga :

$M_x = -\frac{(3a^2 - h^2\;k)}{(2k+3)\,h^2} \cdot PL_1$ (constante)

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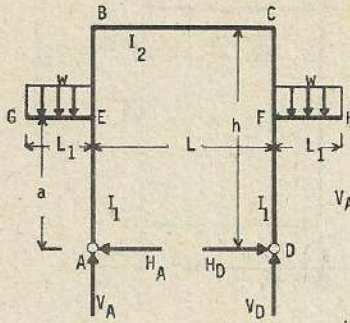
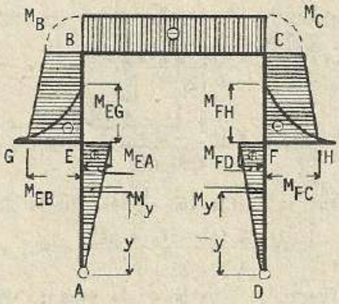


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = wL_1$$



$$H_A = H_D = \frac{3wL_1^2\left[k\,(h^2 - a^2) + h^2\right]}{2\,(2k+3)\,h^3}$$

$$M_B = M_C = -\frac{(2a^2 - h^2)\,k}{2\,(2k+3)\,h^2}\,wL_1^2 \qquad M_{EG} = M_{FH} = -\frac{wL_1^2}{2}$$

$$M_{EB} = M_{FC} = -\frac{(2k+3)h^3 - 3a\left[k\,(h^2 - a^2) + h^2\right]}{2\,(2k+3)\,h^3}.wL_1^2$$

$$M_{EA} = M_{FD} = \frac{3waL_1^2\left[k\,(h^2 - a^2) + h^2\right]}{2\,(2k+3)\,h^3}$$

Esfuerzo de flexión en la columna : $M_y = \dfrac{3wL_1^2\left[k\,(h^2 - a^2) + h^2\right]}{2\,(2k+3)\,h^3}\,y \longrightarrow \Big|\, y \leq a$

$$M_y = \frac{3wL_1^2\left[k\,(h^2 - a^2) + h^2\right]}{2\,(2k+3)\,h^3}.y - \frac{wL_1^2}{2}\Big|\, y \geq a$$

Esfuerzo de flexión en la viga : $M_x = -\dfrac{(3a^2 - h^2)\,k}{2(2k+3)h^2}\,.\,wL_1^2$ (constante)

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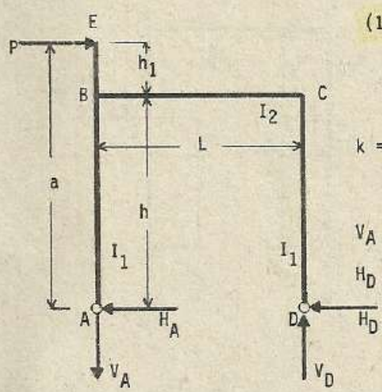
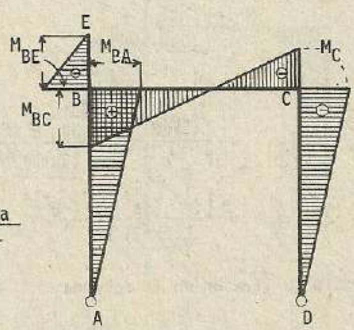


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{Pa}{L}$$

$$H_D = P - H_A$$



$$H_A = \frac{P\,(2kh + 3a)}{2h(2k+3)} \quad ; \quad H_D = P - H_A$$

$$M_{BA} = H_A h \quad ; \quad M_{BC} = M_{BA} + Ph_1 = H_A h + Ph_1$$

$$M_{CB} = M_{CD} = -\,(P - H_A)\,h \qquad M_{BE} = -Ph_1$$

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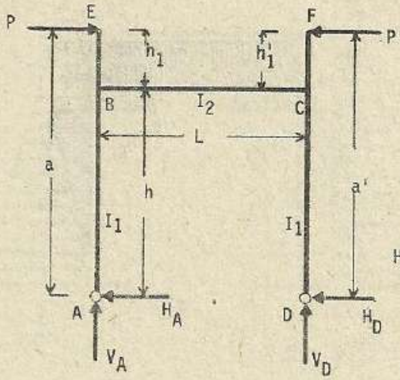

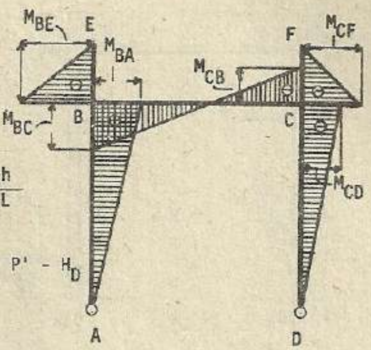

$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = P - P' - H_D$$

$$H_D = \frac{(P-P')\,2kh + 3\,(Pa + P'a') - 6\,P'\,h}{2h\,(2k+3)}$$

$$V_A = V_D = \frac{Pa - P'\,a'}{L}$$

$M_{BA} = H_A h$; $M_{BC} = M_{BA} + Ph_1$; $M_{CB} = M_{CD} + P'\,h_1'$; $M_{CD} = -H_D h$

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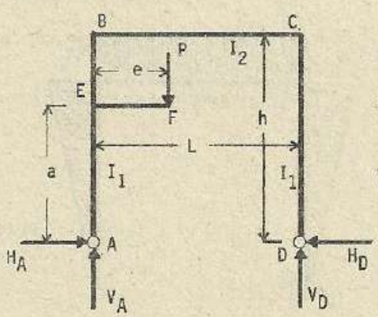
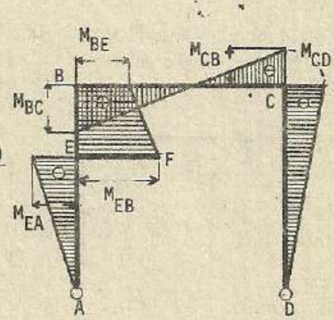


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = \frac{P\,(L-e)}{L}$$

$$V_D = \frac{Pe}{L}$$



$$H_A = H_D = H = 3\,Pe\,\frac{k\,(h^2 - a^2) + h^2}{2h^3\,(2k+3)}$$

$M_{BE} = Pe - Hh = M_{BC}$ $\qquad$ $M_{EA} = -\,H_A a$ $\qquad$ $M_{CB} = -\,Hh = M_{CD}$

$M_{FB} = -\,H_A a + Pe$

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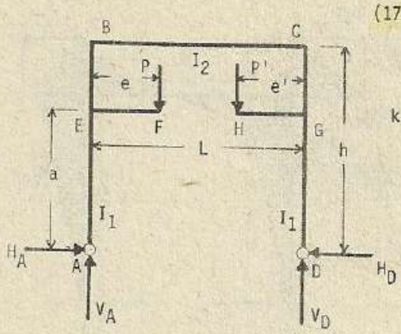
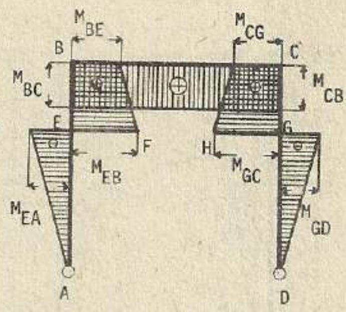


$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = \frac{P'\,e' + P\,(L-e)}{L}$$

$$V_D = \frac{Pe + P'\,(1-e')}{L}$$

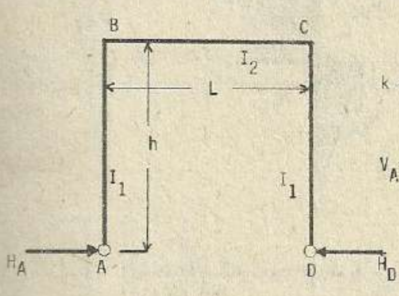
$$H_A = H_D = H = 3\,(Pe+P'e')\cdot\frac{k\,(h^2-a^2)+h^2}{2h^3\,(2k+3)} \quad ; \quad M_{BE} = Pe - Hh = M_{BC}$$

$$M_{CB} = P'\,e' - Hh = M_{CG} \quad ; \quad M_{EA} = -H_A a \quad ; \quad M_{GD} = -H_D a$$

$$M_{EB} = -H_A a + pe \quad ; \quad M_{GC} = -H_D a + P'e'$$

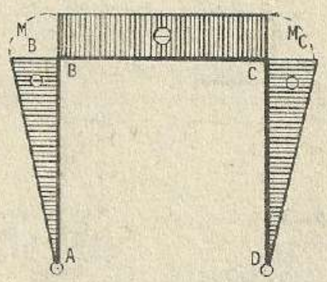
Esfuerzos por cambio de Temperatura

(174)



$$k = \frac{I_2 h}{I_1 L}$$

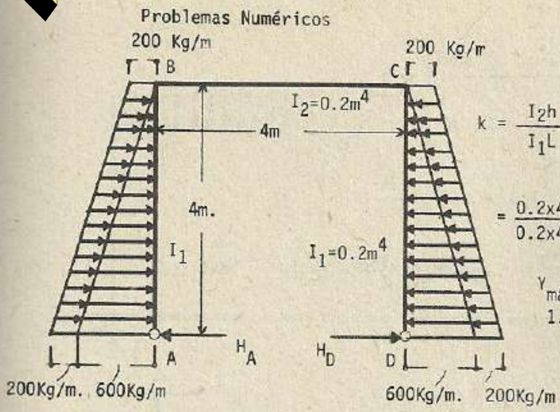
$$V_A = V_D = 0$$



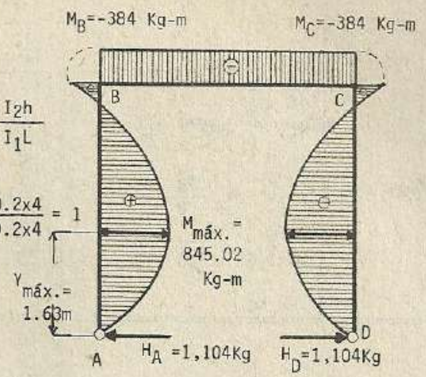
$$H_A = H_D = \frac{3E\varepsilon t I_2}{(3+2k)h^2}$$

$$M_B = M_C = -\frac{3E\varepsilon t I_2}{(2k+3)\ \ h^2}$$

Nota : El presente problema corresponde al caso de aumento de temperatura. En caso de descenso, los esfuerzos se anotan con los signos contrarios.



$$k = \frac{I_2 h}{I_1 L} = \frac{0.2 \times 4}{0.2 \times 4} = 1$$



De acuerdo a (153) :

$$H_A = H_D = \frac{3w_o h}{4} \cdot \frac{k+2}{2k+3} + \frac{wh}{20} \cdot \frac{11k+20}{2k+3}$$

$$= \frac{3}{4} \times 200 \times 4 \times \frac{1+2}{2+3} + \frac{600 \times 4}{20} \times \frac{11+20}{2+3} = 1{,}104 \text{ Kg.}$$

$$V_A = V_D = 0$$

$$M_B = M_C = -\frac{w_o h^2}{4} \frac{k}{2k+3} - \frac{wh^2}{60} \frac{7k}{2K+3} = -\frac{200 \times 4 \times 4}{4} \cdot \frac{1}{2+3} - \frac{600 \times 4 \times 4}{60} \cdot \frac{7}{2+3} = -384. \text{ Kg -m}$$

Similares resultados se obtienen descomponiendo las cargas como uniformemente distribuida y de forma triangular :

w_o = 200 Kg6m ; $W = \frac{600 \times 4}{2} = 1{,}200$ Kg.

. Carga uniformemente distribuida : $H'_A = H'_D = \frac{3\ (k+2)}{4\ (2k+3)} \cdot wh$

$$= \frac{3x\ (1+2)}{4x\ (2+3)} \times 200 \times 4 = 360 \text{ Kg}$$

. Carga triangular :

$$H''_D = H''_D = \frac{11k+20}{10\ (2k+3)} \cdot W = \frac{11+20}{10\ (2+3)} \times 1{,}200 = 744 \text{ Kg.}$$

$$\therefore \quad H_A = H_D = 360 + 744 = 1{,}104 \text{ Kg.}$$

. Carga uniformemete distribuida : $M'_B = M'_C = -\frac{k}{4\ (2k+3)} \cdot w_o h^2$

$$= -\frac{1}{4\ (2+3)} \times 200 \times 4 \times 4 = -160 \text{ Kg -m}$$

. Carga triangular :

$$M''_B = M''_C = -\frac{7k}{30(2k+3)} \cdot Wh = -\frac{7}{30\ (2+3)} \times 1200 \times 4 = -224 \text{ Kg -m}$$

$$\therefore \quad M_B = M_C = -160 - 224 = -384 \text{ Kg -m}$$

Si consideramos como (y_m) la distancia que media entre (A) y el punto donde el esfuerzo de flexión es máxima ($M_{máx.}$) :

$$V = H_A - w_o y_m - W \cdot \frac{(2hy_m - y_m^2)}{h^2} = 0$$

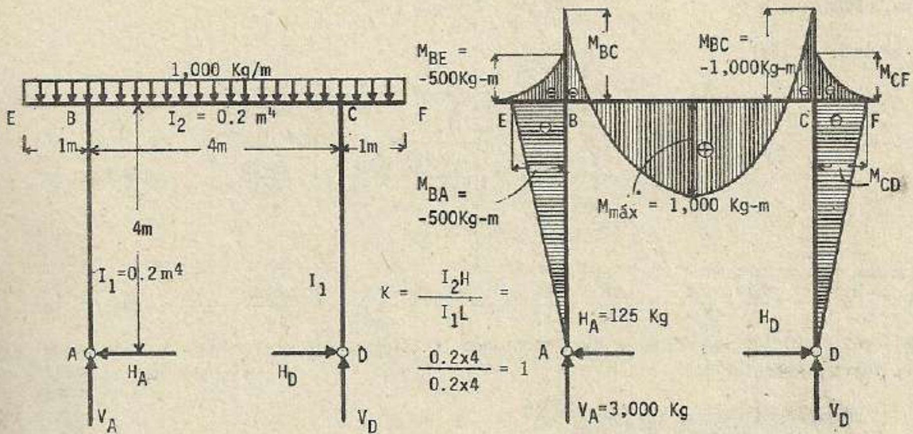
$$1{,}104 - 200y_m - 1{,}200\times\frac{(2.4.y_m - y_m^2)}{4\times 4} = 0$$

$$75y_m^2 - 800y_m + 1{,}104 = 0 \qquad y_m = 1.63\ \text{m.}$$

$$M_{max.} = H_A y_m - w_0\frac{y_m^2}{2} - \frac{Wy_m^2}{3h^2}\cdot(3h-y_m)$$

$$= 1{,}104\text{x}1.63 - 200\times\frac{1.63\text{x}1.63}{2} - \frac{1.200\text{x}1.63\text{x}1.63}{3\text{x}4\text{x}4}\times(3\text{x}4-1.63)$$

$$= 845.02\ \text{Kg.}$$



$$K = \frac{I_2H}{I_1L} = \frac{0.2\text{x}4}{0.2\text{x}4} = 1$$

$$V_A = V_D = \frac{w(L+2L_1)}{2} = \frac{1.000(4+2\text{x}1)}{2} = 3{,}000\text{Kg}$$

Carga en (B-C) - 142 -

$$H'_A=H'_D=\frac{wL^2}{4(3+2K)h} = \frac{1{,}000\text{x}4\text{x}4}{4(3+2)\text{x}4} = 200\text{Kg}$$

Carga por volado - 165 -

$$H''_A=H''_D = \frac{3wL_1^2}{2(2K+3)h} = \frac{3\text{x}1{,}000\text{x}1\text{x}1}{2(2+3)\times 4} = 75\text{Kg}$$

$\therefore\ H_A=H_D = 200 - 75 = 125$Kg Nota: tener cuidado con los signos.

$$M_{BE} = M_{CF} = -\frac{wL_1^2}{2} = -\frac{1{,}000\text{x}1\text{x}1}{2} = -500\text{Kg-m}$$

$$M_{BA} = M_{CD} = -H_Ah = -125\times 4 = -500\ \text{Kg-m}$$

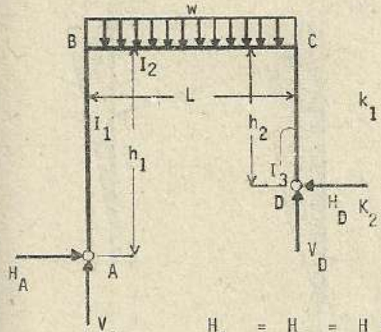
$$M_{BC} = M_{CB} = -500 - 500 = -1{,}000\text{Kg-m}$$

Máximo esfuerzo de flexión o doblado ($M_{máx.}$)

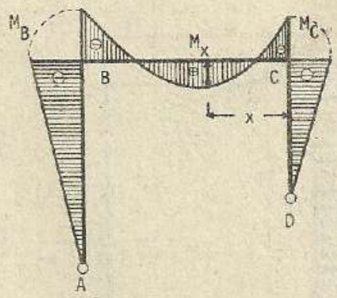
$$M_{máx.} = V_A\cdot\frac{L}{2} - H_Ah - \frac{w}{2}(L_1+\frac{L}{2})^2 = 3{,}000\times\frac{4}{2} - 125\times 4 - \frac{1{,}000}{2}(1+\frac{4}{2})^2$$

$$= 1{,}000\ \text{Kg-m}$$

(175)



$$k_1 = \frac{I_2 h_1}{I_1 L} \qquad K_2 = \frac{I_2 h_2}{I_3 L}$$



$$H_A = H_D = H = \frac{w h_1 L^2}{4\,(k_1 h_1^2 + K_2 h_2^2) + 6h_1\,(h_1 + h_2)}$$

$$V_A = \frac{wL}{2} + H \cdot \frac{h_1 - h_2}{L} \; ; \quad V_D = \frac{wL}{2} - H \cdot \frac{h_1 - h_2}{L} \; ; \quad M_B = -Hh_1 \; ; \quad M_C = -Hh_2$$

$$M_x = M_o - \left[M_C + \frac{x}{L} \cdot (M_B - M_C) \right] \quad \text{siendo} \quad M_o = \frac{wx}{2} \cdot (L - x)$$

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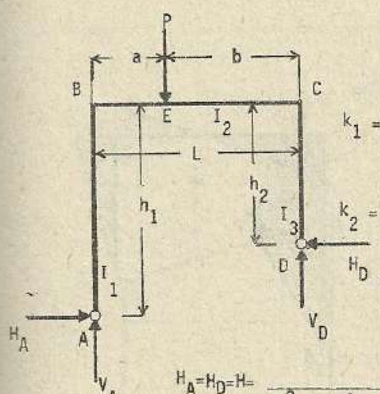
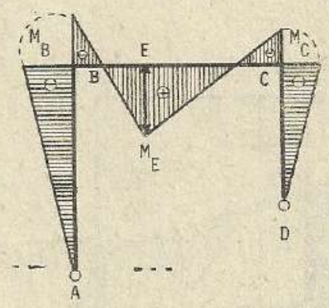
(176)



$$k_1 = \frac{I_2\,h_1}{I_1 L} \qquad k_2 = \frac{I_2\,h_2}{I_3 L}$$

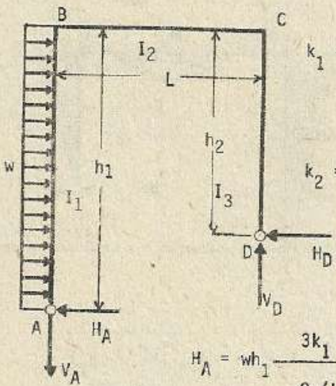


$$H_A = H_D = H = \frac{3abP\,(ah_1 + bh_2)}{2L^2\,(k_1 h_1^2 + k_2 h_2^2) + 3ah_1^2\,(L+a) + 3bh_2^2(L+a) + 3h_1 h_2\,(a^2 + b^2)}$$

$$V_A = P \cdot \frac{b}{L} + H\,\frac{h_1 - h_2}{L} \; ; \qquad V_D = P \cdot \frac{a}{L} - H\,\frac{h_1 - h_2}{L} ; \quad M_B = -Hh_1 \; ; \quad M_C = -Hh_2$$

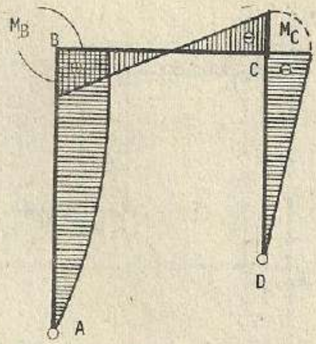
$$M_E = -V_A a - Hh_1 = V_D b - Hh_2$$

(177)



$$k_1 = \frac{I_2 h_1}{I_1 L}$$

$$k_2 = \frac{I_2 h_2}{I_3 L}$$



$$H_A = wh_1 \frac{3k_1 h_1^2 + 8k_2 h_2^2 + 6h_1(h_1 + 2h_2)}{8(k_1 h_1^2 + k_2 h_2^2) + 12h_1(h_1 + h_2)} ; \quad H_D = wh_1 - H_A$$

$$-V_A = V_D = \frac{wh_1\left(h_2 - \frac{h_1}{2}\right) + H_A(h_1 - h_2)}{L} ; \quad M_B = -H_A h_1 + \frac{wh_1^2}{2}$$

$$M_C = -H_D h_2$$

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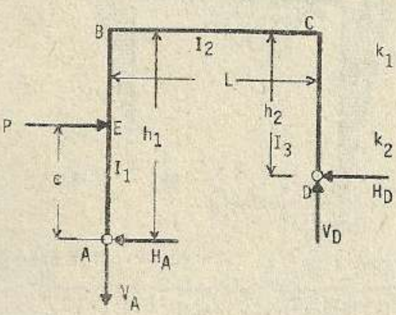
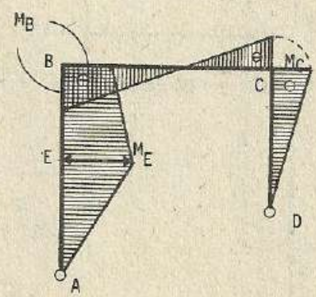
(178)



$$k_1 = \frac{I_2 h_1}{I_1 L}$$

$$k_2 = \frac{I_2 h_2}{I_3 L}$$

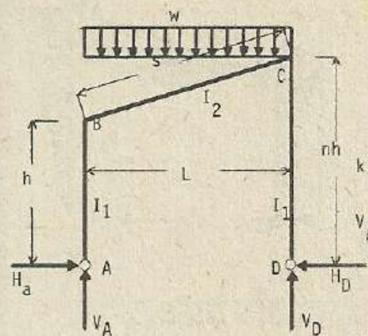


$$H_A = P \frac{k_1(2h_1^3 + c^3 - 3ch_1^2) + 2k_2 h_1 h_2^2 + 3k_1^2(h_1 - c + h_2)}{h_1^2(2k_1 + 3) + h_2(3k_1^2 + 2k_2 h_1 h_2)} ; \quad H_D = P - H_A$$

$$-V_A = V_D = \frac{P(h_2 + c - h_1) + H_A(h_1 - h_2)}{L}$$

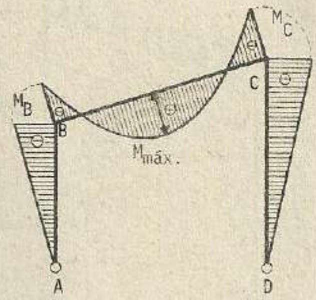
$$M_B = -H_A h_1 + P(h_1 - c) ; \quad M_C = -H_D h_2 ; \quad M_E = -H_A c$$

(179)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_D = \frac{wL}{2}$$



$$H_A = H_D = H = \frac{wL^2}{8h} \cdot \frac{1+n}{\{1+n+n^2+k\ (1+n^3)\}}$$

$$M_B = -Hh \quad ; \quad M = -nHh \quad ; \quad M_{máx.} = \frac{wL^2}{8} - Hh\left(\frac{1+n}{2}\right)$$

Cuando la carga uniformemente distribuida acciona perpendicularmente en la cara de proyección de la viga :

$$- \ V_A = V_D = \frac{wh^2\ (n^2-1)}{2L} \ ; \quad H_A\ \frac{wh}{8} \cdot \frac{\alpha}{\beta} \ ; \quad H_D = wh\ (n-1) - H_A ; \quad M_B = H_A h$$

$$M_C = -nh\ \{ wh\ (n-1) - H_A \} \qquad \alpha = 8\ n^3\ k\ (n-1) + 7n^3 - 2n^2 - 3n - 1$$

$$\beta = 1 + n + n^2 + k\ (1 + n^3)$$

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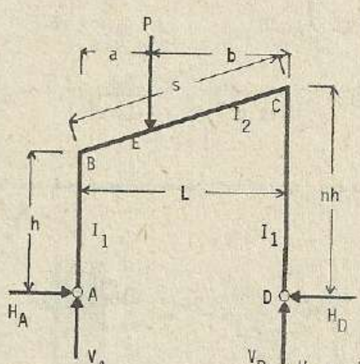
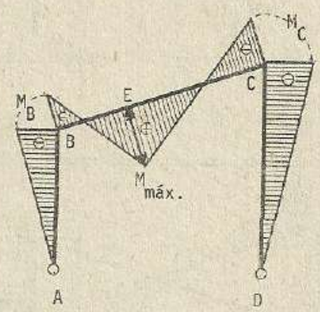
(180)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = \frac{Pb}{L}$$

$$V_D = \frac{Pa}{L}$$



$$H_A = H_D = H - \frac{Pab\ \{ L\ (2+n) + a\ (n-1) \}}{2hL^2\ \{1 + n + n^2 + k\ (1+n^3)\}}$$

$$M_{máx} = \frac{Pab}{L} - H\{ h + \frac{a}{L}\ (nh - h) \}$$

$$\text{Si } a = b = \frac{L}{2} \quad ; \quad V_A = V_D = \frac{P}{2} \quad ; \quad H_A = H_D = H = \frac{3PL}{16h} \cdot \frac{(1+n)}{\beta}$$

$$\beta = 1 + n + n^2 + k\ (1+n^3)$$

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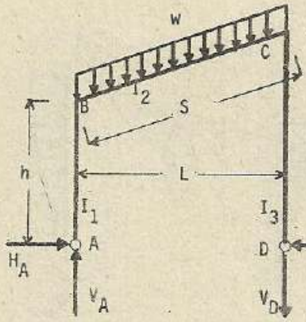
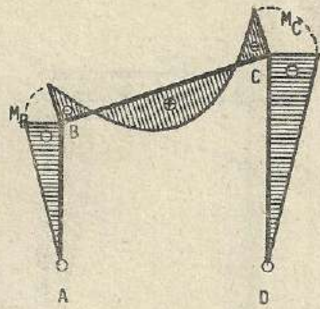


$$\lambda_1 = \frac{I_1}{I_2}$$

$$\lambda_2 = \frac{I_3}{I_2}$$

$$k' = \frac{I_2 nh}{I_3 S}$$



$$\gamma = \frac{1}{1 + n + n^2 + k + n^2\ k'}$$

$H_A = H_D = H = -\frac{\gamma}{8h}\ WL\ (1+n)$ W : Carga total

$M_B = Hh$; $M_C = Hhn$

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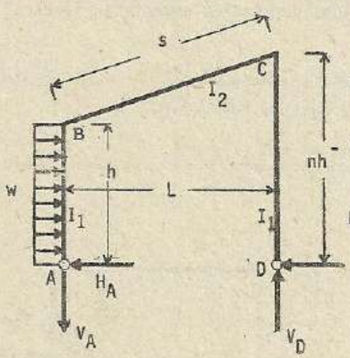
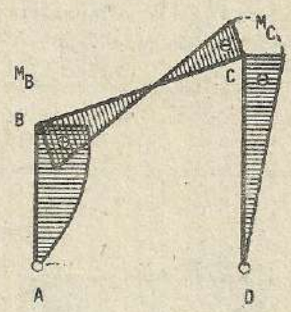
(182)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_D = \frac{wL}{2}$$

$$H_A = wh - H_D$$



$$H_D = \frac{wh}{8} \cdot \frac{(5k + 4 + 2n)}{\{1 + n + n^2 + k\ (1+n^3)\}}$$

$M_B = H_A\ h - \frac{wh^2}{2}$; $M_C = -nH_D h$

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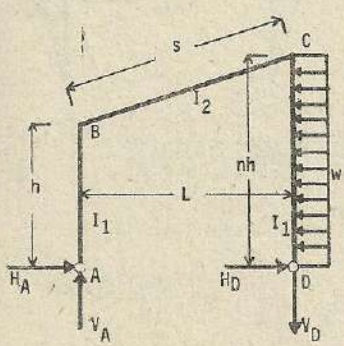
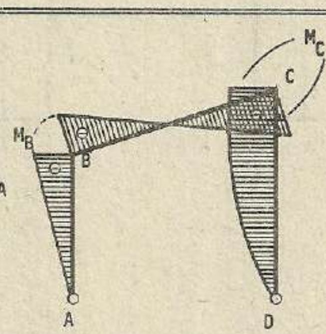
(183)



$$k = \frac{I_2 h}{I_1 s}$$

$$H_D = wnh - H_A$$

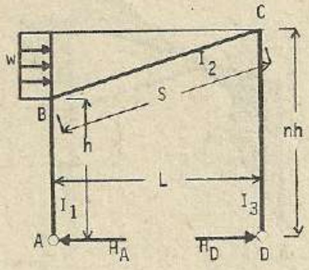
$$V_A = -V_D = \frac{wn^2h^2}{2L}$$



$$H_A = -\frac{wn^2h}{8}\ \frac{\{5n^2k + 2\ (1+2n)\}}{\{1 + n + n^2+k\ (1+n^3)\}}$$

$M_B = H_A h$; $M_C = VL - nH_A h$

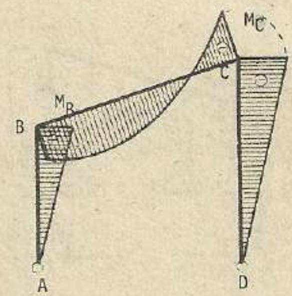
(184)



$\lambda_1 = \frac{I_1}{I_2}$

$\lambda_2 = \frac{I_3}{I_2}$

$k' = \frac{I_2 nh}{I_3 S}$



$\gamma = \frac{1}{1 + n + n^2 + k + n^2k'}$ $\qquad H = -\gamma W \left(\lambda_1 + \frac{7 + 4n + n^2}{8}\right)$

$H_A = W + H_D$; $M_B = (W + H_D)\,h$; $M_C = H_D\,nh$ $\qquad W$: Carga total

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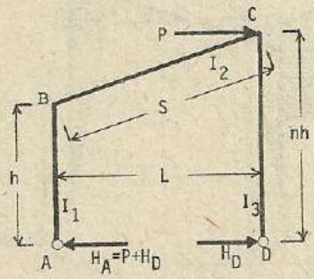
(185)



$\lambda_1 = \frac{I_1}{I_2}$; $\lambda_2 = \frac{I_3}{I_2}$

$\gamma = \frac{1}{1 + n + n^2 + k + n^2k'}$

$H_D = -\frac{\gamma}{2} P\,(2 + n + 2k')$; $H_A = P + H_D$

$M_B = (P + H_D)\,h$ $\qquad M_C = H_D\,nh$

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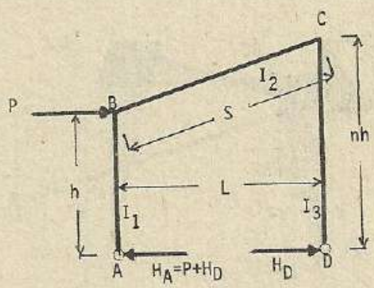
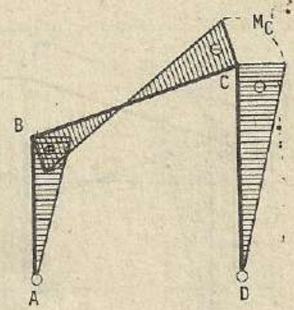
(186)



$\lambda_1 = \frac{I_1}{I_2}$

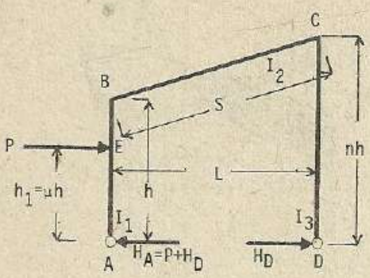
$\lambda_2 = \frac{I_3}{I_2}$



$\gamma = \frac{1}{1 + n + n^2 + k + n^2k'}$ $\qquad H_D = -\frac{\gamma}{2} P\,(2 + n + 2k')$

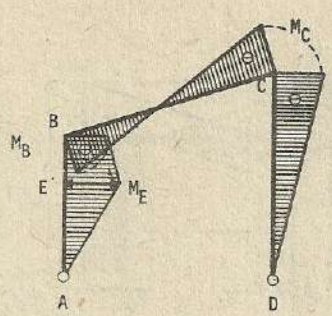
$H_A = P + H_D$ $\qquad M_B = (P + H_D)\,h$ $\qquad M_C = H_D\,nh$

(187)



$$\lambda_1 = \frac{I_1}{I_2}$$

$$\lambda_2 = \frac{I_3}{I_2}$$



$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

$$H_D = -\frac{\gamma}{2} P \mu \{ k (\mu^2 - 3\mu + 4) + n + 2 \} \quad ; \quad H_A = P + H_D$$

$$M_B = (P\mu + H_D)\, h \quad ; \quad M_C = H_D \cdot nh$$

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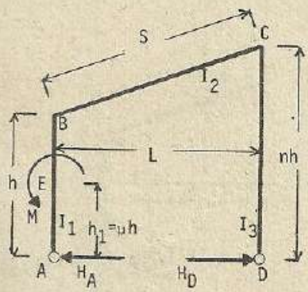
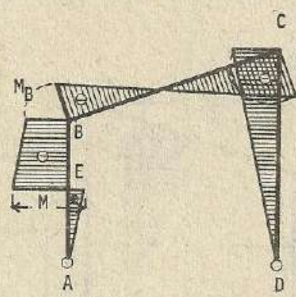


$$\lambda_1 = \frac{I_1}{I_2}$$

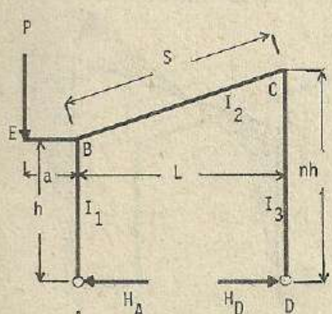
$$\lambda_2 = \frac{I_3}{I_2}$$



$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

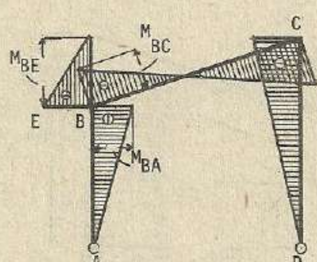
$$H_A = H_D = H = \frac{\gamma}{h} M (2 + n + 3k - 3k'\mu^2) \quad ; \quad M_B = Hh - M \quad ; \quad M_C = Hnh$$

(189)



$$\lambda_1 = \frac{I_1}{I_2}$$

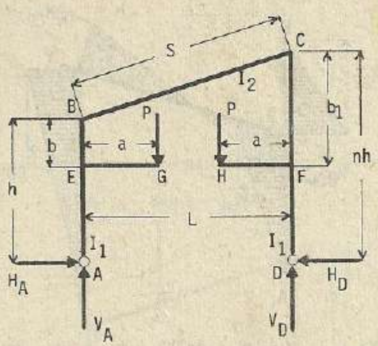
$$\lambda_2 = \frac{I_3}{I_2}$$



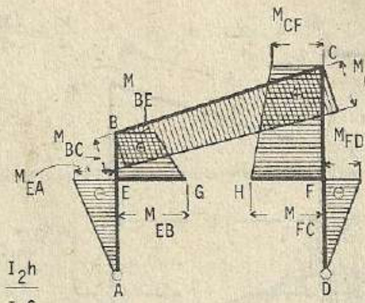
$$\gamma = \frac{1}{1 + n + n^2 + k + n^2 k'}$$

$$H_A = H_D = H = \frac{\gamma}{2h} Pa (2 + n) \quad ; \quad M_{BA} = Hh \quad ; \quad M_{BC} = Hh - Pa \quad ; \quad M_C = Hnh$$

(190)



$$k = \frac{I_2 h}{I_1 S}$$



$$V_A = V_D = P$$

$$H_A = H_D = H = \frac{3Pa}{2h^3} \cdot \frac{k\,(2bh - b^2 + 2b_1\,nh - b_1^2) + h^2\,(1+n)}{1 + n + n^2 + (1+n^3)\,k}$$

$M_{BC} = M_{BA} = -Hh + Pa$; $M_{CB} = M_{CD} = -Hnh + Pa$; $M_{EA} = M_{FD} = -H\,(h-b)$

$M_{EB} = M_{FC} = -H\,(h-b) + Pa$

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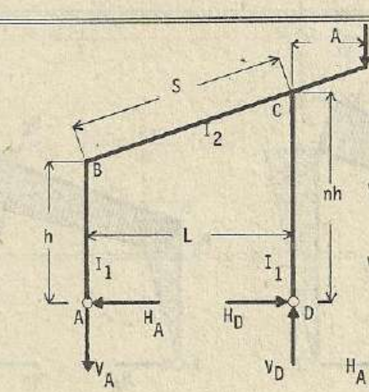
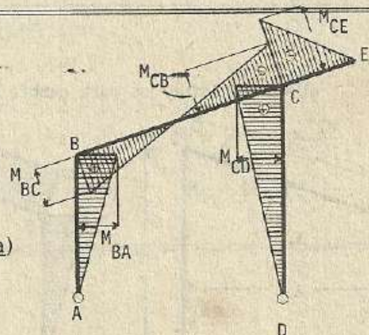
(191)



$$k = \frac{I_2 h}{I_1 S}$$

$$V_A = \frac{Pa}{L}$$

$$V_D = \frac{P\,(L+a)}{L}$$



$$H_A = H_D = H = \frac{Pa}{2h} \cdot \frac{1 + 2n}{1 + n + n^2 + (1+n^3)\,k}$$

$M_{CD} = Hnh$; $M_{CB} = Hnh - Pa$; $M_{BA} = M_{BC} = +Hh$

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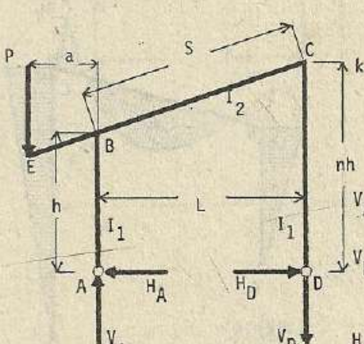
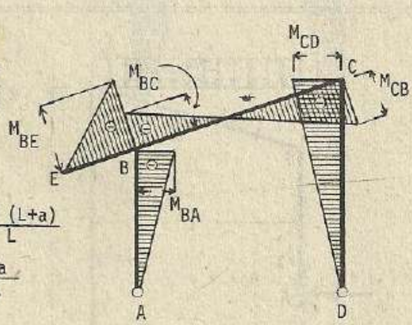
(192)



$$k = \frac{I_2 h}{I_1 S}$$

$$V_A = \frac{P\,(L+a)}{L}$$

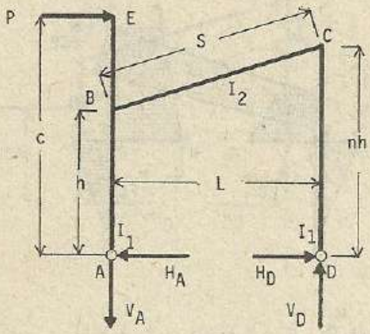
$$V_D = \frac{Pa}{L}$$



$$H_A = H_D = H = \frac{Pa}{2h} \cdot \frac{2 + n}{1 + n + n^2 + (1+n^3)\,k}$$

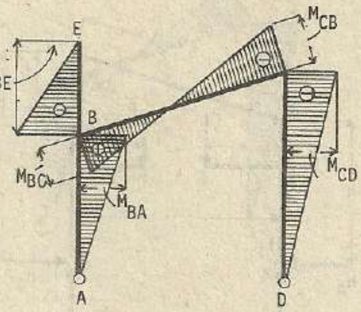
$M_{BA} = Hh$; $M_{BC} = Hh - Pa$; $M_{CB} = M_{CD} = Hnh$

(193)



$$k = \frac{I_2 h}{I_1 S}$$

$$H_D = P - H_A$$



$$H_A = \frac{Pc}{2h} \cdot \frac{n\,(2nh + 2n^2hk + c) + 2c - 2h}{1 + n + n^2 + k\,(1+n^3)}$$

$$V_A = V_D = V = \frac{Pc}{L} \quad ; \quad M_{BA} = H_A\,h \quad ; \quad M_{BC} = VL - (P-H_A)\,h$$

$$M_{CB} = M_{CD} = -\,(P - H_A)nh \quad ; \quad M_{BE} = -\,(c-h)\,P$$

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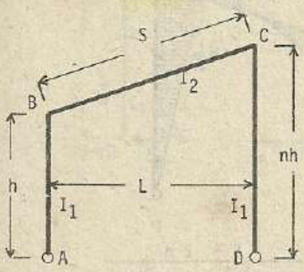
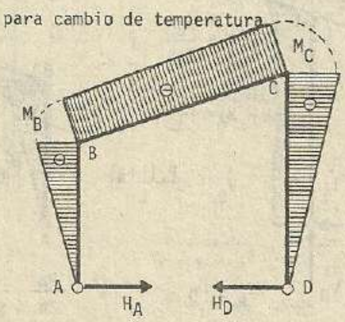
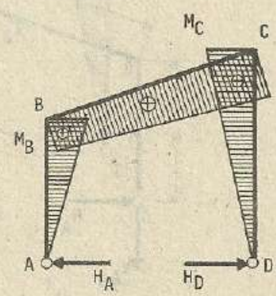
(194) : Esfuerzos para cambio de temperatura





Por aumento de temperatura



Por descenso

$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_D = H = \frac{3E\varepsilon I_2\,tL}{h^2 S\,\{k\,(n^2+1) + (n^2+n+1)\}} \quad ; \quad M_B = Hh \quad ; \quad M_C = Hnh$$

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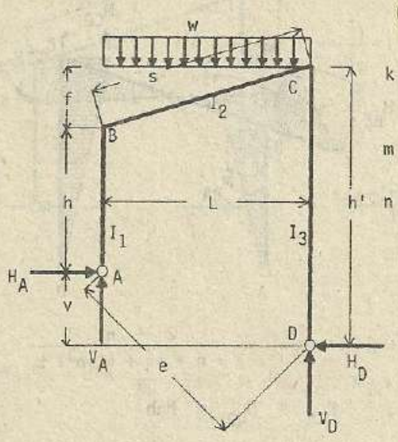
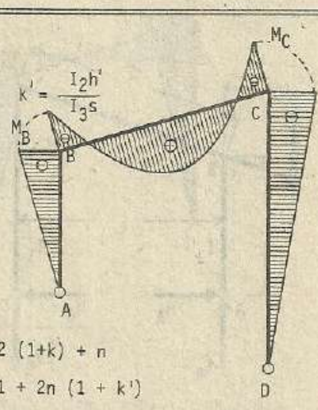
(195)



$$k = \frac{I_2 h}{I_1 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$



$$\beta = 2\,(1+k) + n$$

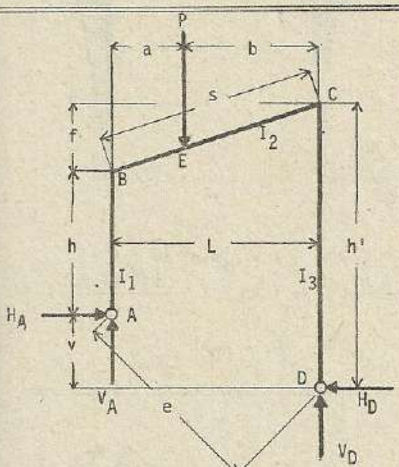
$$\gamma = 1 + 2n\,(1 + k')$$

$$N = \beta + n\gamma = 2\,\{1 + k + n + n^2\,(1 + k')\}$$

$$H_A = H_D = H = \frac{(1+n)\,wL^2}{4\,Nh} \quad ; \quad V_A = \frac{wL}{2} - \frac{Hv}{L} \quad ; \quad V_D = \frac{wL}{2} + \frac{Hv}{L}$$

$$M_B = -\,Hh \quad ; \quad M_C = -\,Hh'$$

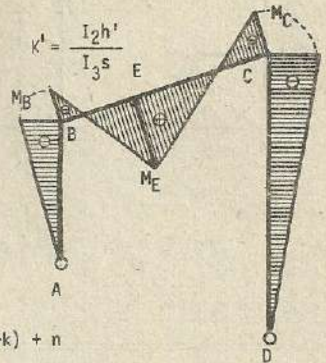
(196)



$$k = \frac{I_2 h}{I_1 s} \qquad K' = \frac{I_2 h'}{I_3 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$



$$\beta = 2\ (1+k) + n$$

$$\gamma = 1 + 2n\ (1+k')$$

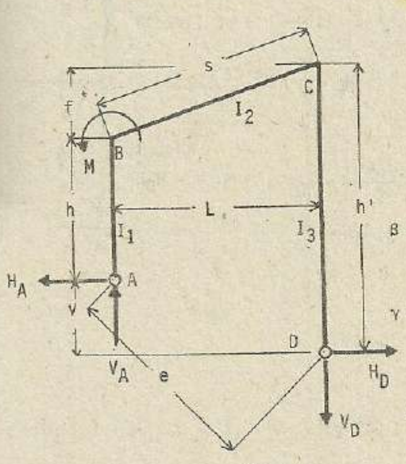
$$N = \beta + n\gamma = 2\ \{1 + k + n + n^2\ (1+k')\}$$

$$H_{BC} = -\frac{Pab}{2L^2}\,(L+b) \quad ; \quad H_{CB} = -\frac{Pab}{2L^2}\,(L+a) \quad ; \quad H_A - H_D - H = -\,2\,\frac{H_{BC} + H_{CB}}{Nh}$$

$$V_A = \frac{Pb - Hv}{L} \quad ; \quad V_D = P - V_A \quad ; \quad M_B = -\,Hh \quad ; \quad M_C = -\,Hh'$$

(197)

$$k = \frac{I_2 h}{I_1 s} \quad ; \quad k' = \frac{I_2 h'}{I_3 s}$$

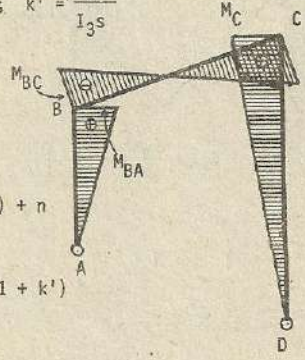


$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$

$$\beta = 2\ (1 + k) + n$$

$$\gamma = 1 + 2n\ (1 + k')$$

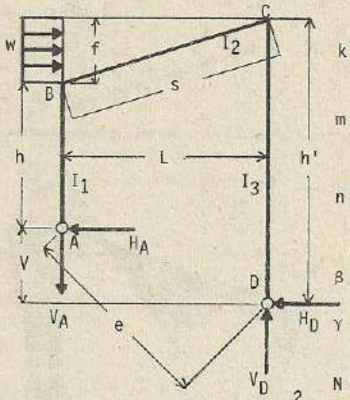


$$N = \beta + n\gamma = 2\ \{1 + k + n + n^2\ (1 + k')\}$$

$$H_A = H_D = H = \frac{(2+n)\ M}{Nh} \qquad V_A = V_D = V = \frac{M + Hv}{L}$$

$$M_{BA} = Hh \qquad M_{BC} = M_{BA} - M \qquad M_C = Hh'$$

(198)



$$k = \frac{I_2 h}{I_1 s}$$

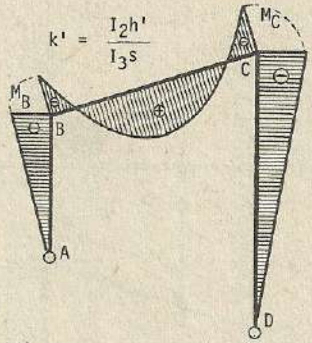
$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$

$$\beta = 2\,(1+k) + n$$

$$\gamma = 1 + 2n\,(1 + k')$$

$$k' = \frac{I_2 h'}{I_3 s}$$



$$N = \beta + n\gamma = 2\,\{1 + k + n + n^2\,(1 + k')\}$$

$$H_A = wf - H_D \quad ; \quad H_D = \frac{wf^2}{4h} \cdot \frac{4\,m\,\beta + 1 + N}{N} \quad ; \quad V_A = V_D = V = \frac{wf^2\,(1+2m)}{2\,L} + \frac{H_D v}{L}$$

$$M_B = (wf - H_D)\,h \quad ; \quad M_C = -H_D h'$$

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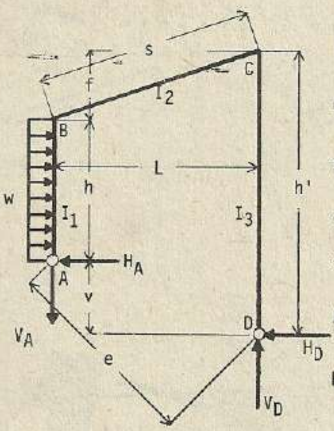
(199)



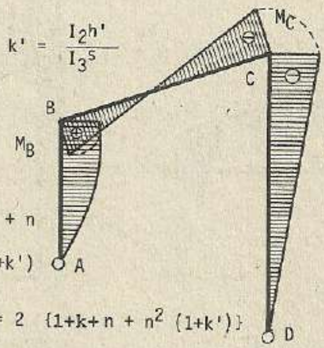
$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$

$$\beta = 2\,(1 + k) + n$$

$$\gamma = 1 + 2n\,(1+k')$$

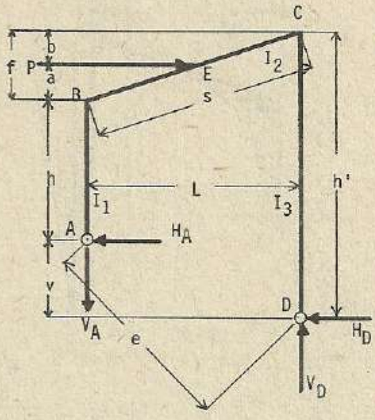


$$N = \beta + n\,\gamma = 2\,\{1+k+n + n^2\,(1+k')\}$$

$$H_A = wh - H_D \qquad H_D = \frac{wh\,(2\beta + k)}{4N}$$

$$V_A = V_D = v = \frac{wh^2}{2L} + \frac{H_D v}{L} \quad ; \quad M_B = \frac{wh^2}{2} - H_D h \; ; \quad M_C = -H_D h'$$

(200)



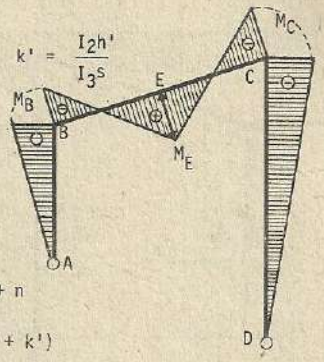
$$k = \frac{I_2 h}{I_1 s} \quad ; \quad k' = \frac{I_2 h'}{I_3 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$

$$\beta = 2\,(1+k) + n$$

$$\gamma = 1 + 2n\,(1 + k')$$



$$N = \beta + n\gamma = 2\,\{1 + k + n + n^2\,(1+k')\}$$

$$H_{BC} = -\frac{Pab}{2f^2}\,(f+b) \quad ; \quad H_{CB} = -\frac{Pab}{2f^2}\,(f+a)$$

$$H_A = P - H_D \quad ; \quad H_D = \frac{\beta P}{N} - 2\,\frac{H_{BC} + nH_{CB}}{Nh} \quad ; \quad V_A = V_D = \frac{P\,(h+a) + H_D v}{L}$$

$$M_B = (P - H_D)\,h \qquad M_C = -H_D h'$$

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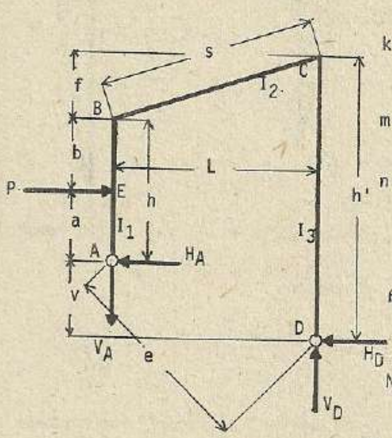
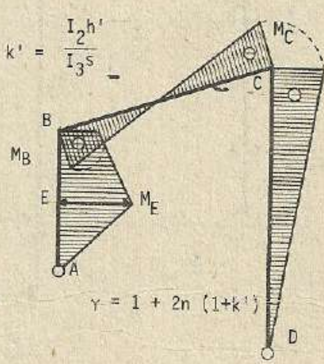
(201)



$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$



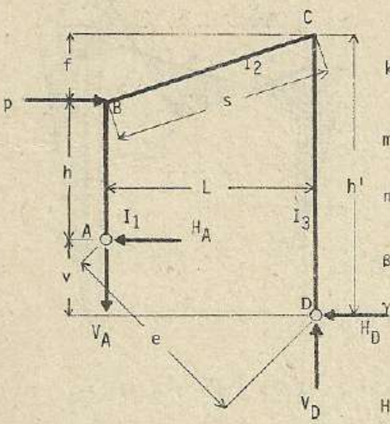
$$\beta = 2\,(1+k) + n \qquad \gamma = 1 + 2n\,(1+k')$$

$$N = \beta + n\gamma = 2 \times \{1 + k + n + n^2\,(1 + k')\}$$

$$H_{BA} = -\frac{Pab}{2h^2}\,(h + b) \qquad H_D = \frac{\beta\,Pa - 2\,k\,H_{BA}}{Nh}$$

$$H_A = P - H_D \quad ; \quad V_A = V_D = V = \frac{Pa + H_D v}{L} \quad ; \quad M_B = Pa - H_D h \quad ; \quad M_C = -H_D h'$$

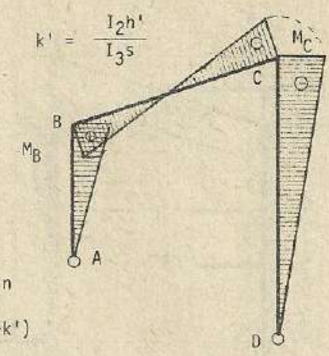
(202)



$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$



$$\beta = 2\,(1+k) + n$$

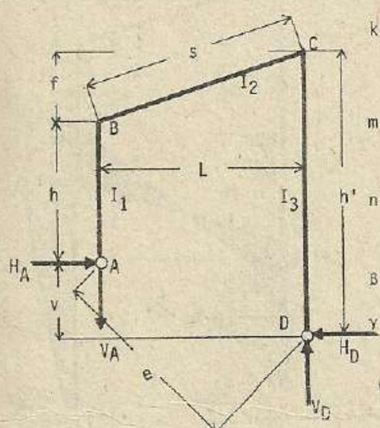
$$\gamma = 1 + 2n\,(1+k')$$

$$N = \beta + n\gamma = 2\,\{1 + k + n + n^2\,(1+k')\}$$

$$H_D = \frac{\beta P}{N} \;;\quad H_A = P - H_D \;;\quad V_A = V_D = \frac{Ph + H_D v}{L}$$

$$M_B = (P - H_D)\,h \qquad M_C = -\,H_D h'$$

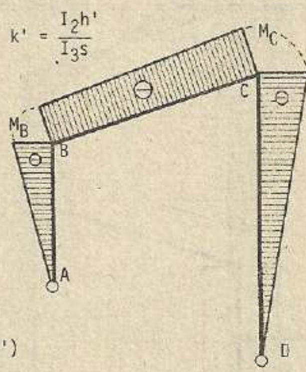
Esfuerzo por cambio de temperatura (203)



$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$

$$m = \frac{h}{f}$$

$$n = \frac{h'}{h}$$



$$\beta = 2\,(1+k) + n$$

$$\gamma = 1 + 2n\,(1+k')$$

$$N = \beta + n\gamma = 2\,\{1 + k + n + n^2\,(1+k')\}$$

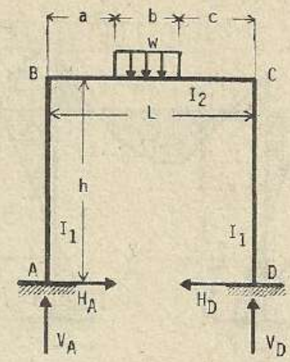
$$H_A = H_D = H = \frac{T}{N}\;\frac{L^2 + v^2}{L^2} \;;\quad T = \frac{6E\varepsilon I_2 tL}{sh^2}$$

$$V_A = V_D = \frac{Hv}{L}$$

$$M_B = -\,Hh \qquad M_C = -\,Hh'$$

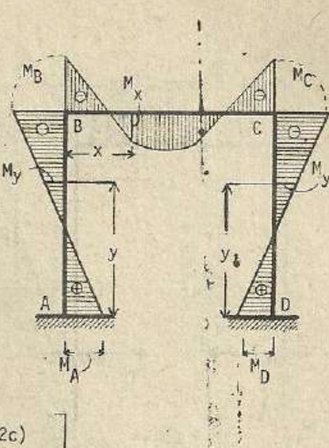
Nota :

El presemte problema corrresponde al caso de aumento de temperatura.
En caso de descenso, los esfuerzos se anotan con los signos contrarios.



(204)

$$k = \frac{I_2 h}{I_1 L}$$



$$V_A = \frac{\left[3\,abc + (3L-2c)c^2 + (L+c-\frac{b}{2})b^2 + 3kL^2\,(b+2c)\right]}{(6k+1)L^3}\,wb$$

$$V_D = \frac{\left[3abc + (3L - 2a)a^2 + (L+a - \frac{b}{2})b^2 + 3kL^2\,(b+2a)\right]}{(6k+1)L^3}\,wb$$

$$H_A = H_D = \frac{(6ac + 3bL-2b^2)}{4\,(k+2)hL}\,wb$$

$$M_A = \frac{wb}{12L^2(k+2)(6k+1)}\left[6a(3cL+2ab-4c^2) + b(10bL-3L^2-6b^2) + 6ka(7cL+ab-2c^2)\right.$$
$$\left. + 3bk(5L^2 - 2bL-b^2)\right]$$

$$M_D = \frac{wb}{12L^2(k+2)(6k+1)}\left[6c(3aL+2cb-4a^2) + b\,(10bL-3L^2-6b^2) + 6kc(7aL+bc-2a^2)\right.$$
$$\left. + 3bk\,(5L^2 - 2bL-b^2)\right]$$

$M_B = M_A - H_A h$; $M_C = M_D - H_D h$

Momento por flexión en la columna AB : $M_y = M_A - H_A y$

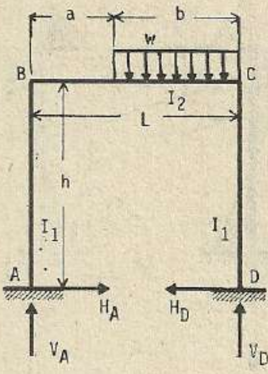
columna CD : $M_y = M_D - H_D y$

Momento por flexión en la viga :

$M_x = M_A - H_A h + V_A x$ $\qquad x \leqslant a$

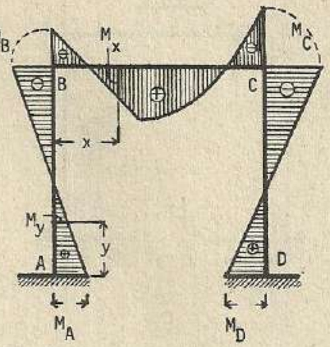
$M_x = M_A - H_A h + V_A x - \dfrac{w(x-a)^2}{2}$ $\qquad a \leqslant x \leqslant a+b$

$M_x = M_A - H_A h + V_A x - \dfrac{wb(2x-2a-b)}{2}$ $\qquad x \geqslant a+b$



(205)

$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{3L-2b}{4\,(k+2)\,hL} \cdot wb^2 \qquad V_A = \frac{(L - \frac{b}{2})\,b + 3kL^2}{(6k+1)\;L^3}\; wb^2$$

$$V_D = \frac{(3L-2a)a^2 + (L+a-\frac{b}{2})b^2 + 3kL^2\,(b+2a)}{(6k+1)L^3} \cdot wb$$

$$M_A = \frac{(9a^2 + 4ab + b^2) + 3k\,(7a^2 + 2b^2 + 8ab)}{12\,L^2\,(k+2)\,(6k+1)} \cdot wb^2 \qquad M_B = M_A - H_A h$$

$$M_D = \frac{(10\,bL - 3L^2 - 6b^2) + 3k\,(5a^2 + 2b^2 + 8ab)}{12L^2\,(k+2)\,(6k+1)} \cdot wb^2 \qquad M_C = M_D - H_D h$$

Momento de flexión en la columna AB : $M_y = M_A - H_A y$

columna CD : $M_y = M_D - H_D y$

Momento de flexión en la viga :

$$M_x = M_A - H_A h + V_A x \qquad x \leq a$$

$$M_x = M_A - H_A h + V_A x - \frac{w(x-a)^2}{2} \qquad x \geq a$$

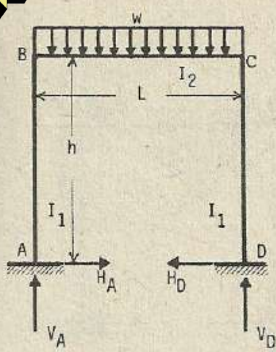
Si $a = b = \frac{L}{2}$ $\qquad V_D = \frac{(72k+13)}{32\,(6k+1)} \cdot wL \qquad V_A = \frac{3\,(8k+1)}{32\,(6k+1)} \cdot wL$

$$H_A = H_D = \frac{1}{8\,(k+2)\,h} \cdot wL^2 \qquad M_A = \frac{51k+14}{192(k+2)(6k+1)} \cdot wL^2$$

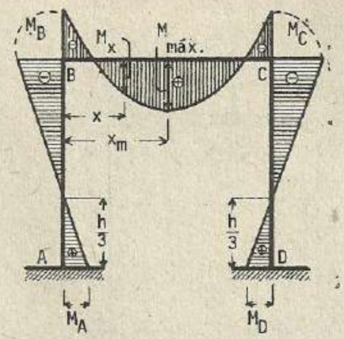
$$M_D = \frac{45k+2}{192\,(k+2)\,(6k+1)} \cdot wL^2 \qquad M_B = -\frac{(93k+10)}{192(k+2)(6k+1)} \cdot wL^2$$

$$M_C = -\frac{(99k+22)}{192\,(k+2)(6k+1)} \cdot wL^2$$

(206)



$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{wL^2}{4\,(k+2)h} \quad ; \quad V_A = V_D = \frac{wL}{2} \quad ; \quad M_A = M_D = \frac{wL^2}{12(k+2)} \quad ; \quad M_B = M_C = -\frac{wL^2}{6(k+2)}$$

Momento por flexión en la columna :

$$M_y = \frac{wL^2}{12\,(k+2)} - \frac{wL^2 y}{4\,(k+2)\,h}$$

Momento por flexión en la viga :

$$M_x = -\frac{wL^2}{6\,(k+2)} + \frac{wL}{2}x - \frac{wx^2}{2}$$

$$x_m = \frac{L}{2} \qquad M_{máx.} = \frac{(3k+2)}{24\,(k+2)} \cdot wL^2$$

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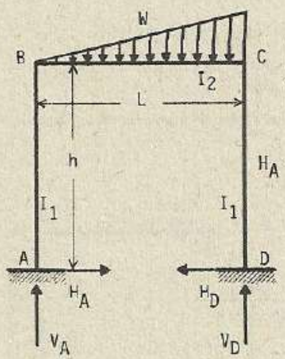
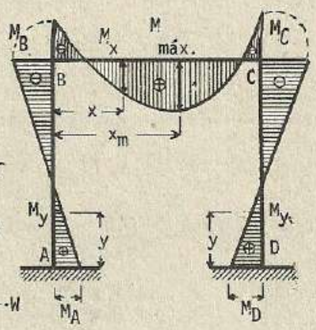


$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{WL}{4\,(k+2)\,h}$$

$$V_A = \frac{(20k+3)}{10(6k+1)} \cdot W$$

$$V_D = \frac{(40k+7)}{10\,(6k+1)} \cdot W$$

$$M_A = \frac{(31k+7)}{60\,(k+2)\,(6k+1)} \cdot WL$$

$$M_D = \frac{(29k+3)}{60\,(k+2)\,(6k+1)} \cdot WL \qquad M_B = -\frac{(59k+8)}{60\,(k+2)\,(6k+1)} \cdot WL$$

$$M_C = -\frac{(61k+12)}{60\,(k+2)\,(6k+1)} \cdot WL$$

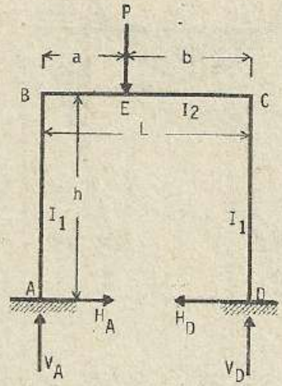
Esfuerzo de momento por flexión en la columna AB :

$$M_y = \frac{(31k+7)WL}{60\,(k+2)\,(6k+1)} - \frac{WLy}{4(k+2)h}$$

Columna DC : $$M_y = \frac{(29k+3)WL}{60\,(k+2)(6k+1)} - \frac{WLy}{4\,(k+2)h}$$

Viga BC : $$M_x = -\frac{(59k+8)}{60\,(k+2)(6k+1)}WL + \frac{(20k+3)}{10\,(6k+1)}Wx - \frac{Wx^3}{3L^2}$$

$$x_m = L \cdot \sqrt{\frac{2k+0.3}{6k+1}}$$



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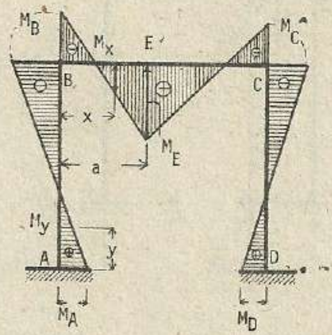
$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{3Pab}{2(k+2)hL} \quad ; \quad V_A = \frac{Pb}{L} + \frac{Pab(L-2a)}{(6k+1)L^3} \; ; \; V_D = \frac{Pa\,(6kL^2 + 3aL - 2a^2)}{L^3\,(6k+1)}$$

$$M_A = \frac{(5k-1)L + 2a(k+2)}{2\,(k+2)\,(6k+1)\,L^2}\,Pab \; ; \qquad M_D = \frac{(7k+3)L - 2a\,(k+2)}{2\,(k+2)\,(6k+1)\,L^2}\cdot Pab$$

$$M_B = -\frac{(13k+4)L - 2a(k+2)}{2\,(k+2)(6k+1)\,L^2}\cdot Pab \; ; \qquad M_C = -\frac{11kL + 2a\,(k+2)}{2\,(k+2)\,(6k+1)\,L^2}\cdot Pab$$

Momento por flexión en la columna AB ;

$$M_y = \frac{(5k-1)L - 2a(k+2)}{2\,(k+2)\,(6k+1)^2}\cdot Pab - \frac{3Paby}{2(k+2)hL}$$

Columna CD :

$$M_y = \frac{(7k+3)L - 2a(k+2)}{2\,(k+2)\,(6k+1)\,L^2}\,Pab - \frac{3Paby}{2(k+2)hL}$$

Viga BC :

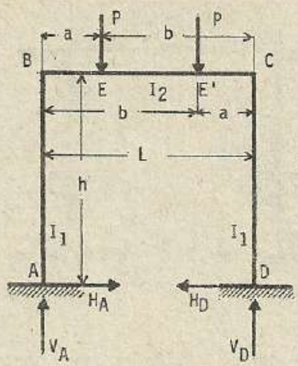
$$M_x = -\frac{(13k+4)L + 2a(k+2)}{2(k+2)\,(6k+1)\,L^2}\cdot Pab + \frac{Pb}{L}\,x + \frac{Pab(L-2a)x}{(6k+1)\,L^3} \quad — \quad x \leq a$$

$$M_x = -\frac{(13k+4)L + 2a(k+2)}{2\,(k+2)\,(6k+1)\,L^2}\,Pab + \frac{Pb}{L}\,x + \frac{Pab\,(L-2a)x}{(6k+1)L^3} - P\,(x-a) \quad — \quad x \geq a$$

$$M_E = -\frac{(13k+4)L + 2a(k+2)}{2\,(k+2)\,(6k+1)L^2}\,Pab + \frac{Pab}{L} + \frac{Pa^2b\,(L-2a)}{(6\,k+1)\,L^3}$$

Si $a = b = \frac{L}{2}$: $H_A = H_D = \frac{3PL}{8\,(k+2)h}$; $V_A = V_D = \frac{P}{2}$

$M_A = M_D = \frac{PL}{8\,(k+2)}$; $M_B = M_C = -\frac{PL}{4(k+2)}$; $M_E = \frac{(k+1)}{4\,(k+2)}\cdot PL$



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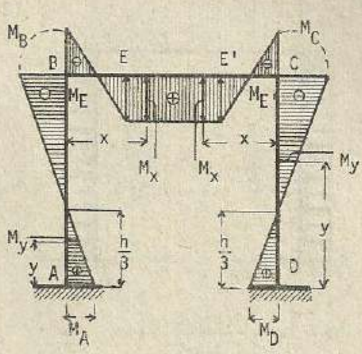
$$k = \frac{I_2 h}{I_1 L}$$



$$H_A = H_D = \frac{3Pab}{(k+2)\ hL} \qquad V_A = V_D = P \qquad M_A = M_D = \frac{Pab}{(k+2)L}\ ;\ M_B = M_C = -\frac{2Pab}{(k+2)L}$$

$$M_E = Pa - \frac{2Pab}{(k+2)L}\ ;$$ Momento por flexión en la columna :

$$M_y = \frac{Pab}{(k+2)\ L} - \frac{3Paby}{(k+2)\ hL}$$

En la viga : $M_x = -\dfrac{2Pab}{(k+2)L} + Px \qquad x \leq a$

Esfuerzo de momento por flexión entre las cargas : $M_x = Pa - \dfrac{2Pab}{(k+2)\ L}$ (constante)

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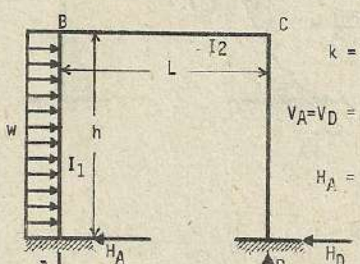
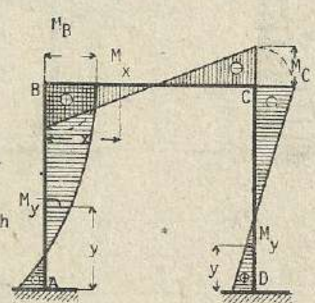


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{kwh^2}{(6k+1)L}$$

$$H_A = \frac{(2k+3)}{8\ (k+2)}\cdot wh$$



$$H_D = \frac{(6k+13)}{8(k+2)}\cdot wh \qquad M_A = -\frac{(30k^2+73k+15)}{24(k+2)\ (6k+1)}\cdot wh^2$$

$$M_D = \frac{(18k^2+35k+9)}{24\ (k+2)\ (6k+1)}\cdot wh^2\ ;\quad M_B = \frac{k\ (6k+23)}{24\ (k+2)(6k+1)}wh^2 \qquad M_C = -\frac{k\ (18k+25)}{24(k+2)(6k+1)}\cdot wh^2$$

Esfuerzo de momento por flexión en la columna AB :

$$M_y = -\frac{(30k^2+73k+15)}{24(k+2)(6k+1)}wh^2 + \frac{6k+13}{8(k+2)}\cdot why - \frac{wy^2}{2}$$

Columna CD : $M_y = \dfrac{(18k^2+35k+9)}{24(k+2)(6k+1)}\cdot wh^2 - \dfrac{(2k+3)}{8(k+2)}\cdot wh\ y$

Viga BC : $M_x = \dfrac{k\ (6k+23)}{24\ (k+2)(6k+1)}wh^2 - \dfrac{k}{(6k+1)\ L}\cdot wh^2 x$

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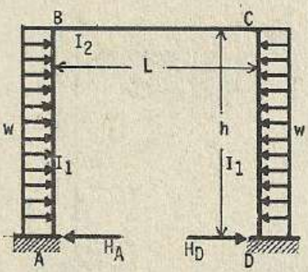
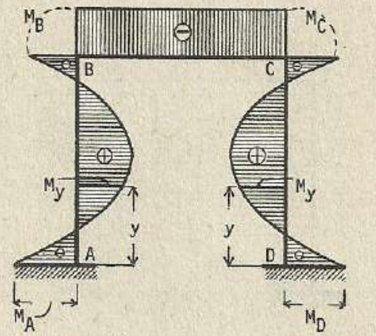


$k = \frac{I_2 H}{I_1 L}$

$V_A = V_D = 0$



$H_A = H_D = \frac{(2k+5)}{4(k+2)} \cdot wh$

$M_A = M_D = -\frac{(k+3)}{12(k+2)} \cdot wh^2$; $M_B = M_C = -\frac{k}{12\ (k+2)} wh^2$

Esfuerzo de flexión en la columna : $M_y = -\frac{(k+3)}{12(k+2)} \cdot wh^2 + \frac{(2k+5)}{4(k+2)} \cdot why - \frac{wy^2}{2}$

Esfuerzo de flexión en la viga : $M_x = -\frac{k}{12(k+2)} wh^2$

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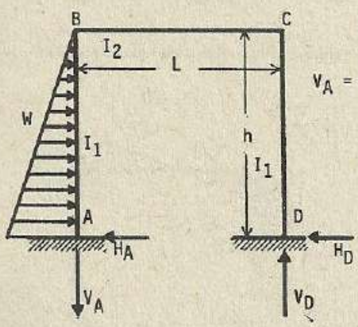
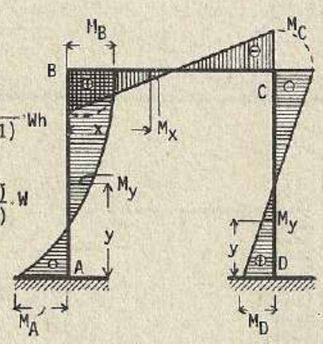
(212)



$k = \frac{I_2 h}{I_1 L}$

$V_A = V_D = \frac{k}{2(6k+1)} Wh$

$H_A = \frac{(17k+36)}{20(k+2)} W$



$H_D = \frac{(3k+4)}{20(k+2)} \cdot W$

$M_A = -\frac{(63k^2+151k+28)}{60(k+2)(6k+1)} \cdot Wh$; $M_D = \frac{(27k^2+49k+12)}{60(k+2)(6k+1)} \cdot Wh$; $M_B = \frac{k\ (3k+28)}{60(k+2)(6k+1)} \cdot Wh$

$M_C = -\frac{k\ (27k+32)}{60(k+2)\ (6k+1)} \cdot Wh$

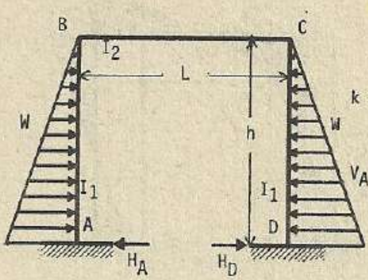
Momento por flexión en la columna AB :

$M_y = -\frac{(63k^2+151k+28)}{60(k+2)\ (6k+1)} \cdot Wh + \frac{(17k+36)}{20(k+2)} \cdot Wy - \frac{Wy^2}{3h^2} \cdot (3h-y)$

Columna CD : $M_y = \frac{(27k^2+49k+12)}{60\ (k+2)\ (6k+1)} \cdot Wh - \frac{(3k+4)}{20\ (k+2)} \cdot Wy$

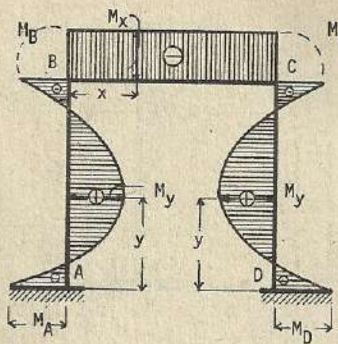
Viga BC : $M_x = \frac{k\ (3k+28)}{60\ (k+2)\ (6k+1)} \cdot Wh - \frac{k}{2(6k+1)} \cdot Whx$

(213)



$k = \frac{I_2 h}{I_1 L}$

$V_A = V_D = 0$



$H_A = H_D = \frac{(7k+16)}{10\ (k+2)} \cdot W$; $M_A = M_D = -\frac{(3k+8)}{30(k+2)} \cdot Wh$; $M_B = M_C = -\frac{k}{15\ (k+2)} \cdot Wh$

Esfuerzo de momento por flexión en la columna :

$M_y = -\frac{(3k+8)}{30(k+2)} \cdot Wh + \frac{(7k+16)}{10(k+2)} \cdot Wy - \frac{Wy^2}{3h^2} \cdot (3h-y)$

En la viga : $M_x = -\frac{k}{15(k+2)} \cdot Wh$

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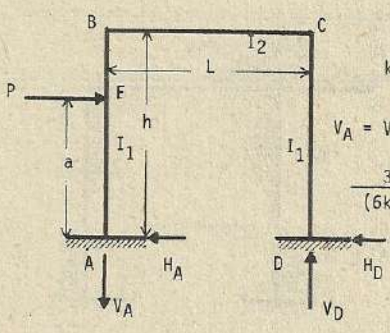
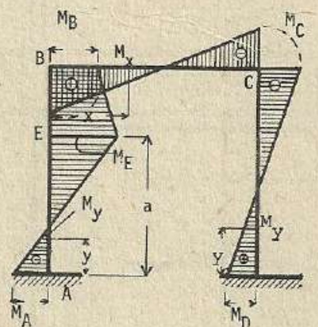
(214)



$k = \frac{I_2 h}{I_1 L}$

$V_A = V_D = \frac{3a^2 k}{(6k+1)hL} P$



$H_A = \frac{2\ (k+2)h^3 - a^2[3h\ (k+1) - a(2k+1)]}{2(k+2)h^3} \cdot P$; $H_D = \frac{a^2[3k(k+1) - a\ (2k+1)]}{2\ (k+2)h^3} \cdot P$

$M_A = -Pa + \frac{[(15k^2+26k+3)h - a\ (k+1)\ (6k+1)]}{2\ (k+2)\ (6k+1)\ h^2} \cdot Pa^2$

$M_D = \frac{[(9k^2+14k+3)h - a\ (k+1)\ (6k+1)]}{2\ (k+2)\ (6k+1)h^2}\ Pa^2$; $M_B = \frac{k\ [a(6k+1) - h\ (3k-5)]}{2(k+2)\ (6k+1)\ h^2} \cdot Pa^2$

$M_C = -\frac{k[(9k+7)h - (6k+1)a]}{2(k+2)\ (6k+1)h^2} Pa^2$; $M_E = M_A + H_A a$

Esfuerzo de flexión en la columna AB : $M_y = M_A + H_A y$ $\quad y \leq a$

$M_y = M_A + H_A y - P\ (y-a)$ $\quad y \geq a$

columna CD : $M_y = M_D - H_D y$

viga BC : $M_x = M_B - V_A x$

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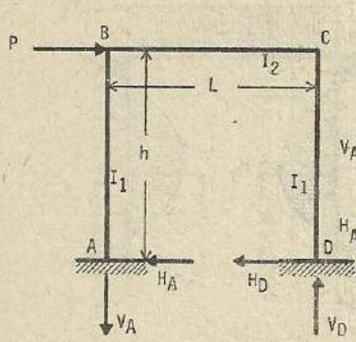
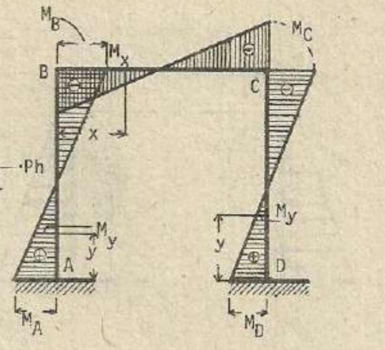


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = \frac{3k}{(6k+1)\,L}\cdot Ph$$

$$H_A = H_D = \frac{P}{2}$$



$$M_A = -\frac{(3k+1)}{2\,(6k+1)}\cdot Ph \;;\quad M_D = \frac{(3k+1)}{2(6k+1)}\cdot Ph \;;\quad M_B = \frac{3k}{2(6k+1)}\cdot Ph;\quad M_C = -\frac{3k}{2(6k+1)}\cdot Ph$$

Esfuerzo de momento por flexión en la columna AB :

$$M_y = -\frac{(3k+1)}{2\,(6k+1)}\cdot Ph + \frac{P}{2}\,y$$

columna CD : $M_y = \dfrac{(3k+1)}{2\,(6k+1)}\cdot Ph - \dfrac{P}{2}\,y$

Viga BC : $M_x = \dfrac{3k}{2(6k+1)}\cdot Ph - \dfrac{3k}{(6k+1)L}\cdot Phx$

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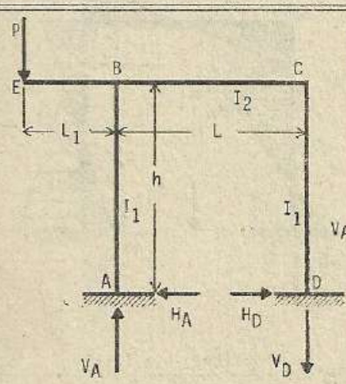
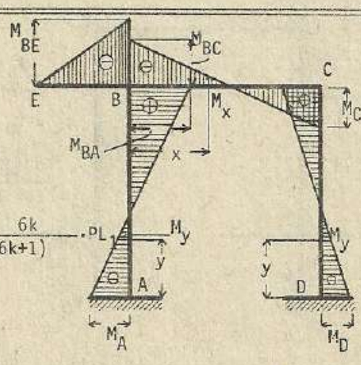


$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = P + \frac{6k}{(6k+1)}\cdot PL_1$$



$$V_D = \frac{6k}{(6k+1)}\cdot PL_1 \;;\quad H_A = H_D = \frac{3}{2(k+2)h}\cdot PL_1$$

$$M_A = -\frac{(5k-1)}{2\,(k+2)(6k+1)}\cdot PL_1 \;;\quad M_D = \frac{7k+3}{2\,(k+2)(6k+1)}\cdot PL_1 \;;\quad M_{BE} = -\,PL_1$$

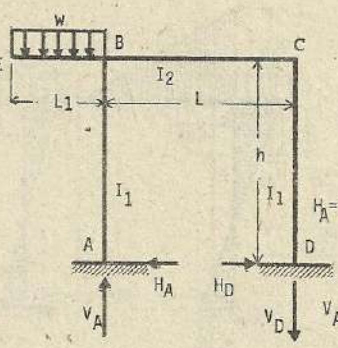
$$M_{BA} = \frac{(13k+4)}{2(k+2)\,(6k+1)}\cdot PL_1 \;;\quad M_{BC} = \frac{k(12k+13)}{2(k+2)(6k+1)}\cdot PL_1 \;;\quad M_C = \frac{11k}{2\,(k+2)\,(6k+1)}\cdot PL_1$$

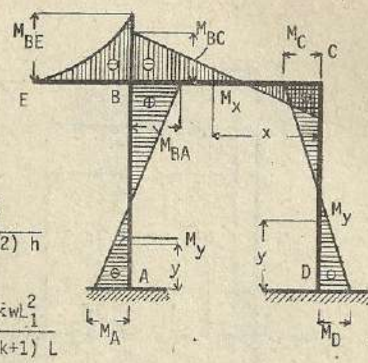
Esfuerzo de flexión en la columna AB : $M_y = -\dfrac{5k-1}{2(k+2)(6k+1)}\cdot PL_1 + \dfrac{3}{2\,(k+2)\,h}\,PL_1 y$

columna DC : $M_y = -\dfrac{7k+3}{2(k+2)\,(6k+1)}\,PL_1 + \dfrac{3}{2\,(k+2)h}\,PL_1 y$

Viga BC : $M_x = -\dfrac{k\,(12k+13)}{2(k+2)\,(6k+1)}\cdot PL_1 - Px - \dfrac{6k}{(6k+1)\,L}\cdot PL_1 x$

(217)





$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_D = \frac{3wL_1^2}{4\,(k+2)\,h}$$

$$V_A = wL_1 + \frac{3kwL_1^2}{(6k+1)\,L}$$

$$V_D = \frac{3kwL_1^2}{(6k+1)L} \quad ; \quad M_A = -\frac{(5k-1)\,wL_1^2}{4\,(6k+1)\,(k+2)}$$

$$M_D = -\frac{(7k+3)wL_1^2}{4(6k+1)(k+2)} \quad ; \quad M_{BE} = -\frac{wL_1}{2} \quad ; \quad M_{BA} = \frac{(13k+4)\,wL_1^2}{4(6k+1)\,(k+2)}$$

$$M_{BC} = -\frac{k\,(12k+13)wL_1^2}{4\,(6k+1)\,(k+2)} \quad ; \quad M_C = \frac{11kwL_1^2}{4\,(6k+1)\,(k+2)}$$

Esfuerzo de flexión en la columna AB : $M_y = -\dfrac{(5k-1)\,wL_1^2}{4(6k+1)\,(k+2)} + \dfrac{3wL_1^2}{4(k+2)\,h}\cdot y$

Columna CD : $M_y = -\dfrac{(7k+3)wL_1^2}{4(6k+1)(k+2)} + \dfrac{3wL_1^2}{4\,(k+2)\,h}\cdot y$

Viga BC : $M_x = \dfrac{11kwL_1^2}{4(6k+1)\,(k+2)} - \dfrac{3kwL_1^2}{(6k+1)L}\cdot x$

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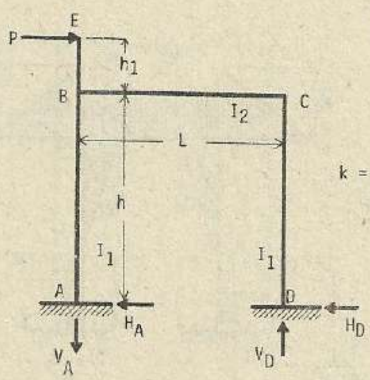
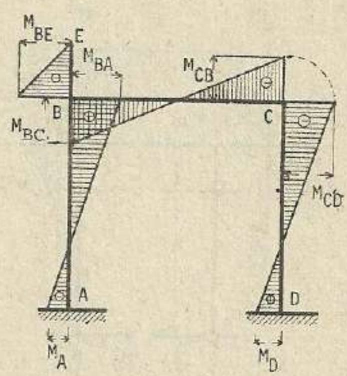
(218)





$$k = \frac{I_2 h}{I_1 L}$$

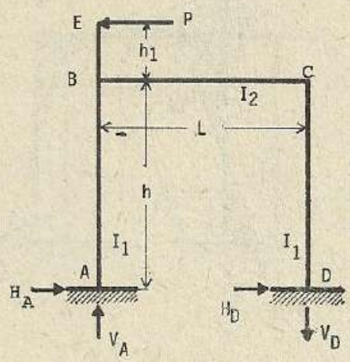
$$V_A = V_D = \frac{P\,(h+h_1)}{L} + \frac{M_{AB}-M_{DC}}{L} \quad ; \quad H_A = \frac{M_{BA}-M_{AB}}{h} \quad ; \quad H_D = \frac{M_{DC}-M_{CD}}{h}$$

$$M_{AB} = -\frac{P}{2}\cdot\frac{h_1(1-5k) + h\,(3k^2+7k+2)}{(k+2)\,(6k+1)} \quad ; \quad M_{BC} = \frac{Pk}{2}\cdot\frac{h_1(12k+13) + 3h\,(k+2)}{(k+2)\,(6k+1)}$$

$$M_{BA} = M_{BC} - Ph_1 \quad ; \quad M_{CD} = M_{CB} = -\frac{pk}{2}\cdot\frac{11h_1 + 3h\,(k+2)}{(k+2)\,(6k+1)}$$

$$M_{DC} = \frac{P}{2}\cdot\frac{h_1(7k+3) + h\,(3k^2+7k+2)}{(k+2)\,(6k+1)}$$

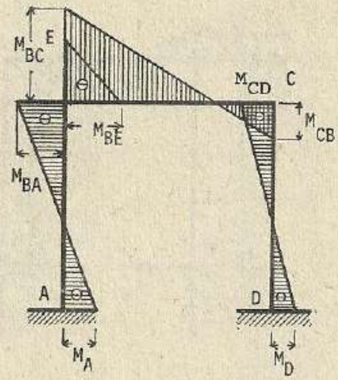
(219)



$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = \frac{M_{BA} - M_{AB}}{h}$$

$$H_D = \frac{M_{CD} - M_{DC}}{h}$$



$$V_A = V_D = \frac{P\,(h+h_1)}{L} + \frac{M_{AB} - M_{DC}}{L}$$

$$M_{AB} = \frac{P}{2} \cdot \frac{h_1(1-5k) + h\,(3k^2+7k+2)}{(k+2)\,(6k+1)} \quad ; \quad M_{BA} = M_{BC} - Ph_1$$

$$M_{BC} = -\frac{Pk}{2} \cdot \frac{h_1(12k+13) + 3h\,(k+2)}{(k+2)\,(6k+1)} \quad ; \quad M_{CD} = M_{CB} = \frac{Pk}{2} \cdot \frac{11h_1 + 3h\,(k+2)}{(k+2)\,(6k+1)}$$

$$M_{DC} = -\frac{P}{2} \cdot \frac{h_1(7k+3) + h\,(3k^2+7k+2)}{(k+2)\,(6k+1)}$$

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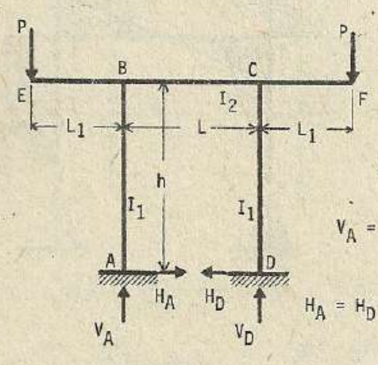
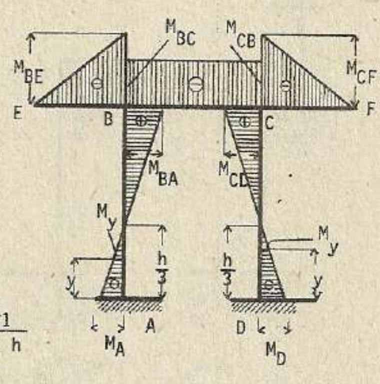
(220)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = P$$

$$H_A = H_D = \frac{3PL_1}{(k+2)\,h}$$



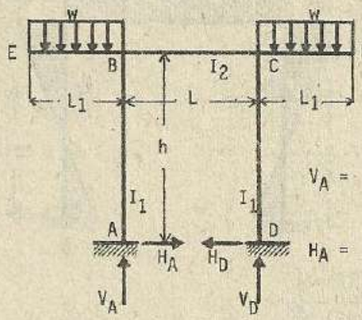
$$M_A = M_D = -\frac{PL_1}{(k+2)} \quad ; \quad M_{BE} = M_{CF} = PL_1 \quad ; \quad M_{BA} = M_{CD} = \frac{2PL_1}{(k+2)}$$

$$M_{BC} = M_{CB} = -\frac{kPL_1}{(k+2)}$$

Esfuerzo de flexión en la columna : $M_y = -\frac{PL_1}{(k+2)} + \frac{3PL_1}{(k+2)h} \cdot y$

En la viga : $M_x = -\frac{kPL_1}{(k+2)}$ (constante)

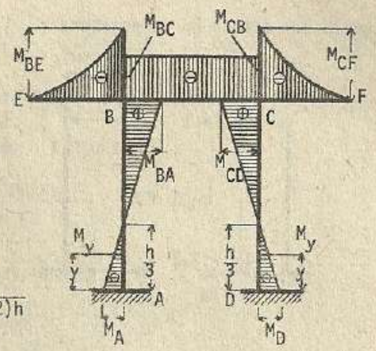
(221)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = wL_1$$

$$H_A = H_D = \frac{3wL_1^2}{2(k+2)h}$$



$$M_A = M_D = -\frac{wL_1^2}{2\,(k+2)} \quad ; \quad M_{BE} = M_{CF} = -\frac{wL_1^2}{2} \quad ; \quad M_{BA} = M_{CD} = \frac{wL_1^2}{k+2}$$

$$M_{BC} = M_{CB} = -\frac{kwL_1^2}{2(k+2)}$$

Esfuerzo de flexión en la columna : $M_y = -\frac{wL_1^2}{2(k+2)} + \frac{3wL_1^2}{2(k+2)\,h}\cdot y$

en la viga : $M_x = -\frac{kwL_1^2}{2(k+2)}$ (constante)

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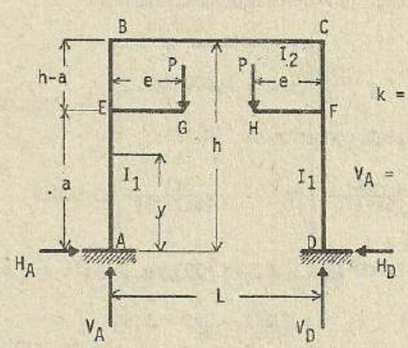
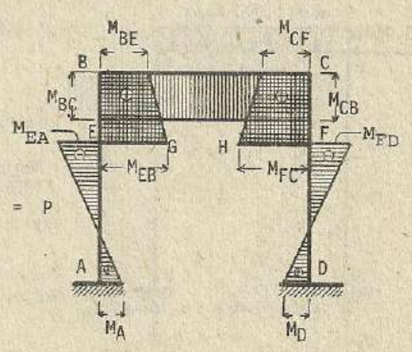
(222)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = V = P$$



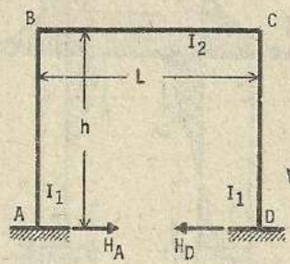
$$H_A = H_D = H = \frac{3Pe}{h^3}\cdot\frac{2ak(h-a) + a\,(2h-a)}{k+2} \quad ; \quad M_{BC} = M_{BA} = M_{CB} = M_{CD} = -Hh + M_A + Pe$$

$$M_{AB} = M_{DC} = \frac{Pe}{h^2}\cdot\frac{k\,(4ah - 3a^2 - h^2) + (6ah - 2h^2 - 3a^2)}{k+2}$$

Esfuerzo de flexión en la columna AE : $M_y = M_A - Hy$

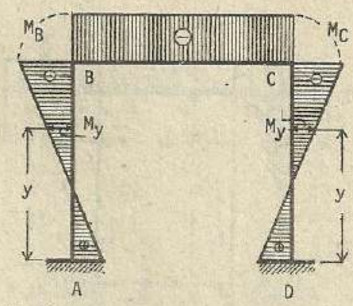
columna EB : $M_y = M_A + Pe - Hy$

Esfuerzo por cambio de temperatura : (223)



$$k = \frac{I_2 h}{I_1 L}$$

$$V_A = V_D = 0$$



$$H_A = H_D = \frac{3E\varepsilon t I_2(2k+1)}{k\,(k+2)\,h^2}$$

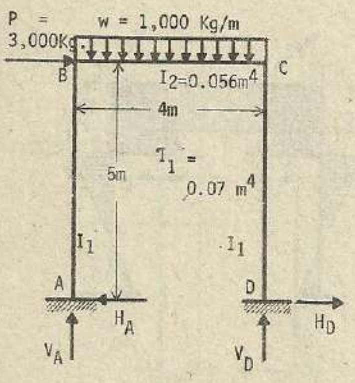
$$M_A = M_D = \frac{3E\varepsilon t I_2(k+1)}{k\,(k+2)\,h} \quad ; \quad M_B = M_C = -\frac{3E\varepsilon t I_2}{(k+2)\,h}$$

Esfuerzo de flexión en la columna : $M_y = \dfrac{3E\varepsilon t I_2}{k(k+2)h}\cdot\left[(k+1) - (2k+1)\cdot\dfrac{y}{h}\right]$

en la viga : $M_x = -\dfrac{3E\varepsilon t I_2}{(k+2)\,h}$ (constante)

Nota : El presente problema corresponde al caso de aumento de temperatura, en caso de descenso, los esfuerzos se anotan con los signos contrarios.

Solución numérica :



$$k = \frac{I_2 h}{I_1 L} = \frac{0.056\text{x}5}{0.\ 07\text{x}4} = 1$$

Debido a la carga uniformemente distribuida (C.U.D)

$$V'_A = \frac{wL}{2} = \frac{1{,}000\text{x}4}{2} = 2{,}000\ \text{Kg}$$

Debido a la carga concentrada (C.C.) (215)

$$V''_A = -\frac{3k}{(6k+1)L}\cdot Ph = -\frac{3\text{x}1}{(6\text{x}1+1)4}\ \text{x}3{,}000\text{x}5$$

$\therefore$ V_A = 2,000 Kg - 1,607Kg =393 Kg. (hacia arriba)

V_D = wL - V_A = 1,000x4 - 393 = 3,607 (hacia arriba)

Debido a C.U.D (206) : $H'_A = \dfrac{wL^2}{4(k+2)\,h} = \dfrac{1{,}000\text{x}4\text{x}4}{4(1+2)\text{x}5} = 267$ Kg (hacia la derecha) +

Debido a C.C. (215) : $H''_A = -\dfrac{P}{2} = -\dfrac{3{,}000}{2} = -1{,}500$ Kg (hacia la izquierda) -

$\therefore$ H''_A = 267 - 1,500 = - 1,233 Kg (hacia la izquierda) -

H_D = P - H_A = 3,000 - 1233 = 1,767 Kg (hacia la derecha +)

Debido a C.U.D. (206) : $M' = \dfrac{wL^2}{12(k+2)} = \dfrac{1{,}000\text{x}4\text{x}4}{12\text{x}3} = 444$ Kg-m

Debido a la C.C. : $M''_A = -\frac{(3k+1)}{2\,(6k+1)}\,Ph = -\frac{(3x1+1)}{2(6x1+2)}\,x3{,}000x5 = -\,4{,}286$ Kg-m

$\therefore\ M_A = 444 - 4{,}286 = -2{,}842$ Kg-m

$M_D = \frac{(3k+1)}{2(6k+1)}\,Ph + \frac{wL^2}{12\,(k+2)} = 4{,}286 + 444 = 4{,}730$ Kg-m.

Debido a la C.U.D : $M'_B = -\frac{1}{6(k+2)}\,wL^2 = -\frac{1{,}000x4x4}{6\,(1+2)} = -\,889$ Kg-m

Debido a la C.C. : $M''_B = \frac{3k}{2\,(6k+1)}\,Ph = \frac{3}{2\,(6x1+1)}\,x3{,}000x5 = 3{,}214$ Kg-m

$\therefore\ M_B = -\,889 + 3{,}214 = 2{,}325$ Kg-m

$M_C = -\frac{1}{6\,(k+2)}\cdot wL^2 - \frac{3k}{2\,(6k+1)}\cdot Ph = -\,889 - 3{,}214 = -\,4{,}103$ Kg-m

Considerando como (x_m) la distancia entre (B) y el punto donde la cortante es cero :

$V_A = wx_m \longrightarrow 393 = 1{,}000x_m \quad \therefore\ x_m = 0.393$ m.

$M_{máx.} = M_B + V_A x_m - w\frac{x_m^2}{2}$

$2{,}325 + 393 \times 0.393 - 1{,}000 \times \frac{0.393x0.393}{2} = 2{,}402$ Kg-m

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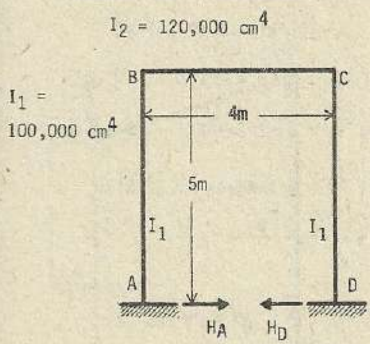
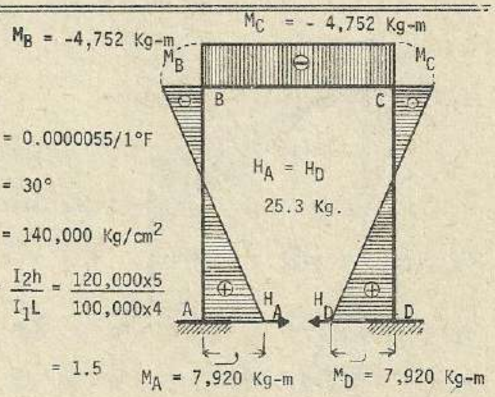


$\varepsilon = 0.0000055/1°F$

$t = 30°$

$E = 140{,}000$ Kg/cm^2

$k = \frac{I_2 h}{I_1 L} = \frac{120{,}000x5}{100{,}000x4} = 1.5$



$M_A = M_D = \frac{3E\varepsilon tI_2\,(k+1)}{k\,(k+2)\,h}$

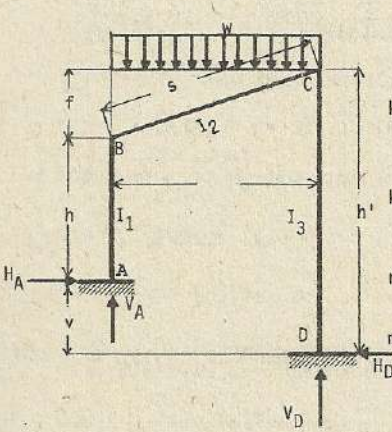
$= \frac{2x140{,}000x0.0000055x30x120{,}000x(1.5+1)}{1.5\,(1.5+2)x500} = 792{,}000$ Kg-cm $= 7{,}920$ Kg-m

$M_B = M_C = -\frac{3E\varepsilon tI_2}{(k+2)h} = -M_A \times \frac{k}{1+k} = -\,7{,}920 \times \frac{1.5}{1+1.5} = -\,4{,}752$ Kg-m

$H_A = H_D = \frac{3E\varepsilon tI_2(2k+1)}{k(k+2)\,h^2} = M_B \times \frac{2k+1}{kh} = 4{,}752 \times \frac{2x1.5 + 1}{1.5 \times 500} = 25.3$ Kg.

Nota : En caso de descenso de temperatura, se modifican los signos.

(224)

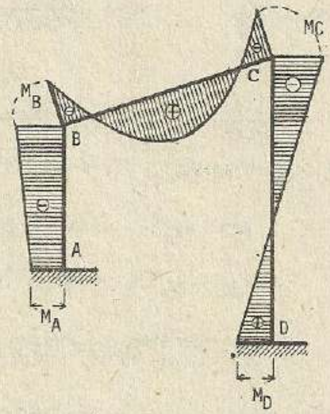


$k = \frac{I_2 h}{I_1 s}$

$k' = \frac{I_2 h'}{I_3 s}$

$n = \frac{h'}{h}$

$m = \frac{h}{f}$



$N = (1+k+k'+3kk')\ (k+n^2k') + 3kk'\ (1+n+n^2)$

$\alpha_1 = 2k + 2n^2k'\ (1+k') + (8+3n)\ k'k$; $\alpha_2 = k\ [\ k + 2n\ (3+2n)\ k'\ ]$

$\alpha_3 = 3k\ [\ 1 + 2\ (2+n)\ k'\ ]$; $\beta_1 = k'\ [\ n^2k' + 2\ (2+3n)\ k\ ]$

$\beta_2 = 2n^2k' + 2k\ (1+k) + n\ (3+8n)\ kk'$; $\beta_3 = 3k'\ [\ n + 2\ (1+2n)\ k\ ]$

$\gamma_1 = nkk'\ (2+4n+3nk')$; $\gamma_2 = nkk'\ (4+2n+3k)$; $\gamma_3 = k\ [\ 2+2k+6kk' + (8+3n)\ k'\ ]$

$\delta_1 = k\ (2k+6kk' - nk')$; $\delta_2 = k\ (2nk'+6kk'-k)$; $\delta_3 = k(1+4k+4k' + 12kk')$

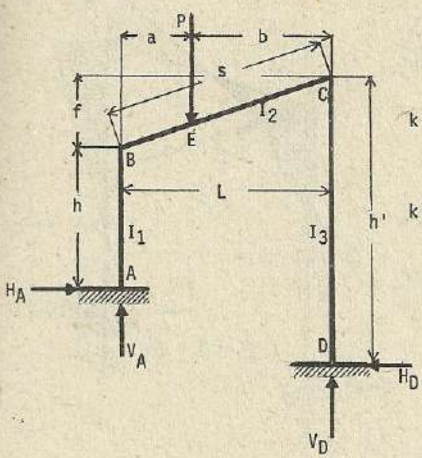
$H_A = H_D = H = \frac{wL^2}{24Nh}\ (\alpha_3 + \beta_3)$; $V_A = \frac{wL}{2} + \frac{M_C - M_B}{L} + \frac{Hf}{L}$

$V_D = \frac{wL}{2} - \frac{M_C - M_B}{L} - \frac{Hf}{L}$; $M_B = -\frac{wL^2}{24N}\ (\alpha_1 + \beta_1)$; $M_A = M_B + Hh$

$M_C = -\frac{wL^2}{24}\ (\alpha_2 + \beta_2)$; $M_D = M_C + Hh'$

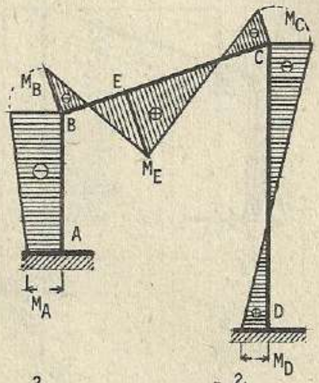
Nota : Los datos deducidos son también utilizables para los problemas que siguen.

(225)



$$k = \frac{I_2 h}{I_1 s}$$

$$k' = \frac{I_2 h'}{I_3 s}$$



$$C_{BC} = -\frac{Pab^2}{L^2} \quad ; \quad C_{CB} = -\frac{Pa^2b}{L^2}$$

$$M_B = \frac{1}{2N}(\alpha_1 C_{BC} + \beta_1 C_{CB}) \quad ; \quad M_A = M_B + Hh \quad ; \quad M_C = \frac{1}{2N}(\alpha_2 C_{BC} + \beta_2 C_{CB})$$

$$M_D = M_C + Hh' \quad ; \quad H_A = H_D = H = -\frac{1}{2Nh}(\alpha_3 C_{BC} + \beta_3 C_{CB})$$

$$V_A = \frac{1}{L}(Pb - M_B + M_C + Hf) \qquad V_D = P - V_A$$

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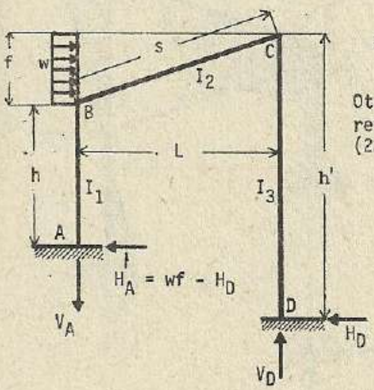
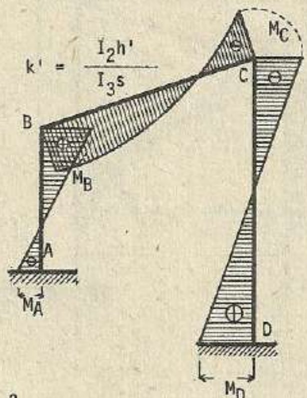
(226)



$$k = \frac{I_2 h}{I_1 s}$$

Otros datos, referencia (224)

$$k' = \frac{I_2 h'}{I_3 s}$$



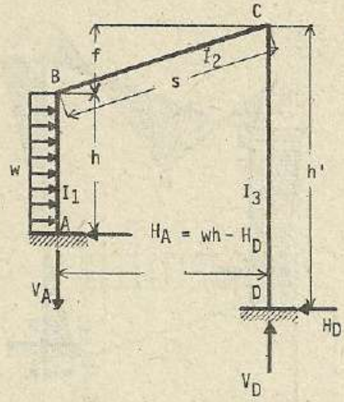
$$M_B = -\frac{wf^2}{24N}\left[12m\,\gamma_1 - (\alpha_1 + \beta_1)\right]$$

$$M_C = -\frac{wf^2}{24N}\left[12m\,\gamma_2 + (\alpha_2 + \beta_2)\right] \quad ; \quad H_D = \frac{wf^2}{12Nh}\left[12m\gamma_3 + (\alpha_3 + \beta_3)\right] ; \quad H = wf - H_D$$

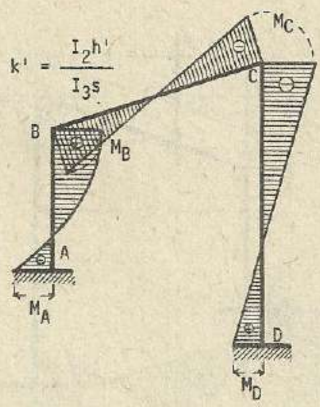
$$M_A = M_B - (wf - H_D)\,h \quad ; \quad M_D = M_C + H_D h'$$

$$V_A = V_D = V = \frac{1}{L}\left(w \cdot \frac{f^2}{2} + M_B - M_C - H_D f\right)$$

(227)



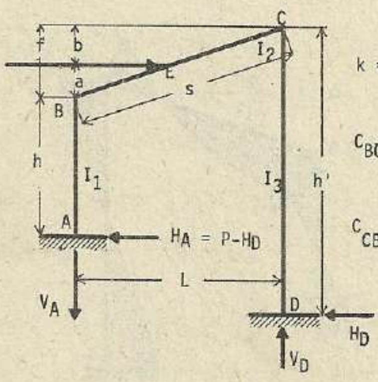
$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$



$$M_B = \frac{wh^2}{24N}(4\gamma_1 - \delta_1) \quad ; \quad M_A = M_B + H_D h - \frac{wh^2}{2} \quad ; \quad M_C = -\frac{wh^2}{24N}(4\gamma_2 + \delta_2)$$

$$M_D = M_C + H_D h' \quad ; \quad H_D = \frac{wh}{24N}(4\gamma_3 + \delta_3) \quad ; \quad V_A = V_D = V = \frac{1}{L}(M_B - M_C - H_D f)$$

(228)

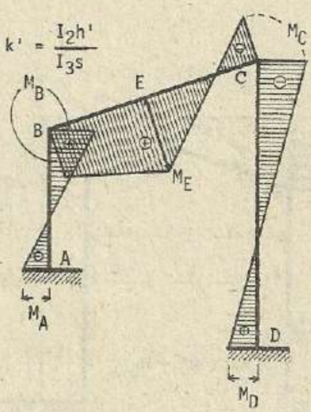


$$k' = \frac{I_2 h'}{I_3 s}$$

$$k = \frac{I_2 h}{I_1 s}$$

$$C_{BC} = -\frac{Pab^2}{f^2}$$

$$C_{CB} = -\frac{Pa^2 b}{f^2}$$



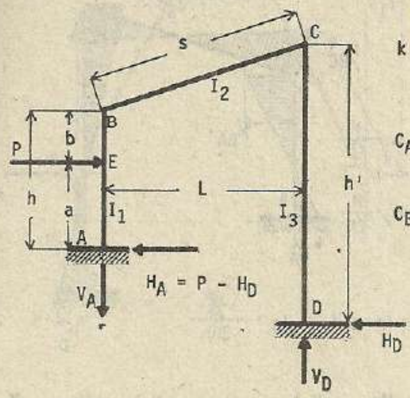
$$M_B = \frac{1}{2N}(\alpha_1 C_{BC} + \beta_1 C_{CB} + \gamma_1 Ph)$$

$$M_C = \frac{1}{2N}(\alpha_2 C_{BC} + \beta_2 C_{CB} - \gamma_2 Ph) \qquad H_D = \frac{1}{2Nh}(\alpha_3 C_{BC} - \beta_3 C_{CB} + \gamma_3 Ph)$$

$$H_A = P - H_D \quad ; \quad M_A = M_B - (P - H_D)h$$

$$M_D = M_C + H_D h' \qquad V_A = V_D = V = \frac{1}{L}(M_B - M_C + Pa - H_D f)$$

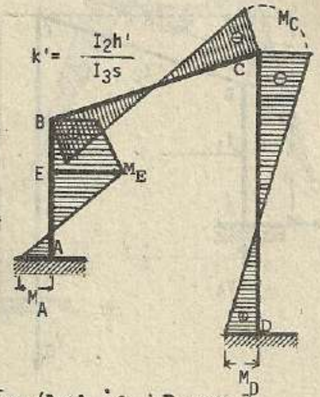
(229)



$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$

$$C_{AB} = -\frac{Pab^2}{h^2}$$

$$C_{BA} = -\frac{Pa^2 b}{h^2}$$



$$M_B = \frac{1}{2N}\left[\gamma_1 (Pa + C_{AB} + C_{BA}) + \delta_1 C_{BA}\right]$$

$$M_C = -\frac{1}{2N}\left[\gamma_2 (Pa + C_{AB} + C_{BA}) - \delta_2 C_{BA}\right] \quad ; \quad H_D = \frac{1}{2Nh}\left[\gamma_3 (Pa + C_{AB} + C_{BA}) - \delta_3 C_{BA}\right]$$

$$H_A = P - H_D \quad ; \quad M_A = M_B + H_D h - Pa \quad ; \quad M_D = M_C + H_D h'$$

$$V_A = V_D = V = \frac{1}{L} \cdot (M_B - M_C - H_D f)$$

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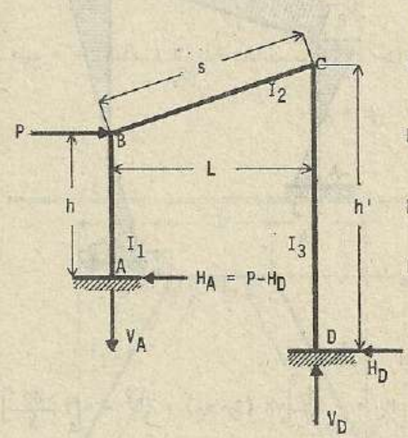
(230)

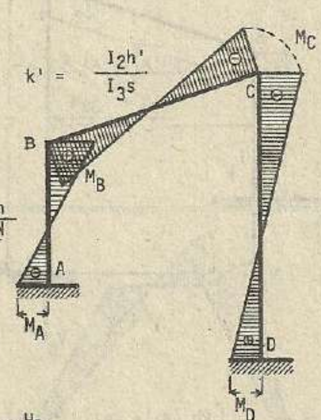


$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$

$$M_B = \gamma_1 \frac{Ph}{2N}$$

$$M_C = -\gamma_2 \cdot \frac{Ph}{2N}$$

$$H_D = \alpha_3 \frac{P}{2N}$$



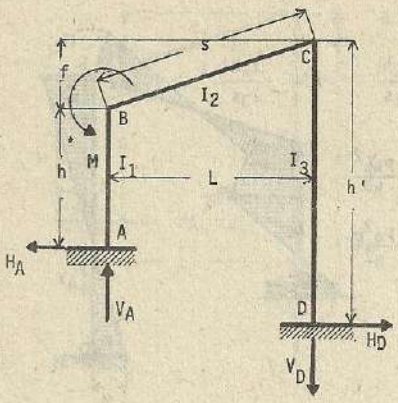
$$H_A = P - H_D$$

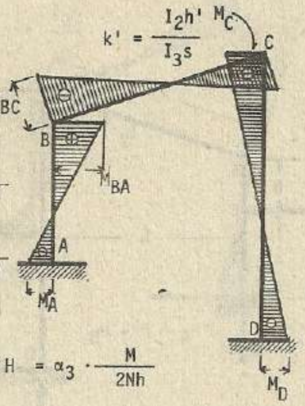
$$M_A = M_B - (P - H_D)h \qquad M_D = M_C + H_D h'$$

$$V_A = V_D = V = \frac{1}{L} \cdot (M_B - M_C - H_D f)$$

(231)

$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$





$$M_{BA} = \alpha_1 \cdot \frac{M}{2N}$$

$$M_C = \alpha_2 \cdot \frac{M}{2N}$$

$$H_A = H_D = H = \alpha_3 \cdot \frac{M}{2Nh}$$

$$M_A = M_{BA} - H_D h \quad ; \quad M_{BC} = M_{BA} - M$$

$$M_D = M_C - H_D h' \quad ; \quad V_A = V_D = \frac{1}{L} \cdot (M + M_C - M_B - H_D f)$$

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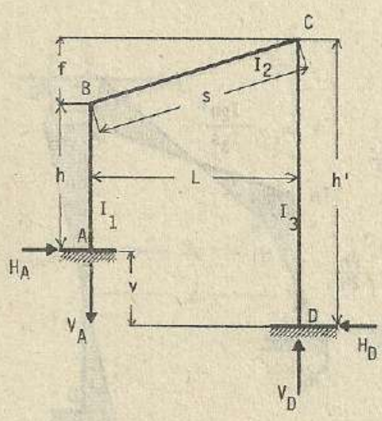
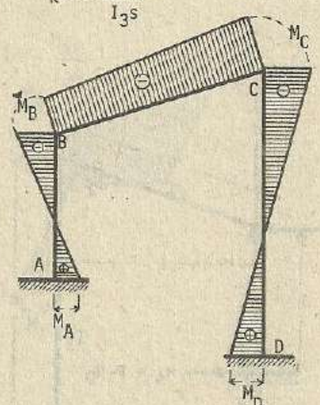
(232)

Esfuerzo por cambio de temperatura

$$k = \frac{I_2 h}{I_1 s} \qquad k' = \frac{I_2 h'}{I_3 s}$$

Otros datos referencia (224)





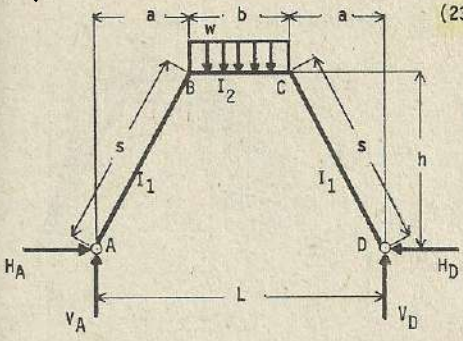
$$M_B = -\frac{T}{N}\left[vh\,(\beta_1 - \alpha_1) + (L^2 - vf)\frac{\delta_1}{k}\right] \; ; \; M_C = -\frac{T}{N}\left[vh\,(\beta_2 - \alpha_2) + (L^2 - vf)\frac{\delta_2}{k}\right]$$

$$H_A = H_D - H = \frac{T}{Nh}\left[vh\,(\beta_3 - \alpha_3) + (L^2 - vf)\frac{\delta_3}{k}\right]$$

$$V_A = V_D = V = \frac{M_B - M_C - Hf}{L} \qquad \text{Aquí} \quad T = \frac{3EI_2 \varepsilon t}{sLh}$$

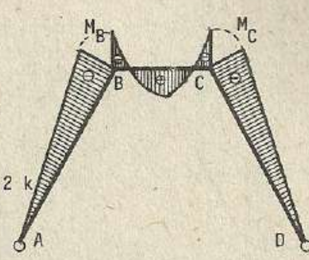
Nota : El presente problema corresponde al incremento de temperatura.
En caso de descenso, los esfuerzos se anotan con los signos contrarios.

(233)



$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$



$$H_A = H_D = H = \frac{wb^2}{2h} \cdot \left(\frac{a}{b} + \frac{1}{2m}\right)$$

$$M_B = M_C = -\frac{wb^2}{4m}$$

$$V_A = V_D = V = \frac{wb}{2}$$

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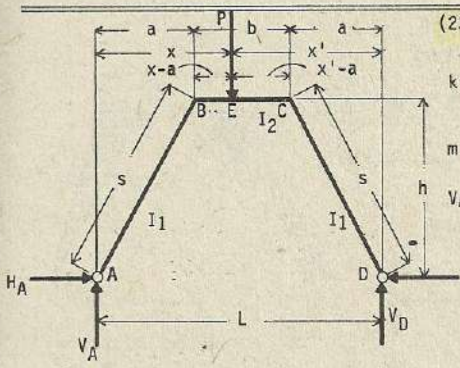
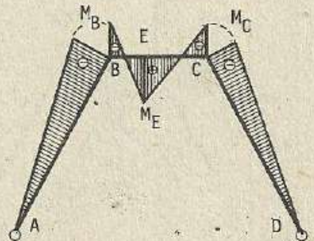
(234)



$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$

$$V_A = \frac{Px'}{L}$$



$$V_D = \frac{Px}{L} \quad ; \quad M_B = V_A a - Hh; \quad M_C = V_D a - Hh$$

$$H_{BC} = -\frac{P\,(x-a)\,(x'-a)}{2b^2} \cdot (b+x'-a) \quad ; \quad H_{CB} = -\frac{P\,(x-a)(x'-a)}{2b^2}\,(b+x-a)$$

$$H_A = H_D = H = \frac{Pa}{2h} - \frac{H_{BC} + H_{CB}}{mh}$$

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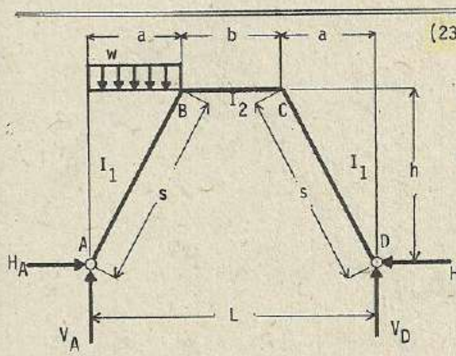
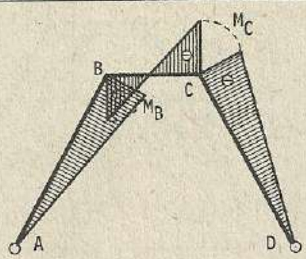
(235)



$$k = \frac{I_2 s}{I_1 b}$$

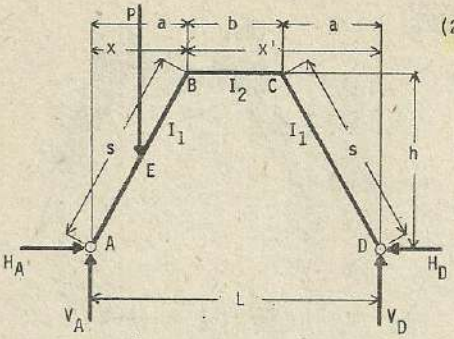
$$m = 3+2k$$



$$V_A = \frac{wa\,(2L-a)}{2L} \quad ; \quad V_D = \frac{wa^2}{2L}$$

$$H_A = H_D = H = \frac{wa}{4h} \cdot \left(1 + \frac{k}{2m}\right); \quad M_B = \frac{wa^2}{4} \cdot \left(\frac{b}{L} - \frac{k}{2m}\right) \quad ; \quad M_C = \frac{wa^2}{4} \cdot \left(-\frac{b}{L} - \frac{k}{2m}\right)$$

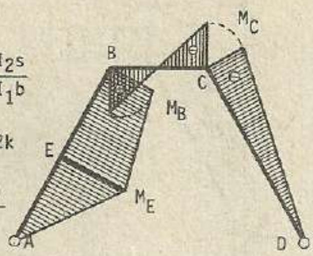
(236)



$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3+2k$$

$$V_A = \frac{Px'}{L}$$



$$V_D = \frac{Px}{L} \quad ; \quad H_{BA} = -\frac{Px\,(a^2-x^2)}{2a^2}$$

$$H_A = H_D = H = \frac{Px}{2h} - \frac{kH_{BA}}{mh} \quad ; \quad M_B = \frac{Pbx}{2L} + \frac{kH_{BA}}{m} \quad ; \quad M_C = -\frac{Pbx}{2L} + \frac{kH_{BA}}{m}$$

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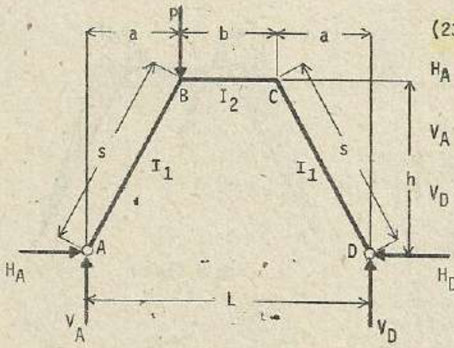
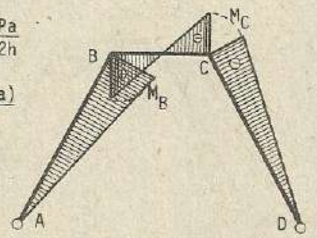
(237)



$$H_A = H_D = \frac{Pa}{2h}$$

$$V_A = \frac{P\,(L-a)}{L}$$

$$V_D = \frac{Pa}{L}$$



$$M_B = \frac{Pab}{2L} \qquad M_C = -\frac{Pab}{2L}$$

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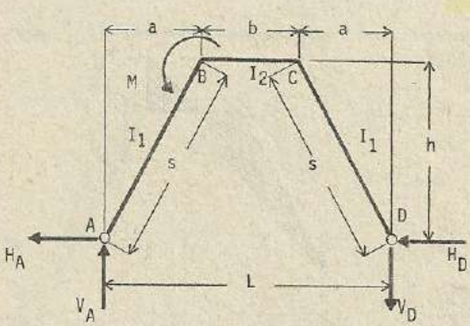
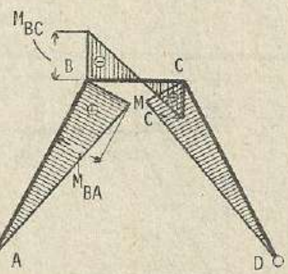
(238)



$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3+2k$$

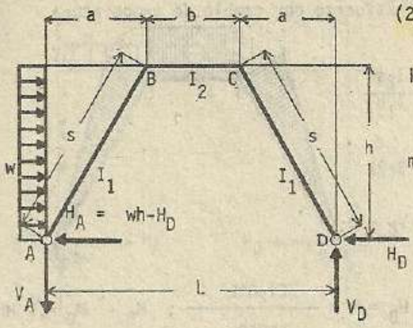


$$H_A = H_D = H = \frac{3M}{2mh}$$

$$M_{BA} = Hh + Va \quad ; \quad M_C = Hh - Va \quad ; \quad V_A = V_D = V = \frac{M}{L}$$

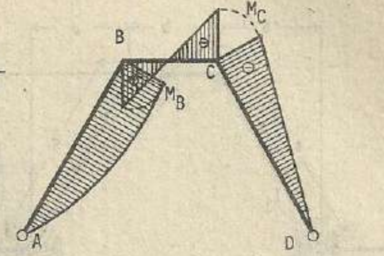
$$M_{BC} = M_{BA} - M$$

(239)



$$k = \frac{I_2 s}{I_1 b}$$

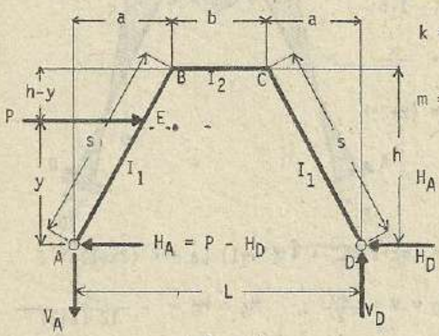
$$m = 3+2k$$



$$H_D = \frac{wh}{4}\left(1+\frac{k}{2m}\right) \quad ; \quad V_A = V_D = V = \frac{mh^2}{2L}$$

$$M_B = \frac{wh^2}{4}\left(\frac{b}{L} - \frac{k}{2m}\right) \quad ; \quad M_C = \frac{wh^2}{4}\left(-\frac{b}{L} - \frac{k}{2m}\right)$$

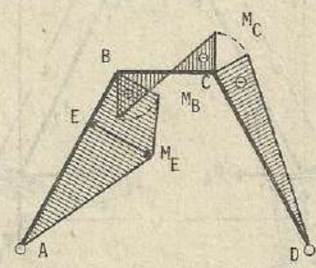
(240)



$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3+2k$$

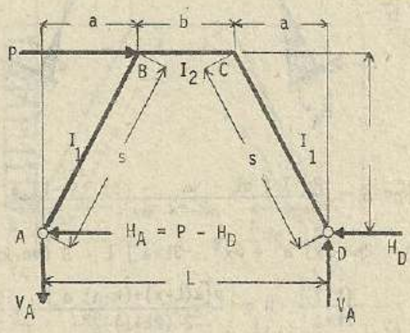
$$H_A = P - H_D$$



$$H_D = \frac{Py}{2h} - \frac{kH_{BA}}{mh} \quad ; \quad V_A = V_D = V = \frac{Py}{L}$$

$$H_{BA} = -\frac{Py\,(h^2 - y^2)}{2h^2} \quad ; \quad M_B = \frac{Pyb}{2L} + \frac{kH_{BA}}{m} \quad ; \quad M_C = -\frac{Pyb}{2L} - \frac{kH_{BA}}{m}$$

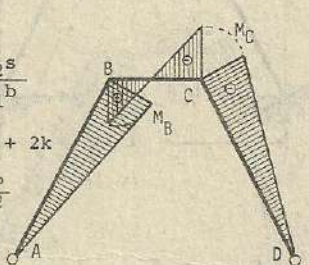
(241)



$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3 + 2k$$

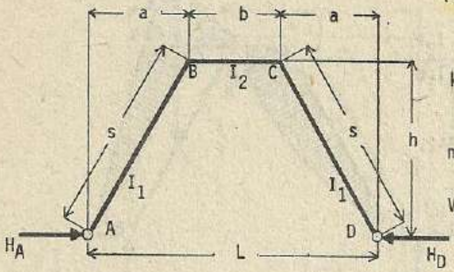
$$H_D = \frac{P}{2}$$



$$V_A = V_D = V = \frac{Ph}{L}$$

$$MB = -\ MC = \frac{Pbh}{2L}$$

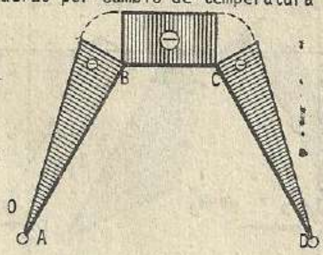
(242) Esfuerzo por cambio de temperatura



$$k = \frac{I_2 s}{I_1 b}$$

$$m = 3+2k$$

$$V_A = V_D = 0$$



$$H_A = H_D = H = \frac{3EI_2 \varepsilon tL}{mh^2 b} \quad ; \quad M_B = M_C = -Hh$$

Nota: El presente problema corresponde al caso donde existe incremento de temparatura.

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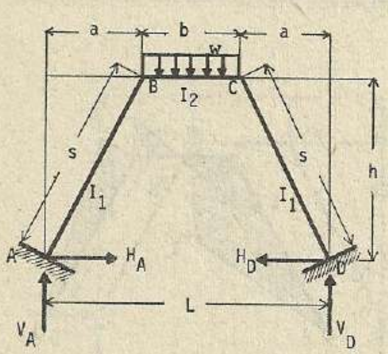
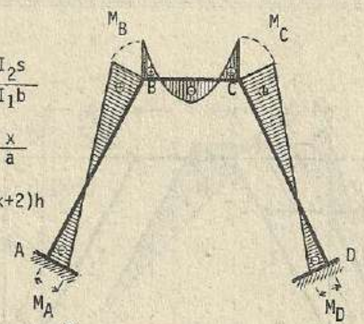
(243)



$$k = \frac{I_2 s}{I_1 b}$$

$$\alpha = \frac{x}{a}$$

$$\beta = (k+2)h$$



$$\gamma = (6k+1)\, L^2 - 4\,(3k+1)\, La + 4\,(2k+1)a^2$$

$$H_A = H_D = H = \frac{wb}{4} \cdot \frac{L + 2(k+1)a}{\beta} \quad ; \quad V_A = V_D = V = \frac{wb}{2} \quad ; \quad M_A = M_D = -\frac{wL^2}{12\,(k+2)}$$

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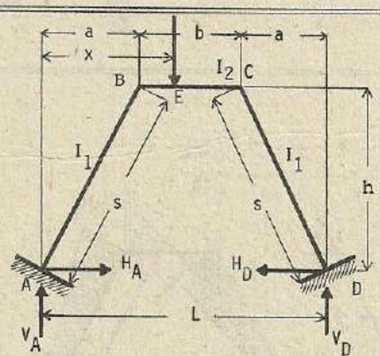
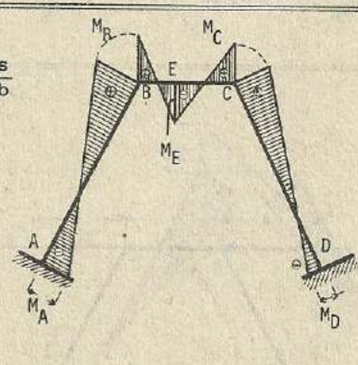
(244)



$$k = \frac{I_2 s}{I_1 b}$$

$$\alpha = \frac{x}{b}$$

$$n = \frac{a}{b}$$



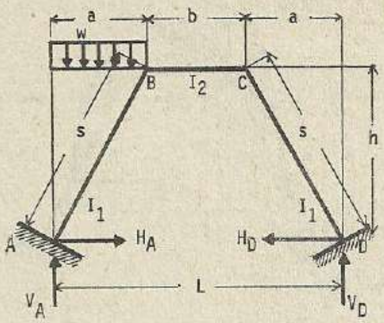
$$H_A = H_D = H = \frac{P}{2} \cdot \frac{3\,(\alpha L - x\alpha \quad - na) + (k-1)\, a}{\beta}$$

$$v = \frac{P}{L\gamma}\left\{ \alpha L^3 + 3\left[(k-n)\, a - \alpha x\right] L^2 + 2\left[(3\alpha + 2n - 2k)\, a^2 + \alpha x^2 - 3kxa\right] L - 8\,(\alpha a - kx)a^2 \right\}$$

$$V_A = \frac{P\,(L-x)}{L} + v \quad ; \quad V_D = \frac{Px}{L} - v \quad ; \quad M_A = -\frac{(k+1)h}{2k+1} \cdot H + \frac{P\left[\alpha(L-x) + (k-n)\, a\right]}{2\,(2k+1)} + \frac{L}{2} v$$

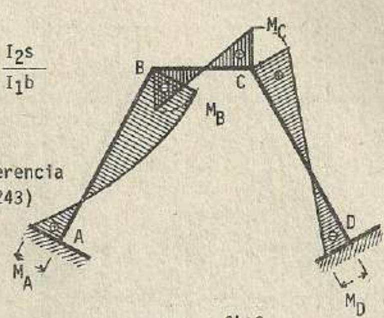
$$M_D = -M_A - vL$$

(245)



$$k = \frac{I_2 s}{I_1 b}$$

referencia (243)



$$H_A = H_D = H = \frac{wa}{8}\cdot\frac{3k+3}{\beta}$$

$$v = \frac{wa^2}{2L}\cdot\frac{(4k+1)L^2-(11k+4)La+4(2k+1)a^2}{\gamma}; \qquad V_A = \frac{wa^2}{2L}(2L-a)+v$$

$$M_A = \frac{wa^2(k+3)}{24(k+2)} + \frac{L}{2}v; \qquad M_D = M_A - vL; \qquad V_D = \frac{wa^2}{2L} - v$$

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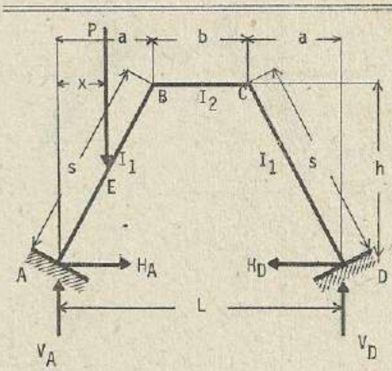
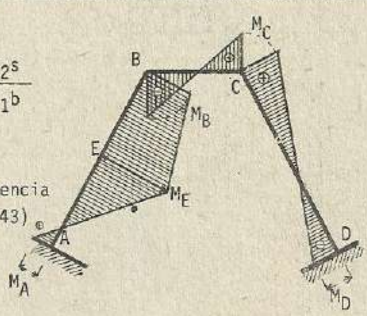
(246)



$$k = \frac{I_2 s}{I_1 b}$$

referencia (243)



$$H_A = H_D = H = \frac{Px}{2}\cdot\frac{3\alpha(k+1)-\alpha^2(2k+1)}{\beta}$$

$$v = \frac{Px}{L}\cdot\frac{(6k-3k\alpha+1)L^2-2(6k-k\alpha^2+2)La+4(2k+1)a^2}{\gamma} \qquad V_A = \frac{P(L-x)}{L}+v$$

$$M_A = -\frac{(k+1)h}{2k+1}\cdot H + \frac{Px(2k-k\alpha-1)}{2(2k+1)} + \frac{L}{2}v; \qquad M_D = M_A - vL; \qquad V_D = \frac{Px}{L} - v$$

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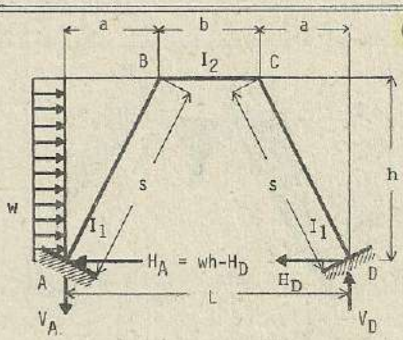
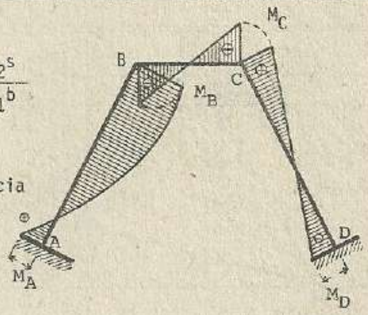
(247)



$$k = \frac{I_2 s}{I_1 b}$$

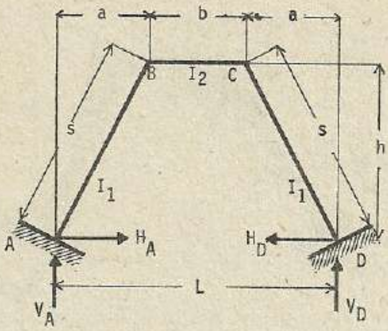
referencia (243)



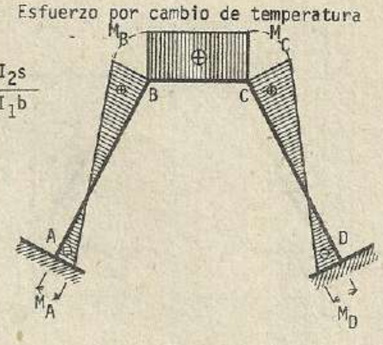
$$H_D = \frac{wh^2}{8}\cdot\frac{2k+3}{\beta}; \qquad V_A = V_D = V = \frac{wh^2}{2}\cdot\frac{k(2L-a)}{\gamma}$$

$$M_A = \frac{wh^2(7k+15)}{24(k+2)} - \frac{L}{2}V \qquad M_D = -\frac{wh^2(5k+9)}{24(k+2)} + \frac{L}{2}V$$

(248) : Esfuerzo por cambio de temperatura



$$k = \frac{I_2 s}{I_1 b}$$



$$H_A = H_D = H = \frac{\pm\, 3(2k+1)\, E\varepsilon t L I_2}{2\,(k+2)\, h^2} \quad ; \quad M_A = M_D = \frac{\pm\, 3(k+1)\, E\varepsilon t L I_2}{s\,(k+2) h}$$

Nota : El presente problema corresponde al incremento de temperatura. En caso de descenso, los esfuerzos se anotan con los signos contrarios.

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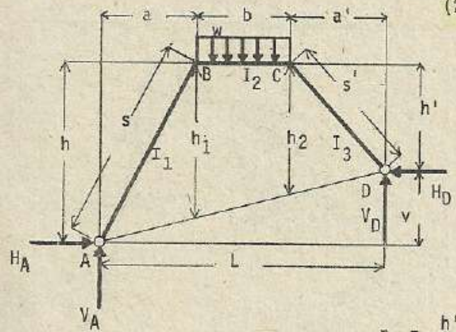
(249)

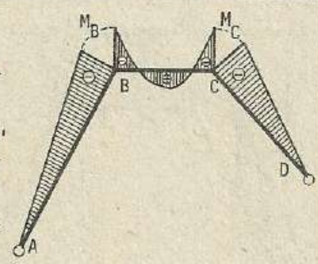


$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$

$$\alpha_1 = \frac{a}{L}$$

$$\alpha_2 = \frac{a'}{L}$$



$$n = \frac{h'}{h} \quad ; \quad n_1 = \frac{h_1}{h} \quad ; \quad n_2 = \frac{h_2}{h} \quad ; \quad m = \frac{n_1}{n_2}$$

$$\vartheta_1 = n + 2m\,(1+k) \quad ; \quad \vartheta_2 = m + 2n\,(1+k') \quad ; \quad \lambda_1 = \vartheta_1\,(1-\alpha_1) + \vartheta_2\,\alpha_2$$

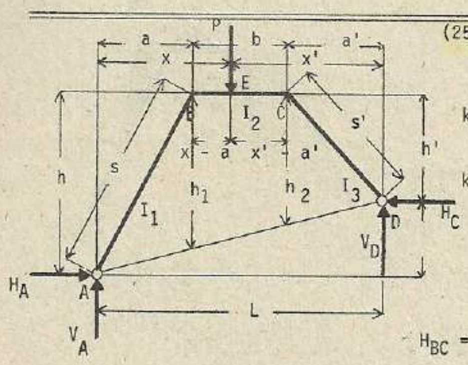
$$N = \frac{n_2}{2}\,(m\vartheta_1 + n\vartheta_2) = n_2\left[m^2\,(1+k) + mn + n^2\,(1+k')\right] \quad ; \quad \lambda_2 = \vartheta_1\alpha_1 + \vartheta_2\,(1-\alpha_2)$$

$$H_A = H_D = H = \frac{wb^2}{8Nh}\left[\,2\,\alpha_1\,\vartheta_1(1+2\,\frac{a'}{b}) + 2\alpha_2\vartheta_2\,(1+2\,\frac{a}{b}) + m + n\right]$$

$$V_A = \frac{wb}{2}\,(1-\alpha_1-\alpha_2) + \frac{Hv}{L} \quad ; \quad V_D = wb - V_A \quad ; \quad M_B = V_A a - Hh \quad ; \quad M_C = V_D a' - Hh'$$

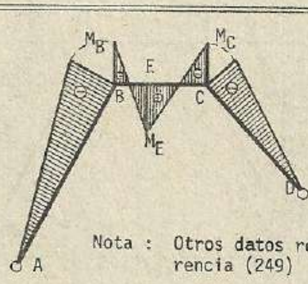
---

(250)



$$k = \frac{I_2 s}{I_1 b}$$

$$k' = \frac{I_2 s'}{I_3 b}$$



Nota : Otros datos referencia (249)

$$H_{BC} = -\frac{P\,(x-a)\,(x'-a')}{2b^2}\,(b+x'-a')$$

$$V_A = \frac{Px' + Hv}{L} \quad ; \quad V_D = \frac{Px - Hv}{L} \quad ; \quad H_{CB} = -\frac{P\,(x-a)\,(x'-a')}{2b^2}\,(b+x-a)$$

$$H_A = H_D = H = \frac{P(\alpha_1 \vartheta_1 x' + \alpha_2 \vartheta_2 x) - 2mH_{BC} - 2nH_{CB}}{2Nh}$$

$$M_B = \alpha_1 Px' - Hh_1 \qquad M_C = \alpha_2 Px - Hh_2$$

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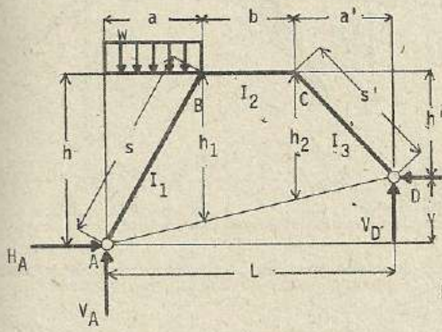
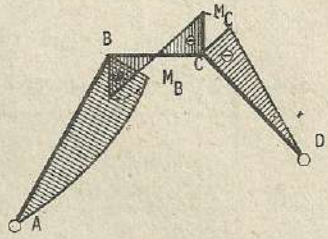
(251)



$$k = \frac{I_2 s}{I_1 b} \qquad k' = \frac{I_2 s'}{I_3 b}$$



$$H_A = H_D = H = \frac{wa^2}{8Nh}\cdot(2\lambda_1 + mk)$$

$$V_A = \frac{wa}{2}\cdot(2-\alpha_1) + \frac{Hv}{L} \; ; \qquad V_D = \frac{wa}{2}\cdot\alpha_1 - \frac{Hv}{L} \; ; \qquad M_B = \frac{wa^2}{2}\cdot(1-\alpha_1) - Hh_1$$

$$M_C = \frac{wa^2}{2}\cdot\alpha_2 - Hh_2$$

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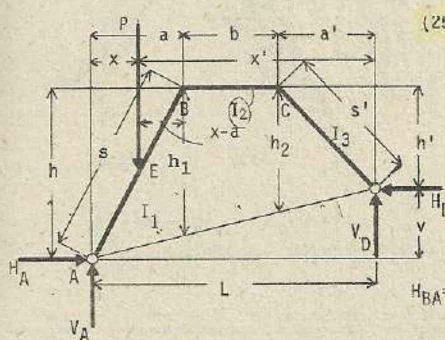
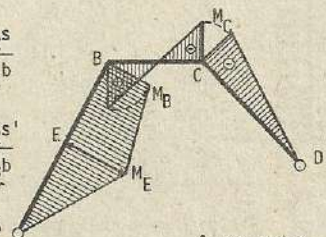
(252)



$$k = \frac{I_2 s}{I_1 b} \qquad k' = \frac{I_2 s'}{I_3 b}$$



$$H_{BA} = -\frac{Px(a^2 - x^2)}{2a^2} \; ; H_A = H_D = H = \frac{\lambda_1 Px - 2mkH_{BA}}{2Nh}$$

$$V_A = \frac{Px' + Hv}{L} \; ; \qquad V_D = \frac{Px - Hv}{L} \; ; \quad M_B = (1-\alpha_1)Px - Hh_1 \; ; \quad M_C = \alpha_2 Px - Hh_2$$

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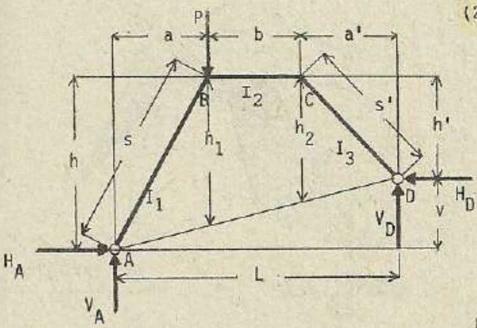
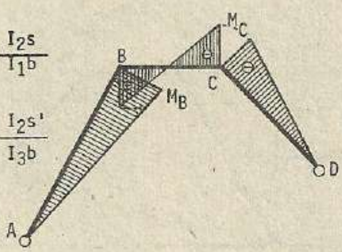
(253)



$$k = \frac{I_2 s}{I_1 b} \qquad k' = \frac{I_2 s'}{I_3 b}$$

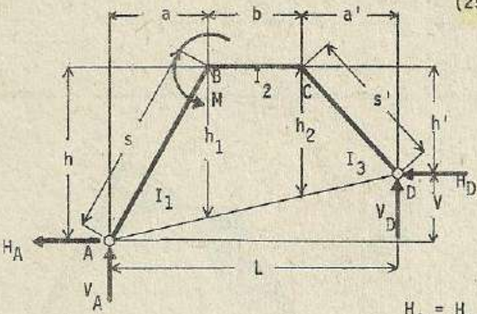


$$H_A = H_D = H = \frac{\lambda_1 Pa}{2Nh}$$

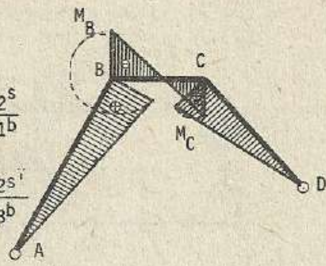
$$V_A = (1-\alpha_1)P + \frac{Hv}{L} \; ; \quad V_D = \alpha_2 \cdot P - \frac{Hv}{L} \; ; \quad M_B = (1-\alpha_2)Pa - Hh_1 \; ; \quad M_C = \alpha_2 Pa - Hh_2$$

Nota : Para otros datos se toma como referencia (249)

(254)



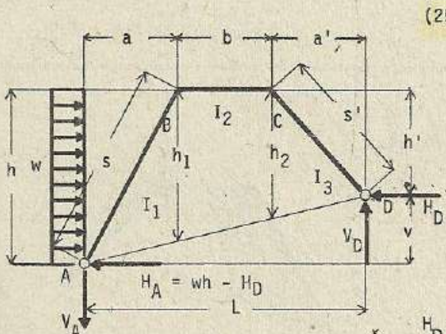
$$k = \frac{I_2 s}{I_1 b} \qquad k' = \frac{I_2 s'}{I_3 b}$$



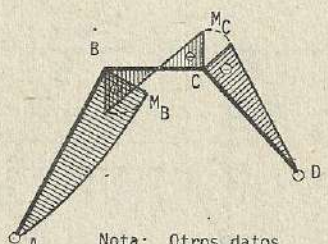
$$H_A = H_D = H = \frac{\lambda_1 Pa}{2Nh} \qquad V_A = (1-\alpha_1)P + \frac{Hv}{L}$$

$$M_B = (1-\alpha_1)Pa - Hh_1 \quad ; \quad M_C = \alpha_2 Pa - Hh_2 \quad ; \quad V_D = \alpha_2 P - \frac{Hv}{L}$$

(255)



$$k = \frac{I_2 s}{I_1 b} \qquad k' = \frac{I_2 s'}{I_3 b}$$

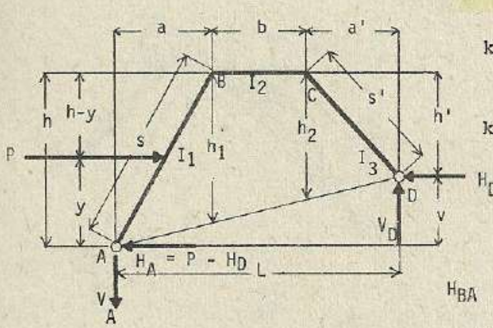


Nota: Otros datos (249)

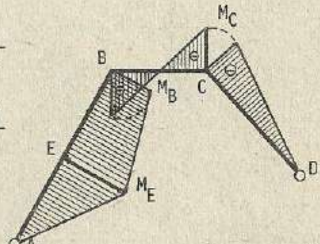
$$H_D = \frac{wh}{8N}\cdot(2\lambda_1 + mk) \ ; \qquad H_A = wh - H_D$$

$$V_A = V_D = v = \frac{wh^2}{2L} - \frac{Hv}{L} \ ; \quad M_B = \frac{wh^2}{2}\cdot(1-\alpha_1) - Hh_1 \ ; \ M_C = \frac{wh^2}{2}\alpha_2 - Hh_2$$

(256)



$$k = \frac{I_2 s}{I_1 b} \qquad k' = \frac{I_2 s'}{I_3 b}$$

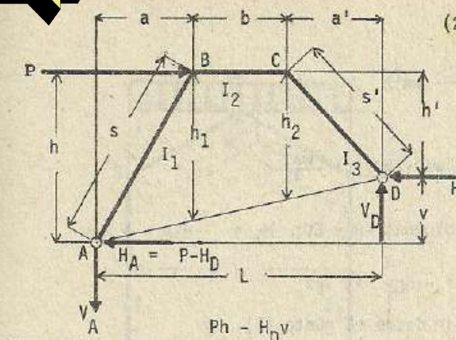


$$H_{BA} = -\frac{Py\,(h^2 - y^2)}{2h^2} \ ; \qquad H_D = \frac{\lambda_1 \quad Py - 2mkH_{BA}}{2Nh}$$

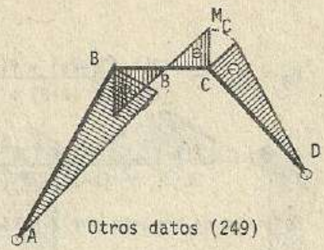
$$H_A = P - H_D \quad ; \quad V_A = V_D = V = \frac{Py - Hv}{L} \ ; \quad M_B = (1-\alpha_1)\,Py - Hh_1$$

$$M_C = \alpha_2\,Py - Hh_2$$

(257)



$$k = \frac{I_2 s}{I_1 b} \qquad k' = \frac{I_2 s'}{I_3 b}$$

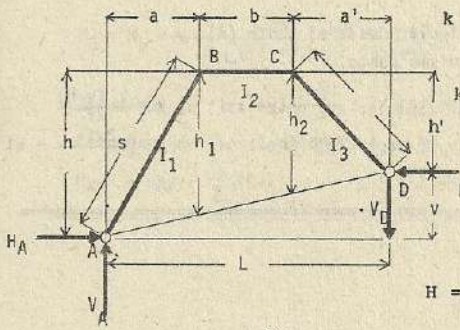


Otros datos (249)

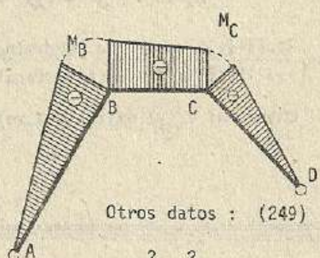
$$H_D = \frac{\lambda_1 P}{2N} \qquad H_A = P - H_D$$

$$V_A = V_D = V = \frac{Ph - H_D v}{L} \; ; \quad M_B = (P - H_D) - Va \; ; \quad M_C = - H_D h' + Va'$$

Esfuerzos por cambio de temperatura : (258)



$$k = \frac{I_2 s}{I_1 b} \qquad k' = \frac{I_2 s'}{I_3 b}$$



Otros datos : (249)

$$H = H_A = H_D = \frac{3E\varepsilon t I_2 \; (L^2 + v^2)}{N n_2 b h^2 L}$$

$$V_A = V_D = V = \frac{Hv}{L} \; ; \quad M_B = -Hh_1 \; ; \quad M_C = Hh_2$$

Nota : En caso de descenso de temperatura, los esfuerzos se anotan con los signos contrarios.

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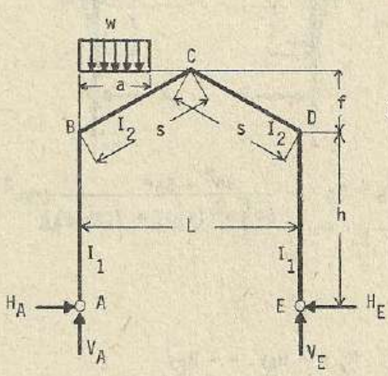
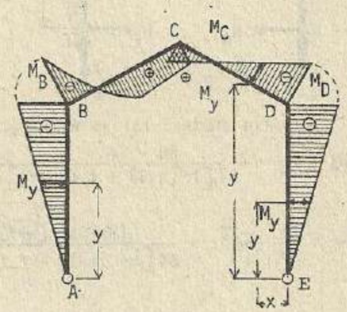
(259)



$$k = \frac{I_2 h}{I_1 s}$$



$$V_A = \frac{a\,(2L-a)}{2L} \cdot w \qquad V_E = \frac{a^2}{2L} \cdot w$$

$$H_A = H_E = H = \frac{2Lh\,(3L-2a) + f\,(3L^2 - 2a^2)}{8L^2 \left[h^2 \;\, (k+3) + f\,(3h+f) \right]} \cdot wa^2$$

$$M_B = M_D = -\frac{2Lh\,(3L-2a) + f(3L^2-2a^2)}{8L^2\left[h^2\,(k+3) + f\,(3h+f)\right]}\,wa^2h$$

$$M_C = -\frac{2Lh\,(3L-2a) + f\,(3L^2-2a^2)}{8L^2\left[\,h^2\,(k+3) + f\,(3h+f)\,\right]}\,wa^2\cdot(h+f) + \frac{wa^2}{4}$$

Esfuerzo de momento por flexión en las columnas AB y ED: $M_y = -H_Ay = -H_Ey$

Momento por flexión en la viga CD : $M_x = -H_Ey + V_Ex$

(y) representa la distancia perpendicular desde el punto (E), y
(x) la distancia lateral desde el mismo punto.

Esfuerzo de flexión en la viga BC :

$$M_x = -H_Ay + V_Ax - \frac{wx^2}{2} \qquad x \leq a$$

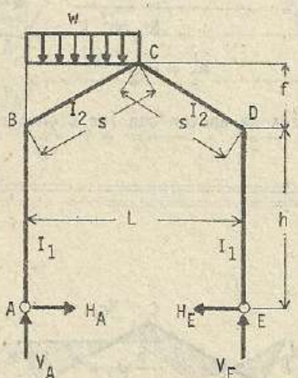
$$M_x = -H_Ay + V_Ax - \frac{wa\,(2x-a)}{2} \qquad x \geq a$$

Aquí, (y) representa la distncia perpendicular desde el punto (A), y
(x) la distancia lateral desde el mismo punto.

Entre (x) e (y) existe una relación constante: $xf = \frac{L}{2}\,(y-h) \quad \therefore \quad x = \frac{L\,(y-h)}{2f}$

o también : $y = \left(\frac{2xf}{L} + h\right)$

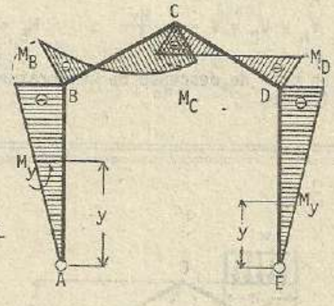
(260)



$$k = \frac{I_2h}{I_1s}$$

$$V_A = \frac{3}{8}\cdot wL$$

$$V_E = \frac{1}{8}\cdot wL$$



$$H_A = H_E = \frac{8h + 5f}{64\left[h^2(k+3) + f\,(3h+f)\right]}\cdot wL^2 \; ; \quad M_B = M_D = -\frac{8h^2 + 5hf}{64\left[h^2\,(k+3) + f(3h+f)\right]}\cdot wL^2$$

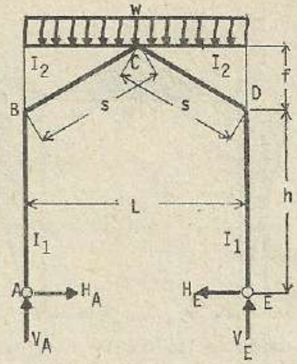
$$M_C = \frac{1}{16}\cdot wL^2 - \frac{(8h+5f)\,(h+f)}{64\left[h^2\,(k+3) + f\,(3h+f)\right]}\cdot wL^2$$

Esfuerzos de flexión en las columnas AB y ED : $M_y = -H_Ay = -H_Ey$

Esfuerzos de flexión en la viga CD : $M_x = -H_Ey + V_Ex$

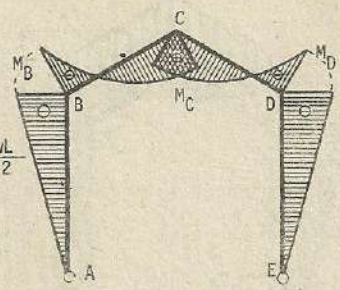
viga BC : $M_x = -H_Ay + V_Ax - \frac{wx^2}{2}$

(261)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{wL}{2}$$



$$H_A = H_E = H = \frac{8h + 5f}{32\left[h^2(k+3) + f\,(3h+f)\right]} \cdot wL^2$$

$$M_B = M_D = -\frac{8h^2 + 5hf}{32\left[h^2(k+3) + f\,(3h+f)\right]} \cdot wL^2 \quad ; \quad M_C = \frac{wL^2}{8} - \frac{(8h+5f)(h+f)}{32\left[h^2(k+3)+f(3k+f)\right]} \cdot wL^2$$

Esfuerzos de flexión en las columnas AB y ED : $M_y = -H_A y = -H_E y$

Esfuerzos de flexión en las vigas BC y DC :

$$M_x = -H_A y + \frac{wx}{2}\cdot(L-x) = -H_A\left(\frac{2fx}{L} + h\right) + \frac{wx}{2}\cdot(L-x)$$

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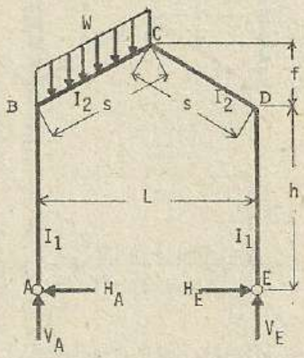
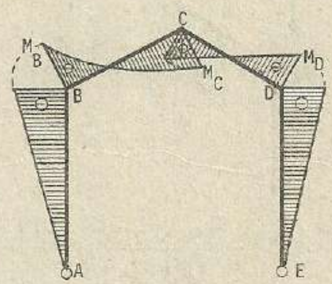
(262)



$$k = \frac{I_2 h}{I_1 s}$$



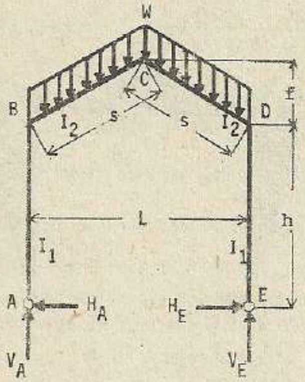
W : Resultante de las fuerzas externas

$$H_A = H_E = H = -\frac{WL}{64} \cdot \frac{8h + 5f}{h^2\,(k+3) + f(3h+f)}$$

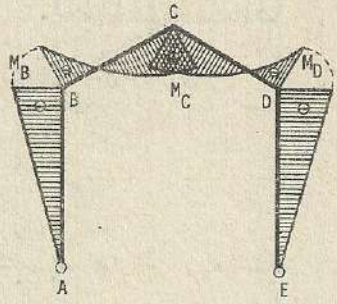
$$V_E = \frac{WL}{4} \qquad V_A = W - V_E$$

Nota : Similar al problema (260)

(263)



$$k = \frac{I_2 h}{I_1 s}$$



W : Resultante de las fuerzas externas

$$H_A = H_E = H = -\frac{WL}{32}\cdot\frac{8h+5f}{h^2(k+3)+f(2h+f)}; \quad V_A = V_E = \frac{WL}{2}$$

Problema similar a (261)

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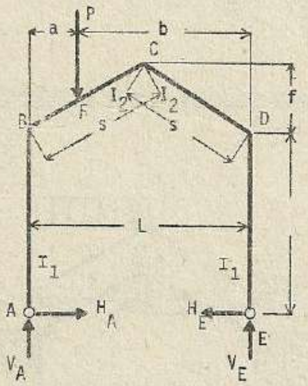
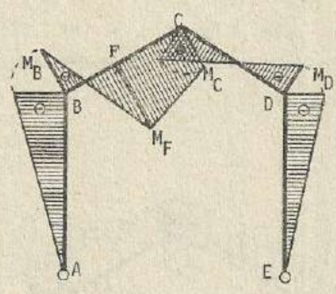
(264)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = \frac{Pb}{L}$$

$$V_E = \frac{Pa}{L}$$



$$H_A = H_E = \frac{a\left[6hbL + f(3L^2 - 4a^2)\right]}{4L^2\left[h^2(k+3) + f(3h+f)\right]}\cdot P \quad ; \quad M_B = M_D = -\frac{ah\left[6hbL + f(3L^2 - 4a^2)\right]}{4L^2\left[h^2(k+3) + f(3h+f)\right]}\cdot P$$

$$M_C = \frac{Pa}{2} - \frac{a\left[6hbL + f(3L^2-4a^2)\right](h+f)}{4L^2\left[h^2(k+3) + f(3h+f)\right]}\cdot P; \quad M_F = \frac{Pab}{L} - \frac{a(hL+2fa)\left[6hbL+f(3L^2-4a^2)\right]}{4L^3\left[h^2(k+3) + f(3h+f)\right]}\cdot P$$

Esfuerzo de flexión en las columnas AB y ED : $M_y = -H_A y = -H_E y$

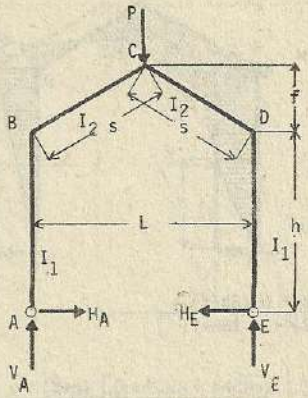
viga CD : $M_x = V_E x - H_E\left(h + \frac{2fx}{L}\right)$

viga BC :

$$M_x = V_A x - H_A\left(h + \frac{2fx}{L}\right) \qquad x \leq a$$

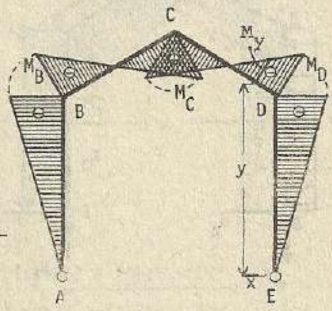
$$M_x = V_A x - H_A\left(h + \frac{2fx}{L}\right) - P(x-a) \qquad x \geq a$$

(265)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{P}{2}$$



$$H_A = H_E = \frac{3h + 2f}{8\left[h^2(k+3) + f(3h+f)\right]} \cdot PL$$

$$M_B = M_D = - \frac{3h^2 + 2hf}{8\left[h^2(k+3)+f(3h+f)\right]} \cdot PL \quad ; \quad M_C = \frac{PL}{4} - \frac{(3h+2f)(h+f)}{8\left[h^2(k+3)+f(3h+f)\right]} \cdot PL$$

Esfuerzo de flexión en las columnas AB o ED : $M_y = - H_A y = - H_E y$

Viga BC o DC : $M_x = \frac{P}{2} x - H_A \left(h + \frac{2fx}{L}\right)$

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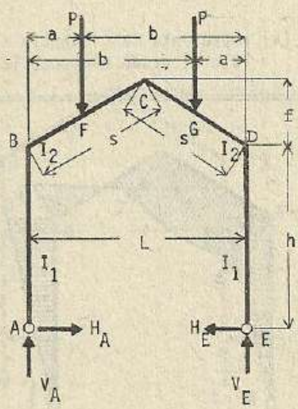
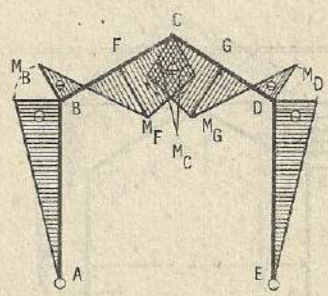
(266)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = P$$



$$H_A = H_E = \frac{a\left[6hbL + f(3L^2-4a^2)\right]}{2L^2\left[h^2(k+3) + f(3h+f)\right]} P$$

$$M_B = M_D = - \frac{ah\left[6hbL + f(3L^2-4a^2)\right]}{2L^2\left[h^2(k+3) + f(3h+f)\right]} \cdot P \quad ; \quad M_C = Pa - H_A(h+f)$$

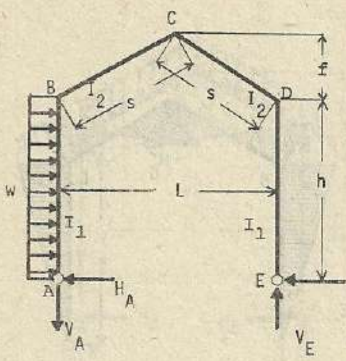
$$M_F = Pa - H_A\left(h + \frac{2af}{L}\right)$$

Esfuerzo de flexión en la columna AB o ED : $M_y = - H_A y = - H_E y$

viga BC o DC : $M_x = Pa - H_A\left(h + \frac{2fx}{L}\right) \quad x \leq a$

$M_x = Pa - H_A\left(h + \frac{2fx}{L}\right) - P(x-a) \quad x \geq a$

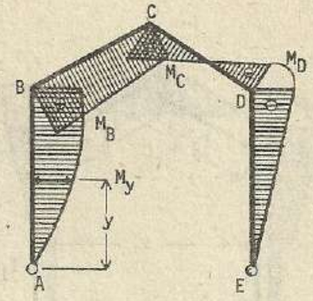
(267)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{h^2}{2L} \cdot w$$

$$H_A = wh - H_E$$



$$H_E = \frac{5hk + 6\,(2h+f)}{16\left[h^2\,(k+3) + f\,(3h+f)\right]} \cdot wh^2$$

$$M_B = \left(1 - \frac{5h^2k + 6\,(2h+f)h}{8\left[h^2(k+3) + f\,(3h+f)\right]}\right) \cdot \frac{wh^2}{2} \quad ; \quad M_C = \left(1 - \frac{\left[5hk + 6(2h+f)\right]\,(h+f)}{4\left[h^2(k+3) + f(3h+f)\right]}\right) \cdot \frac{wh^2}{4}$$

$$M_D = -\frac{5h^2k + 6\,(2h+f)\,h}{16\left[h^2\,(k+3) + f\,(3h+f)\right]} \cdot wh^2$$

Esfuerzo de flexión columna AB : $M_y = H_A y - \frac{wy^2}{2}$

columna ED : $M_y = -\,H_E y$

Esfuerzo de flexión viga BC :

$M_x = H_A\,(h + \frac{2fx}{L}) - wh\,(\frac{h}{2} + \frac{2fx}{L}) - V_A x$ (x) representa la distancia lateral desde el punto (A)

viga CD :

$M_x = -\,H_E\,(h + \frac{2fx}{L}) + V_E x$ aqui. (x) representa la distancia lateral desde el punto (E)

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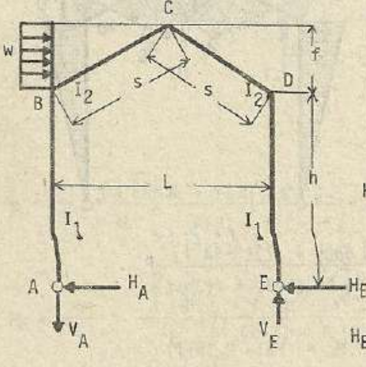
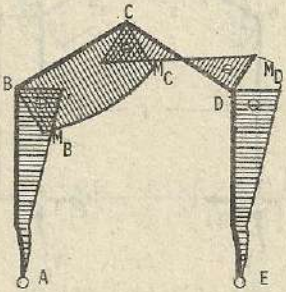
(268)



$$k = \frac{I_2 h}{I_1 s}$$

$$H_A = wh - H_E$$



$$H_E = \frac{8h^2\,(k+3) + 5f\,(4h+f)}{16\left[h^2\,(k+3) + f\,(3h+f)\right]}\, wf$$

$$V_A = V_E = \frac{wf\,(2h+f)}{2L}$$

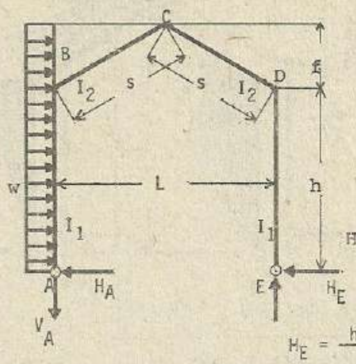
$M_B = (wf - H_E)h$; $M_C = \frac{wf\,(2h+f)}{4} - H_E\,(h+f)$; $M_D = -\,H_E h$

Esfuerzo de flexión columna AB ; $M_y = (wf - H_E)\,y$, $M_y = -\,H_E y$ (columna ED)

Esfuerzo de flexión viga ; BC ; $M_x = (wf - H_E)y - \frac{w\,(y-h)^2}{2} - V_A\,\frac{L\,(y-h)}{2f}$

viga ; (CD) $M_x = -\,H_E y + V_E \cdot \frac{L\,(y-h)}{2f}$

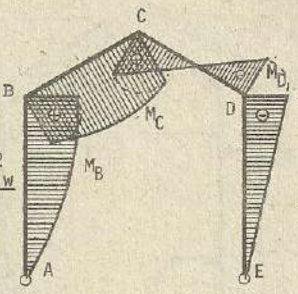
(269)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{(h+f)^2 w}{2L}$$

$$H_A = w(h+f) - H_E$$



$$H_E = \frac{h^3 (5k+12) + 2h^2 f (4k+15) + 20hf^2 + 5f^3}{16 \left[h^2 (k+3) + f (3h+f) \right]} \cdot w$$

$M_B = w \cdot (\frac{h}{2} + f)\, h - H_E h$; $M_C = \frac{w (h+f)^2}{4} - H_E (h+f)$; $M_D = - H_E h$

Esfuerzo de flexión en la columna AB : $M_y = w (h+f) y - H_E y - \frac{wy^2}{2}$

columna ED : $M_y = - H_E y$

Esfuerzo de flexión en la viga BC : $M_x = w (h+f) y - H_E y - \frac{wy^2}{2} - V_A \frac{L (y-h)}{2f}$

viga DC : $M_x = - H_E y + V_E \cdot \frac{L (y-h)}{2f}$

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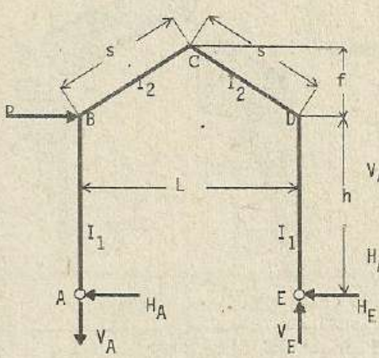
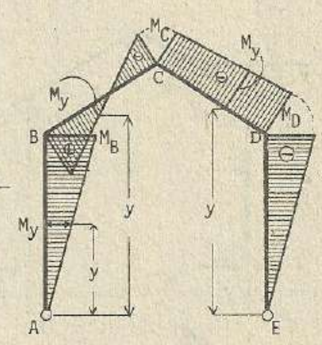
(270)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{Ph}{L}$$

$$H_A = P - H_E$$



$$H_E = \frac{Ph \left[2hk + 3 (2h+f) \right]}{4 \left[h^2 (k+3) + f (3h+f) \right]}$$

$M_B = (P - H_E) h$; $M_C = \frac{Ph}{2} - H_E (h+f)$; $M_D = - H_E h$

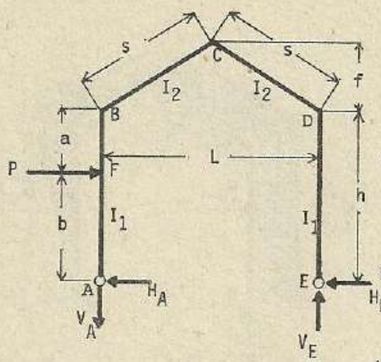
Esfuerzo de flexión en la columna AB : $M_y = Py - H_E y$

columna ED : $M_y = - H_E y$

viga BC : $M_x = Ph - H_E y - \frac{Ph (y-h)}{2f}$

viga CD : $M_x = - H_E y + \frac{Ph (y-h)}{2f}$

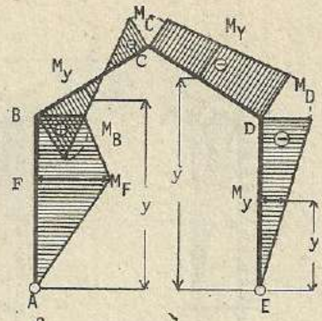
(271)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{Pb}{L}$$

$$H_A = P - H_E$$



$$H_E = \frac{Pb\left[k\left(3h - \frac{b^2}{h}\right) + 3\,(2h+f)\right]}{4\left[h^2(k+3) + f\,(3h+f)\right]}$$

$M_F = H_A b$; $M_B = Pb - H_E h$; $M_C = \frac{Pb}{2} - H_E(h+f)$; $M_D = -H_E h$

Esfuerzo de flexión en la columna AB : $M_y = (P - H_E)\,y$, $y \leq b$

$M_y = Pb - H_E y$ $y \geq b$

columna ED : $M_y = -H_E y$

viga BC : $M_x = Pb - H_E y - V_A \cdot \frac{(y-h)L}{2f}$

viga CD : $M_x = -H_E y + V_E \cdot \frac{(y-h)L}{2f}$

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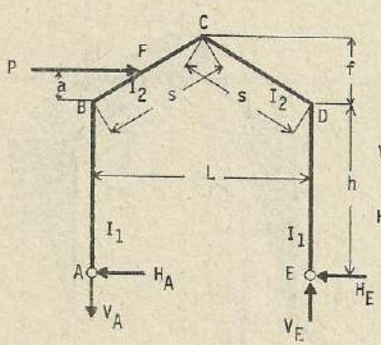
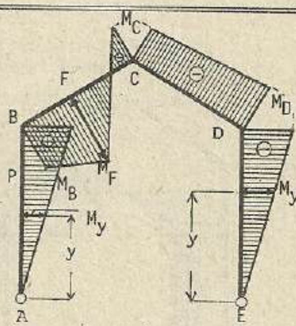
(272)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{h+a}{L} \cdot P$$

$$H_A = P - H_E$$



$$H_E = \frac{2h^2 k + 3\,(h+a)\,(2h+f) - \frac{a^2}{f}\,(3h+a)}{4\left[h^2\,(k+3) + f\,(3h+f)\right]} \cdot P$$

$M_F = H_A\,(h+a) - \frac{P(h+a)\,a}{2f}$; $M_C = \frac{h+a}{2} \cdot P - H_E\,(h+f)$; $M_D = -H_E h$

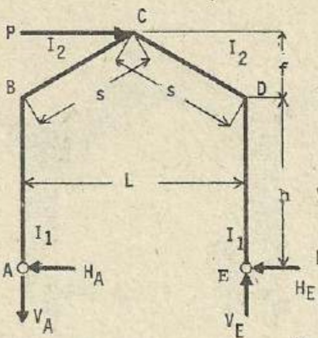
Esfuerzo de flexion en la columna AB ; $M_y = H_A y$; columna ED : $M_y = -H_E y$

viga BC : $M_x = H_A y - V_A \cdot \frac{(y-h)L}{2f}$ $y \leq h+a$

$M_x = H_A y - V_A \cdot \frac{(y-h)L}{2f} - P\,\frac{(y-h-a)\,L}{2f}$ $y \geq h+a$

viga DC ; $M_x = -H_E y + V_E \cdot \frac{(y-h)L}{2f}$

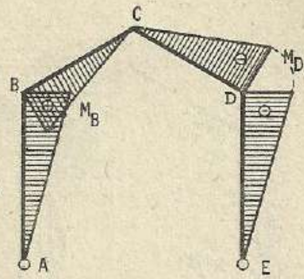
(273)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{(h+f)}{L} \cdot P$$

$$H_A = H_E = \frac{P}{2}$$



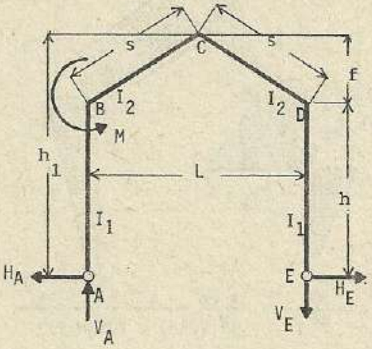
$$M_B = \frac{Ph}{2} \quad ; \quad M_C = 0 \quad ; \quad M_D = -\frac{Ph}{2}$$

Esfuerzo de flexión en la columna AB : $M_y = \frac{P}{2}y$; columna ED : $M_y = -\frac{P}{2}y$

Esfuerzo de flexión en la viga BC : $M_x = \frac{Py}{2} - \frac{P(h+f)(y-h)}{2f}$

viga DC : $M_x = -\frac{Py}{2} + \frac{P(h+f)(y-h)}{2f}$

(274)

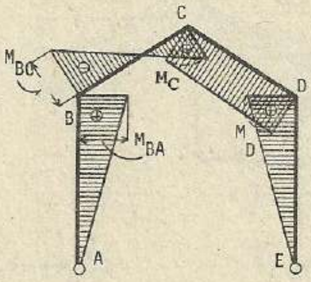


$$k = \frac{I_2 h}{I_1 s}$$

$$n = \frac{f}{h}$$

$$h_1 = (1+n)h$$

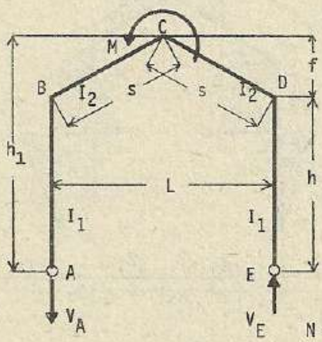
$$N = 3 + k + n(3+n)$$



$$H_A = H_E = H = \frac{3(2+n)M}{4Nh} \quad ; \quad V_A = V_E = V = \frac{M}{L}$$

$$M_{BA} = Hh \quad ; \quad M_{BC} = Hh - M \quad ; \quad M_C = Hh_1 - \frac{M}{2} \quad ; \quad M_D = Hh$$

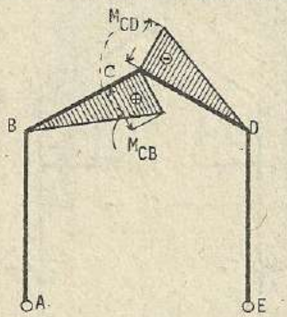
(275)



$$k = \frac{I_2 h}{I_1 s}$$

$$n = \frac{f}{h}$$

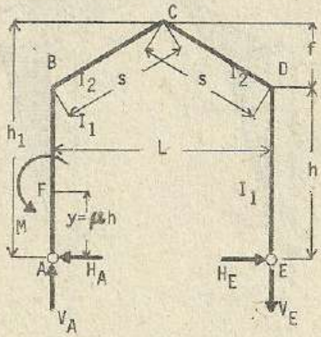
$$h_1 = (1+n)h$$



$$N = 3 + k + n(3+n)$$

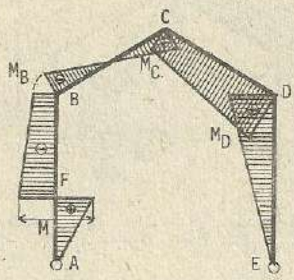
$$H = 0 \quad ; \quad V_A = V_E = V = \frac{M}{L} \quad ; \quad M_B = M_D = 0 \quad ; \quad M_{CB} = -M_{CD} = \frac{M}{2}$$

(276)



$$k = \frac{I_2 h}{I_1 s}$$

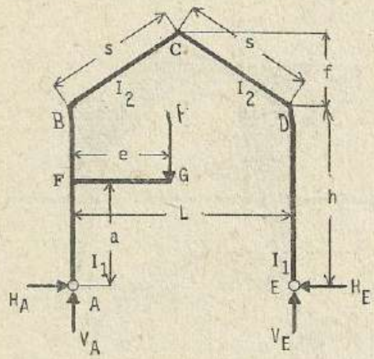
$$V_E = \frac{M}{L}$$



$$H_A = H_E = H = \frac{3}{4}\cdot M \cdot \frac{2h + f + kh(1-\mu^2)}{h^2 (k+3) + f(3h+f)}$$

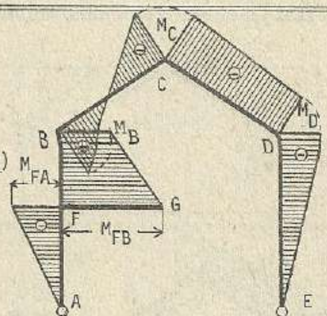
$M_B = Hh - M$; $M_C = H(h+f) - \frac{M}{2}$; $M_D = Hh$

(277)



$$k = \frac{I_2 h}{I_1 s}$$

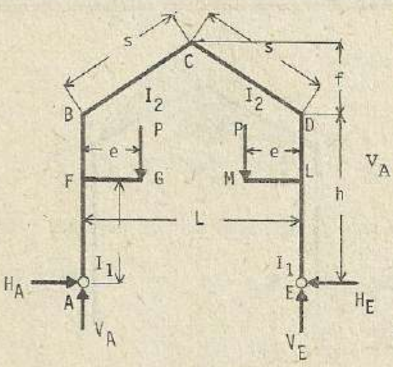
$$V_A = \frac{P(L-e)}{L}$$



$$H_A = H_E = H = \frac{3Pe}{4h} \cdot \frac{k(h^2 - a^2) + h(2h+f)}{h^2 (k+3) + f(3h+f)}$$

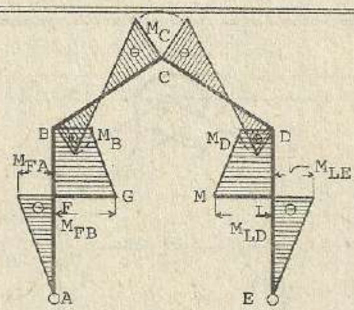
$M_{FA} = - Ha$; $M_{FB} = Pe - Ha$; $M_B = Pe - Hh$; $M_C = \frac{Pe}{2} H(h+f)$; $M_D = Hh$

(278)



$$k = \frac{I_2 h}{I_1 s}$$

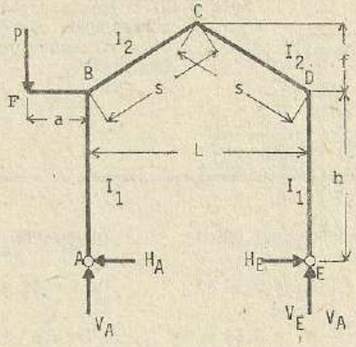
$$V_A = V_E = V = P$$



$$H_A = H_E = H = \frac{3Pe}{2h} \cdot \frac{k(h^2 - a^2) + h(2h+f)}{h^2 (k+3) + f(3h+f)}$$

$M_{FA} = - Ha$; $M_{FB} = Pe - Ha$; $M_B = M_D = Pe - Hh$; $M_C = Pe - H(h+f)$

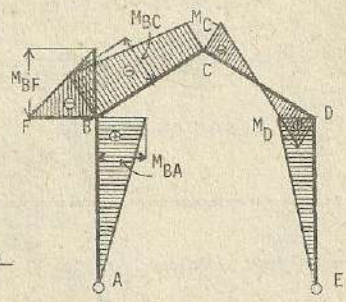
(279)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_E = \frac{Pa}{L}$$

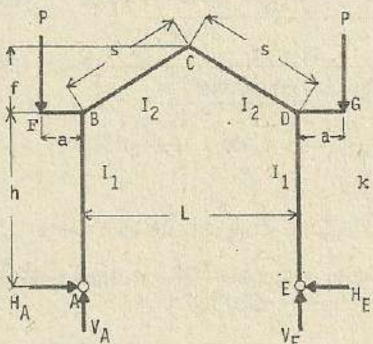
$$V_A = P + V_E = \frac{P\,(L+a)}{L}$$



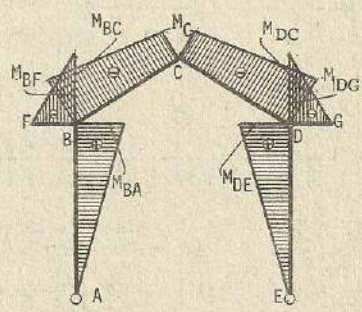
$$H_A = H_E = H = \frac{3}{4} \cdot \frac{Pa\,(2h+f)}{h^2\,(k+3) + f\,(3h+f)} \quad ; \quad M_{BA} = Hh \quad ; \quad M_{BC} = Hh - V_E L$$

$$M_C = H\,(h+f) - V_E \cdot \frac{L}{2} \quad ; \quad M_D = Hh$$

(280)

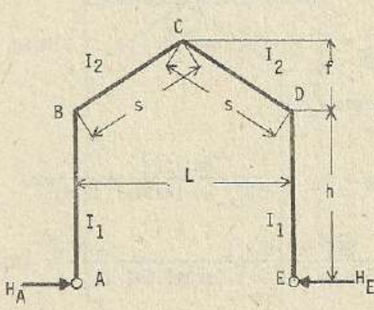


$$k = \frac{I_2 h}{I_1 s}$$

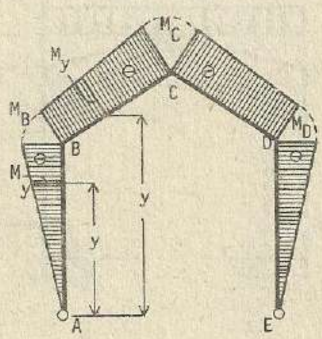


$$H_A = H_E = H = \frac{3}{2} \cdot \frac{Pa\,(2h+f)}{h^2\,(k+3) + f(3h+r)} ; V_A = V_E = P$$

Esfuerzos por cambio de temperatura : **(281)**



$$k = \frac{I_2 h}{I_1 s}$$



$$H_A = H_E = \frac{3E\varepsilon t I_2 L}{2s\left[h^2\,(k+3) + f(3h+f)\right]} \quad ; \quad M_B = M_D = -\frac{3E\varepsilon t I_2 L h}{2s\left[h^2(k+3) + f\,(3h+f)\right]}$$

$$M_C = -\frac{3E\varepsilon t I_2 L\,(h+f)}{2s\left[h^2(k+3) + f\,(3h+f)\right]}$$

Esfuerzos de flexión en las columnas AB y ED :

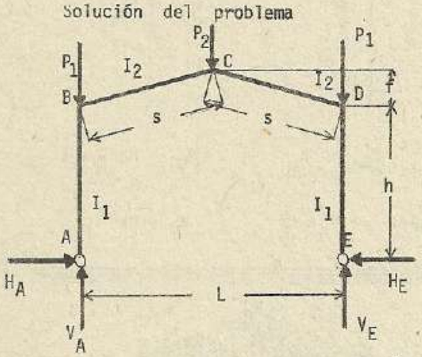
$$M_y = -\frac{3E\epsilon t I_2 L y}{2s\left[h^2(k+3) + f(3h+f)\right]}$$

Nota : el presente problema corresponde cuando hay incremento de temperatura

En las vigas BC y DC :

$$M_x = -\frac{3E\epsilon t I_2 L y}{2s\left[h^2(k+3) + f(3h+f)\right]}$$

Solución del problema



Datos : $P_1 = 1{,}000$ Kg ; $P_2 = 2{,}000$ Kg.

$I_1 = 0.18\ m^2$; $I_2 = 0.35\ m^4$

$s = 8.59$ m ; $L = 16.6$ m

$f = 2.2$ m ; $h = 9.5$ m

$$k = \frac{I_2 h}{I_1 s} = \frac{0.35 \times 9.5}{0.18 \times 8.59} = 2.15$$

$$V_A = V_E = \frac{2{,}000}{2} + 1{,}000 = 2{,}000 \text{ Kg.}$$

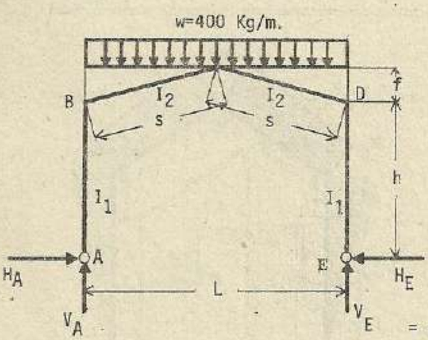
$$H_A = H_E = \frac{3h + 2f}{8\left[h^2(k+3) + f(3h+f)\right]} \cdot P_2 L \quad (265)$$

$$= \frac{3 \times 9.5 + 2 \times 2.2}{8 \times \left[9.5x9.5x5.15 + 2.2x(3x9.5 + 2.2)\right]} \times 2{,}000 \times 16.6 = 257 \text{ Kg.}$$

$$M_B = M_D = -H_A h = -H_E h = -257x9.5 = 2{,}441.5 \text{ Kg - m}$$

$$M_C = -H_A(h+f) + V_A \cdot \frac{L}{2} - P_1 \frac{L}{2} = -257x(9.5+2.2) + 2{,}000x\frac{16.6}{2} - 1{,}000 \times \frac{16.6}{2}$$

$$= 5{,}293.1 \text{ Kg - m}$$



$$k = \frac{I_2 h}{I_1 s} = \frac{0.35x9.5}{0.18x8.59} = 2.15$$

$$V_A = V_E = 400 \times \frac{16.6}{2} = 3.320 \text{ Kg.}$$

desde (261)

$$H_A = H_E = \frac{8h + 5f}{32\left[h^2(k+3)+f(3h+f)\right]} wL^2$$

$$= \frac{8x9.5 + 5x2.2}{32\left[9.5x9.5x5.15 + 2.2x(3x9.5+2.2)\right]} \times 400x16.6^2$$

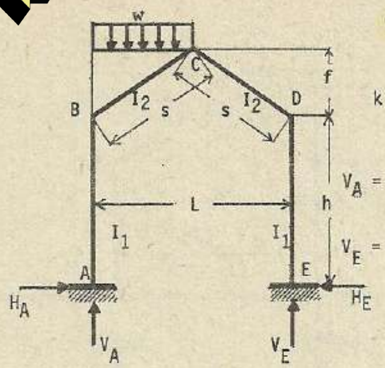
$$= 563 \text{ Kg}$$

$$M_B = M_D = -H_A h = -H_E h = 563x9.5 = -5{,}348.5 \text{ Kg - m}$$

$$M_C = -H_A(h+f) + V_A \cdot \frac{L}{2} - \frac{wL^2}{8} = -563\ (9.5+2.2) + 3{,}320 \times \frac{16.6}{2} - \frac{400x16.6x16.6}{8}$$

$$= 7{,}190.9 \text{ Kg - m}$$

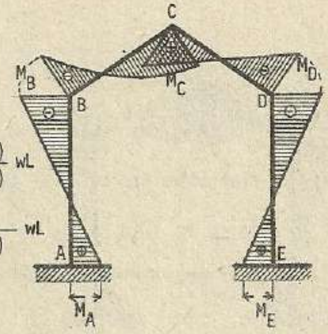
(282)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = \frac{(44k+15)}{32\,(3k+1)}\,wL$$

$$V_E = \frac{(4k+1)}{32\,(3k+1)}\,wL$$



$$H_A = H_E = \frac{[k\,(4h+5f)+f]\,wL^2}{16\,[(hk+f)^2+4k(h^2+hf+f^2)]}; \quad M_A = \left[\frac{hk(8h+15f)+(6h-f)\,f}{(hk+f)^2+4k(h^2+hf+f^2)} - \frac{3}{2(3k+1)}\right]\frac{wL^2}{96}$$

$$M_B = -\left[\frac{hk(16h+15f)+f^2}{(hk+f)^2+4k\,(h^2+hf+f^2)} + \frac{3}{2\,(3k+1)}\right]\cdot\frac{wL^2}{96}$$

$$M_D = -\left[\frac{hk\,(16h+15f)+f^2}{(hk+f)^2+4k\,(h^2+hf+f^2)} - \frac{3}{2(3k+1)}\right]\cdot\frac{wL^2}{96}$$

$$M_E = \left[\frac{hk\,(8h+15f)+6(h-f)f}{(hk+f)^2+4k\,(h^2+hf+f^2)} + \frac{3}{2\,(3k+1)}\right]\cdot\frac{wL^2}{96}$$

$$M_C = -\,H_A\,(h+f) + M_E + V_E\cdot\frac{L}{2}$$

Esfuerzo de flexión en la columna AB : $M_y = M_A - H_A y$, columna ED : $M_y = M_E - H_E y$

viga BC : $M_x = M_A - H_A y + V_A\cdot\frac{(y-h)L}{2f} - \frac{(y-h)^2\,wL^2}{2f^2}$

viga CD : $M_x = M_E - H_E y + V_E\cdot\frac{(y-h)L}{2f}$

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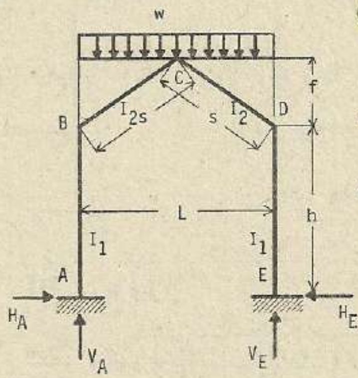
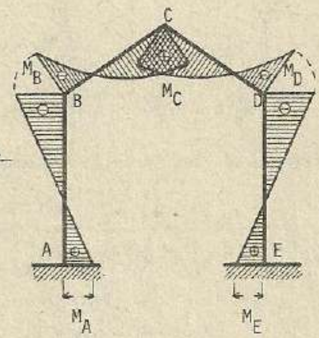
(283)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{wL}{2}$$



$$H_A = H_E = \frac{k\,(4h+5f)+f}{8\,[(hk+f)^2+4k(h^2+hf+f^2)]}\cdot wL^2$$

$$M_A = M_E = \frac{hk\,(8h+15f)+f\,(6h-f)}{48\,[(hk+f)^2+4k\,(h^2+hf+f^2)]}\,.\,wL^2$$

$$M_B = M_D = -\frac{hk\,(16h+15f)+f^2}{48\left[(hk+f)^2+4k\,(h^2+hf+f^2)\right]}wL^2$$

$$H_C = -H_A\,(h+f) + M_A + \frac{wL^2}{8}$$

Esfuerzos de flexión en las columnas AB y ED; $M_y = M_A - H_A y = M_E - H_E y$

vigas BC y CD ; $M_x = M_A - H_A y + \frac{wL^2\,(y-h)}{4f} - \frac{wL^2\,(y-h)^2}{8f^2}$

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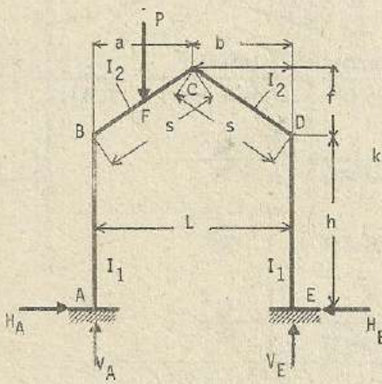
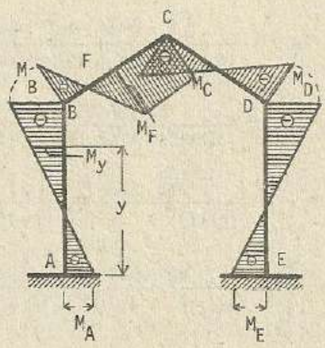
(284)



$$k = \frac{I_2 h}{I_1 s}$$



$$V_A = \frac{Pb\left[3kL^2 + b\,(L+2a)\right]}{(3k+1)L^3}; \qquad V_E = \frac{Pa\left[3kL^2 + a\,(L+2b)\right]}{(3k+1)L^3}$$

$$H_A = H_E = \frac{Pa\left[3kL^2\,(h+f) - 4a^2f(k+1) - 3aL\,(hk-f)\right]}{\left[(hk+f)^2\;\;4k\,(h^2+hf+f^2)\right]L^2}$$

$$\begin{matrix} M_A \\ M_E \end{matrix} = \left[\frac{3kLbh^2 + 3hfL(2a+kL) - f^2L(L-4a) - 4a^2hf(k+2) - 4a^2f^2}{(hk+f^2) + 4k\,(h^2+hf+f^2)} \pm \frac{b\,(b-a)}{(3k+1)}\right]\cdot\frac{Pa}{2L^2}$$

$$M_B = -H_A h + M_A \;; \qquad M_C = -H_A\,(h+f) + M_E + V_E\cdot\frac{L}{2} \;; \qquad M_D = -H_E h + M_E$$

$$M_F = -H_A\left(h + \frac{2af}{L}\right) + M_A + V_A a$$

Esfuerzo de flexión en la columna AB : $M_y = M_A - H_A y$

columna ED : $M_y = M_E - H_E y$

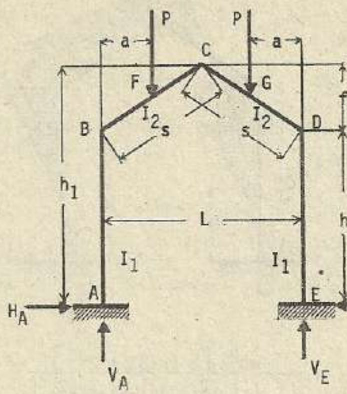
viga BC : $M_x = M_A - H_A y + V_A\cdot\frac{(y-h)\,L}{2f}$ $\qquad y \leq h + \frac{2af}{L}$

$M_x = M_A - H_A y + V_A\cdot\frac{(y-h)L}{2f} - P\left[\frac{(y-h)L}{2f} - a\right]$ $\qquad y \geq h + \frac{2af}{L}$

viga CD :

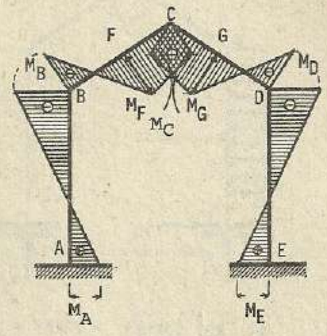
$$M_x = M_E - H_E y + V_E\cdot\frac{(y-h)L}{2f}$$

(285)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = V = P$$



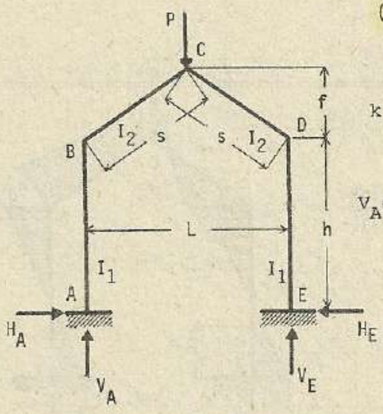
$$H_A = H_E = H = \frac{2Pa}{L^2} \cdot \frac{3L^2k\,(h+f) + 4a^2k\,(k+1) - 3aL\,(hk-f)}{(hk+f)^2 + 4k\,(h^2+hf+f^2)}$$

$$M_{AB} = M_{ED} = \frac{Pa}{L^2} \cdot \frac{1}{(hk+f)^2 + 4k \cdot (h^2+hf+f^2)} \times \Big[2L^2h^2k + 3hLf(2a+Lk) - f^2L\,(L+4a)$$
$$4a^2hf(k+2) - 4a^2f^2 \Big]$$

$M_{BA} = M_{BC} = M_{DC} = M_{DE} = -Hh + M_{AB}$; $M_F = -H\left(h + \frac{2fa}{L}\right) + M_{BA} + Va$

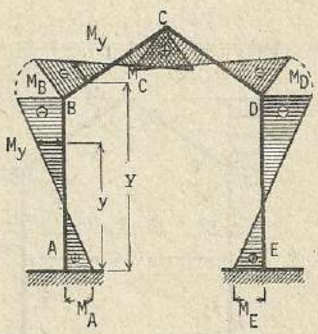
$M_{CD} = M_{CB} = -H\,(h+f) + M_{AB} + V \cdot \frac{L}{2}$

(286)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{P}{2}$$



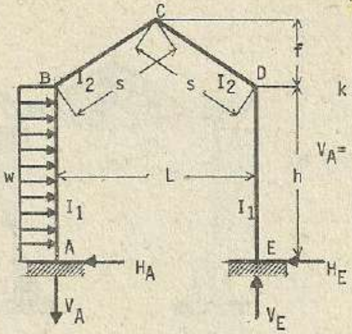
$H_A = H_E = \dfrac{k\,(3h+4f) + f}{4\left[(hk+f)^2 + 4k(h^2+hf+f^2)\right]}$; $M_A = M_E = \dfrac{\left[h^2k + hf\,(2k+1)\right] PL}{4\left[(hk+f)^2 + 4k\,(h^2+hf+f^2)\right]}$

$M_B = M_D = -\dfrac{kh\,(h+f)PL}{2\left[(hk+f)^2 + 4k\,(h^2+hf+f^2)\right]}$; $M_C = -H_A\,(h+f) + M_A + \dfrac{PL}{4}$

Esfuerzos de flexión en las columnas AB y ED ; $M_y = M_A - H_A y = M_E - H_E y$

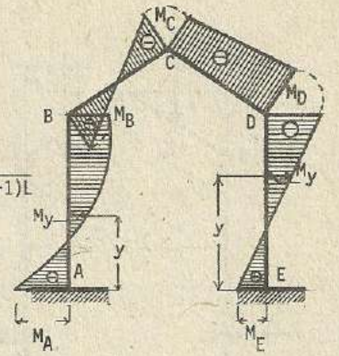
vigas BC y CD : $M_x = M_A - H_A y + \dfrac{P \cdot (y-h)L}{4f}$

(287)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{kwh^2}{2\,(3k+1)L}$$



$$H_A = \frac{wh\left[kh^2\,(3k+13) + 22khf + 4f^2(4k+1)\right]}{4\left[(hk+f)^2 + 4k\,(h^2+hf+f^2)\right]} \;;\; H_E = \frac{wh^2k\left[h\,(k+3) + 2f^2\right]}{4\left[(hk+f)^2 + 4k(h^2+hf+f^2)\right]}$$

$$M_A = -\frac{wh^2}{24}\left[\frac{(12k+6)}{(3k+1)} + \frac{h^2\,k(k+6) + kf\,(15h+16f) + 6f^2}{(hk+f)^2 + 4k\,(h^2+hf+f^2)}\right]; \quad M_B = M_A + \frac{wh^2}{2} - H_E h$$

$$M_E = \frac{wh^2}{24}\left[\frac{(12k+6)}{(3k+1)} - \frac{h^2\,k(k+6) + Kf\,(15h+16f) + 6f^2}{(hk+f)^2 + 4k\,(h^2+hf+f^2)}\right]; \quad M_C = M_E + V_E \cdot \frac{L}{2} - H_E\,(h+f)$$

$$M_D = M_E - H_E h$$

Esfuerzos de flexión en la columna AB : $M_y = M_A + H_A y - \frac{wy^2}{2}$

columna ED : $M_y = H_E - H_E y$

Viga BC : $M_x = M_A + H_A y - wh\,(y - \frac{h}{2}) - V_A\,\frac{(y-h)L}{2f}$

Viga CD : $M_x = M_E - H_E y + V_E \cdot \frac{(y-h)L}{2f}$

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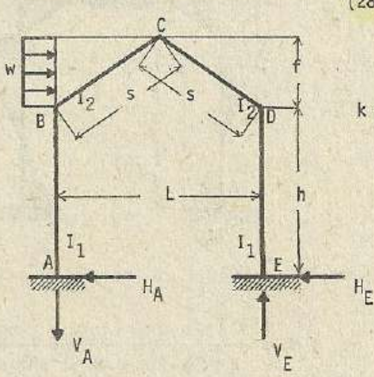
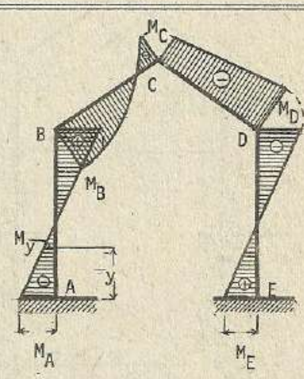
(288)



$$k = \frac{I_2 h}{I_1 s}$$



$$V_A = V_E = \frac{\left[5f + 12k\,(h+f)\right]}{8\,(3k+1)\,L} \cdot wf$$

$$H_A = \frac{\left[2kh^2\,(k+4) + 14khf + f^2\,(11k+3)\right]}{4\left[(hk+f)^2 + 4k\cdot(h^2+hf+f^2)\right]} \cdot wf \;;\quad H_E = \frac{\left[5kf\,(2h+f) + 2kh^2\,(k+4) + f^2\right]}{4\left[(hk+f)^2 + 4k\,(h^2 + hf + f^2)\right]} \cdot wL$$

$$M_A = -\frac{wf}{24}\cdot\left[\frac{12h\,(3k+2) + 3f}{(6k+2)} + \frac{f\,\{\,hk\,(4h+9f) + f(6h+f)\}}{(hk+f)^2 + 4k\,(h^2 + hf + f^2)}\right]$$

$$M_E = \frac{wf}{24}\left[\frac{12h(3k+2)+3f}{(6k+2)} - \frac{f\{hk(4h+9f)+f(6h+f)\}}{(hk+f)^2+4k(h^2+hf+f^2)}\right] \quad ; M_B = M_A + H_A h$$

$$M_C = M_E - H_E(h+f) + V_E\frac{L}{2} \qquad M_D = M_E - H_E h$$

Esfuerzo de flexión en la columna AB : $M_y = M_A + H_A y$

columna ED : $M_y = M_E - H_E y$

Viga BC : $M_x = M_A + H_A y - \frac{w(y-h)^2}{2} - V_A\frac{(y-h)L}{2f}$

Viga CD : $M_x = M_E - H_E y - V_E\frac{(y-h)L}{2f}$

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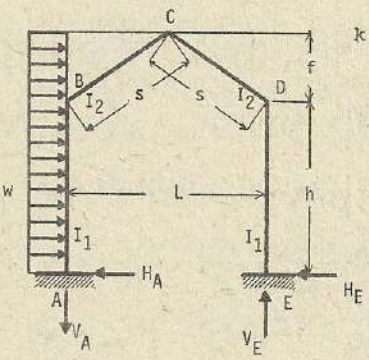
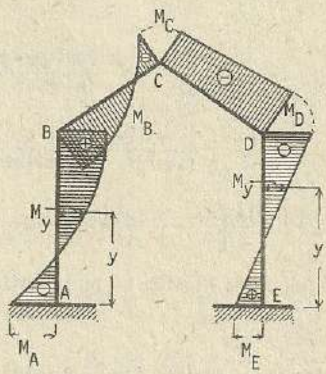
(289)



$$k = \frac{I_2 h}{I_1 s}$$



$$V_A = V_E = \frac{\{f^2(5+12k)+4kh(3f+h)\}}{8(3k+1)L}w$$

$$H_A = \frac{1}{4\{(hk+f)^2+4k(h^2+hf+f^2)\}} \times \left[\{kh^3(3k+13)+f^3(11k+3)+2h^2kf(k+15) \; 2hf^2(15k+2)\}w\right]$$

$$H_E = \frac{\{kh^3(k+3)+f^3(5k+1)+10khf(h+f)+2k^2h^2f\}}{4\{(hk+f)^2+4k(h^2+hf+f^2)\}}w$$

$$M_A = -\frac{w}{24}\left[\frac{12kh(2h+3f)+12h(h+2f)+3f^2}{(6k+2)} + \frac{kh^2\{h^2(k+6)+f(15h+16f)\} + f^2\{kh(4h+9f)+f(6h+f)+6h^2\}}{(hk+f)^2+4k(h^2+hf+f^2)}\right]$$

$$M_E = \frac{w}{24}\left[\frac{12kh(2h+3f)+12h(h+2f)+3f^2}{(6k+2)} - \frac{kh^2\{h^2(k+6)+f(15h+16f)\} + f^2\{hk(4h+9f)+f(6h+f)+6h^2\}}{(hk+f)^2+4k(h^2+hf+f^2)}\right]$$

$$M_B = M_A + \frac{wh^2}{2} - H_E h \quad ; \quad M_C = M_E + V_E\frac{L}{2} - H_E(h+f) \quad ; \quad M_D = M_E - H_E h$$

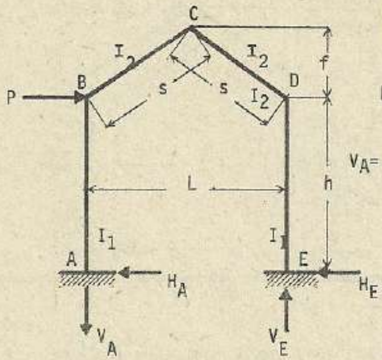
Esfuerzo de flexión en la columna AB : $M_y = M_A + H_A y - \frac{wy^2}{2}$

columna ED : $M_y = M_E - H_E y$

viga BC : $M_x = M_A + H_A y - \frac{wy^2}{2} - V_A\frac{(y-h)L}{2f}$

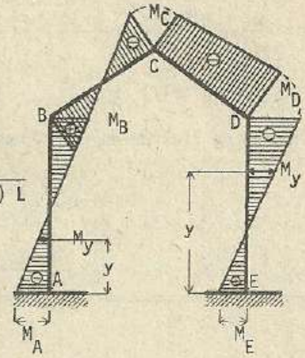
viga CD : $M_x = M_E - H_E y + V_E\frac{(y-h)L}{2f}$

(290)



$$k = \frac{I_2 h}{I_1 s}$$

$$V_A = V_E = \frac{3kPh}{2\,(3k+1)\,L}$$



$$H_A = \frac{\left[kh^2 + 2h^2 + k\,(4h^2+9hf+8f^2)\right]}{2\left[(kh+h)^2 + 4k\,(h^2+hf+f^2)\right]}\,P \quad ; \quad H_E = \frac{kh\left[h\,(k+4) + 3f\right]}{2\left[(kh+f)^2 + 4k\,(h^2+hf+f^2)\right]}\,P$$

$$M_A = -\frac{Ph}{2}\left[\frac{f\,(kh+f+2kf)}{(kh+f)^2 + 4k\,(h^2+hf+f^2)} + \frac{3k+2}{6k+2}\right] \; ; \; M_B = M_A + H_A h \; ; \; M_D = M_E - H_E h$$

$$M_E = \frac{Ph}{2}\left[-\frac{f\,(kh+f+2kf)}{(kh+f)^2 + 4k\,(h^2+hf+f^2)} + \frac{3k+2}{6k+2}\right] \; ; \; M_C = -M_E - H_E\,(h+f) + V_E\,\frac{L}{2}$$

Esfuerzo de flexión en la columna AB : $M_y = M_A + H_A y$

columna ED : $M_y = M_E - H_E y$

Viga BC : $M_x = M_A + H_A y - P\,(y-h) - V_A\,\frac{(y-h)L}{2f}$

Viga CD : $M_x = M_E - H_E y + V_E\,\frac{(y-h)L}{2f}$

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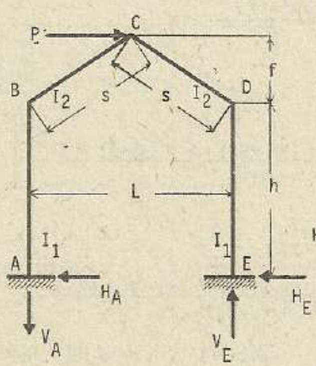
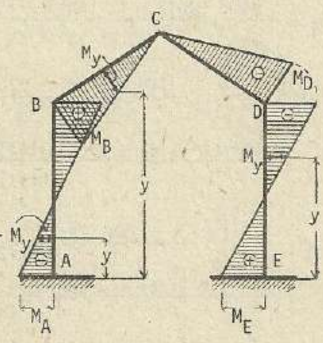
(291)



$$k = \frac{I_2 h}{I_1 s}$$

$$H_A = H_E = \frac{P}{2}$$



$$V_A = V_E = \frac{\left[2f + 3k\,(h+2f)\right]}{2\,(3k+1)\,L}\,P$$

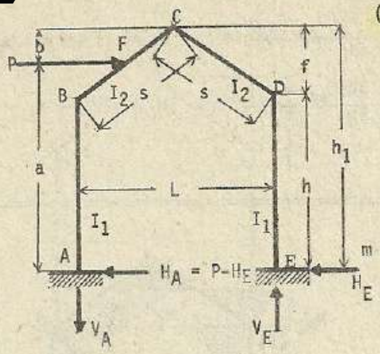
$$M_A = -\frac{(3k+2)}{4(3k+1)}\cdot Ph \quad ; \quad M_E = \frac{(3k+2)}{4(3k+1)}\cdot Ph \quad ; \quad M_B = \frac{3k}{4(3k+1)}\,Ph \quad ; \quad M_C = 0$$

$$M_D = -\frac{3k}{4\,(3k+1)}\cdot Ph$$; Esfuerzo de flexión columna AB : $M_y = M_A + H_A y$

columna ED : $M_y = M_E - H_E y$

Viga BC : $M_x = M_A + H_A y - V_E\,\frac{(y-h)L}{2f}$; viga CD : $M_x = M_E - H_E y + V_E\cdot\frac{(y-h)L}{2f}$

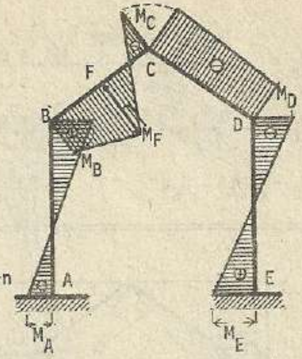
(292)



$$k = \frac{I_2 h}{I_1 s}$$

$$n = \frac{f}{h}$$

$$m = \frac{h_1}{h} = 1+n$$



$N_1 = 4km^2 + (k-n)^2$; $N_2 = 1 + 3k$

$\alpha_0 = (1+k)\,(k-n)^2$; $\alpha_1 = 3k+4nk+n$; $\alpha_2 = 3k + 2nk - n$

$\beta_1 = k+2nk+n$; $\beta_2 = (1+n)\,(k-n)$; $\beta_3 = 2 + 3k$

$\gamma_1 = k\,(4+k+3n)$; $\gamma_2 = k\,(2-k+3n)$; $\gamma_3 = 2kmn + \gamma_1$

$C_{BC} = \frac{-P\,(f-b)\,b^2}{f^2}$ $C_{CB} = -\frac{P\,(f-b)^2 b}{f^2}$; $H_{BC} = -\frac{P\,(f-b)b}{2f^2}\,(2f-b)$

$H_E = \frac{\alpha_0 Ph + \alpha_1(Pa-C_{CB}) - \alpha_2 C_{BC}}{2N_1 h}$ $H_A = P - H_E$

$M_A = -\frac{\beta_1(Pb+C_{CB}) + \beta_2 C_{BC}}{2N_1} - \frac{\beta_3 Ph - 2H_{BC}}{4N_2}$; $M_B = (P - H_E)\,h + M_A$

$M_E = -\frac{\beta_1\,(Pb+C_{CB}) + \beta_2 C_{BC}}{2N_1} + \frac{\beta_3 Ph - 2H_{BC}}{4N_2}$; $M_C = \frac{VL}{L} + M_E - H_E h_1$

$V_A = V_E = V = \frac{1}{L}\,(Pa + M_A - M_E)$; $M_D = M_E - H_E h$

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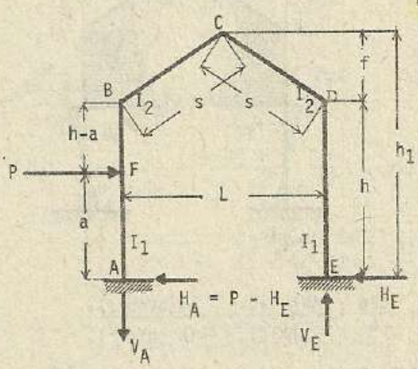
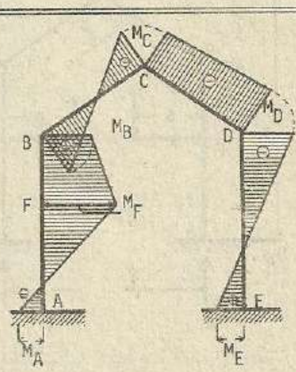
(293)



$$k = \frac{I_2 h}{I_1 s}$$

$$n = \frac{f}{h}$$



Otros datos : referencia (292)

$C_{BA} = -\frac{Pa^2(h-a)}{h^2}$; $C_{AB} = -\frac{P\,(h-a)^2\,a}{h^2}$; $H_E = \frac{\gamma_1\,(Pa+C_{AB}) + \gamma_2 C_{BA}}{2N_1 h}$

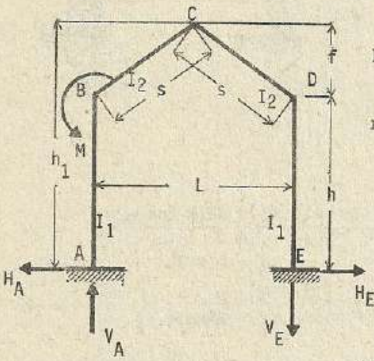
$H_A = P - H_E$; $V_A = V_E = V = \frac{Pa + M_A - M_E}{L}$

$$M_A = \frac{n\,\beta_1\,Pa - 2km^2\,C_{AB} - \gamma_3\,C_{BA}}{2N_1} - \frac{\beta_3\,Pa - 3k(C_{AB}+C_{BA})}{4N_2} \quad ; \quad M_B = Pa + M_A - H_E h$$

$$M_E = \frac{n\,\beta_1\,Pa - 2km^2\,C_{AB} - \gamma_3\,C_{BA}}{2N_1} + \frac{\beta_3 Pa - 3k(C_{AB}+C_{BA})}{4N_2} \quad ; \quad M_C = \frac{VL}{2} + M_E - H_E h_1$$

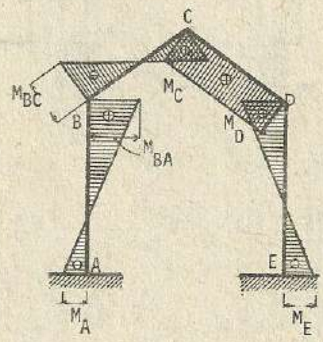
$$M_D = M_E - H_E h$$

(294)



$k = \frac{I_2 h}{I_1 s}$

$n = \frac{f}{h}$



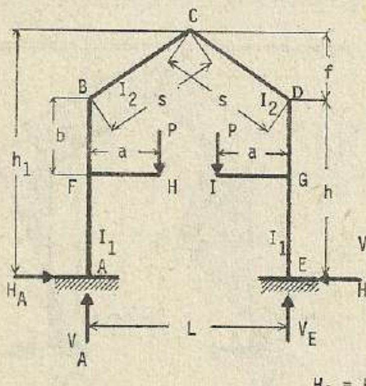
Otros datos : referencia (292)

$$H_A = H_E = H = \frac{3mkM}{N_1 h} \quad ; \quad V_A = V_E = V = \frac{3kM}{N_2 L} \quad ; \quad M_A = -\frac{M}{2}\left[\frac{\alpha_2 - \beta_2}{N_1} - \frac{1}{N_2}\right]$$

$$M_{BA} = M_A + Hh \quad ; \quad M_{BC} = M_{BA} - M \quad ; \quad M_E = -\frac{M}{2}\left[\frac{\alpha_2 - \beta_2}{N_1} + \frac{1}{N_2}\right]$$

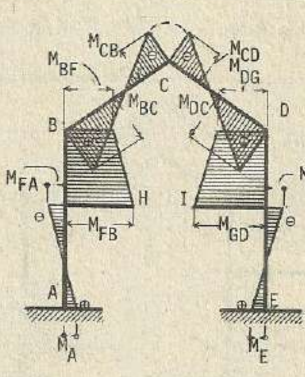
$$M_C = \frac{1}{2}(M_A + M_E - M) + Hh_1 \qquad M_D = M_E + Hh$$

(295)



$k = \frac{I_2 h}{I_1 s}$

$V_A = V_E = V = P$



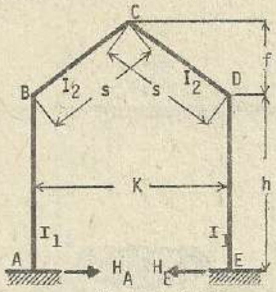
$$H_A = H_E = H = \frac{6Pa}{h} \cdot \frac{hk(h+f+bk) - bk(bk+b+f)}{(hk+f)^2 + 4k(h^2+hf+f^2)}$$

$$M_{AF} = M_{EG} = -\frac{Pa}{h} \cdot \frac{1}{(hk+f)^2 + 4k(h^2+hk+k^2)}\left[h^2k\,(2bk+2h+3f) - bfk(6h+3b+4f) - h\,(3b^2k^2 + 9b^2k+f^2)\right]$$

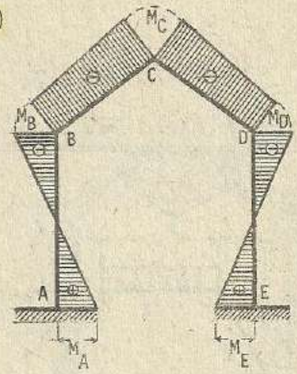
$$M_{FA} = M_{GE} = -H\,(h-b) + M_{AF} \quad ; \quad M_{BC} = M_{DC} = -Hh + M_{AF} + Pa$$

$$M_{FB} = M_{GD} = -H\,(h-b) + M_{AF} + Pa \quad ; \quad M_{CB} = -H\,(h+f) + M_{AB} + Pa + V_E \cdot \frac{1}{2}$$

Esfuerzos debido a cambio de temperatura : (296)



$$k = \frac{I_2 h}{I_1 s}$$



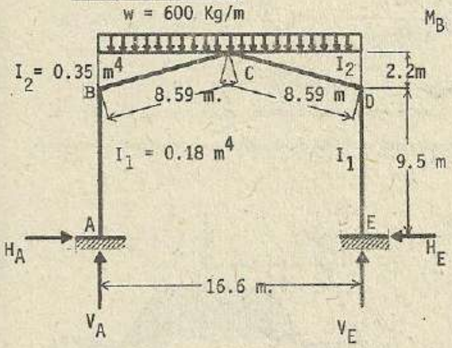
$$H_A = H_E = \frac{6E\varepsilon t I_2 L\ (k+1)}{s\left[(hk+f)^2 + 4k(h^2+hf+f^2)\right]}$$

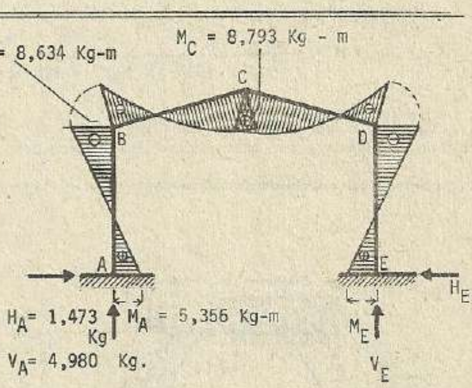
$$M_A = M_E = \frac{3E\varepsilon t I_2 L\ \left[sh\ (k+2)\ + f\right]}{s\left[(hk+f)^2 + 4k(h^2+hf+f^2)\right]}\ ;\quad M_B = M_D = -\frac{3E\varepsilon t I_2 L\ (hk - f)}{s\left[(hk+f)^2 + 4k\ (h^2+hf+f^2)\right]}$$

$$M_C = -\frac{3E\varepsilon t I_2 L\ \left[hk + f\ (2k+1)\right]}{s\left[(hk+f)^2 + 4k\ (h^2+hf+f^2)\right]}$$

Nota : El presente problema corresponde al incremento de temperatura.

Problema numérico





$$k = \frac{I_2 h}{I_1 s} = \frac{0.35 \times 9.5}{0.18 \times 8.59} = 2.15 \quad ; \quad V_A = V_E = \frac{wL}{2} = \frac{600\text{x}16.6}{2} = 4{,}980\ \text{Kg,}$$

$$H_A = H_E = \frac{k\ (4h+5f)\ + f}{8\left[(hk+f)^2 + 4k\ (h^2+hf+f^2)\right]}\ wL^2$$

$$= \frac{2.15\text{x}(4\text{x}9.5+5\text{x}2.2)\ + 2.2}{8\left[(9.5\text{x}2.15+2.2)^2 + 4\text{x}2.15\text{x}(9.5^2+9.5\text{x}2.2\ +2.2^2)\right]} \times 600\text{x}16.6^2 = 1{,}473\ \text{Kg}$$

$$M_A = M_E = \frac{\left[hk\ (8h+15f)\ +f\ (6h-f)\right] wL^2}{48\left[(hk+f)^2 + 4k\ (h^2+hf+f^2)\right]} = \frac{9.5\text{x}2.15\text{x}(8\text{x}9.5+15\text{x}2.2)+2.2\text{x}\ (6\text{x}9.5-2.2)}{48 \times 1{,}509.4046}$$

$$\times\ 600 \times 16.6^2 = 5.356\ \ \text{Kg - m.}$$

$$M_B = M_D = -\frac{hk(16h+15f)\ + f^2}{48\left[(hk+f)^2 + 4k(h^2+hf+f^2)\right]}$$

$$= -\frac{9.5\text{x}2.15\text{x}(16\text{x}9.5 + 15\text{x}2.2)\ + 2.2^2}{48 \times 1{,}509.4046} \times 600 \times 16.6^2 = -8{,}634\ \text{Kg - m}$$

$$M_C = -H_A\,(h+f) + M_A + \frac{wL^2}{8}$$

$$= 1{,}473\ (9.5+2.2) + 5{,}356 + \frac{600\times16.6^2}{8} = 8{,}793\ \text{Kg - m}$$

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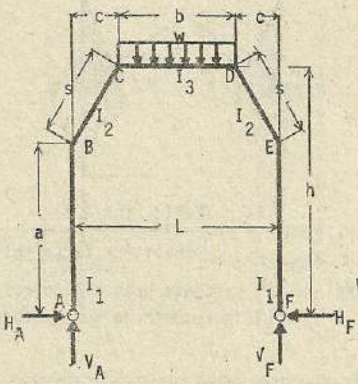
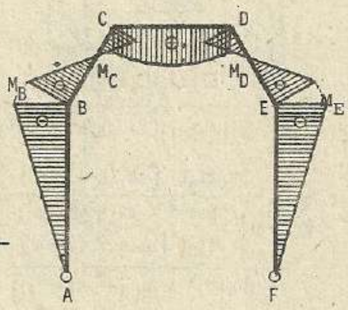
(297)



$$k = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$V_A = V_F = \frac{wb}{2}$$



$$H_A = H_F = H = \frac{wb}{4}\cdot\frac{2\,(a+2h)c + hk_2\,(L+4c)}{2a^2\,(k_1+1) + 2ah + h^2\,(2+3k_2)}\ ; \quad M_{BC} = M_{BA} = M_{EF} = M_{ED} = -\,Ha$$

$$M_{CB} = M_{CD} = M_{DC} = M_{DE} = -\,Hh + \frac{wb}{2}\,c \quad : \quad M_{máx.} = -\,Hh + \frac{wb}{2}\cdot\frac{L}{2}$$

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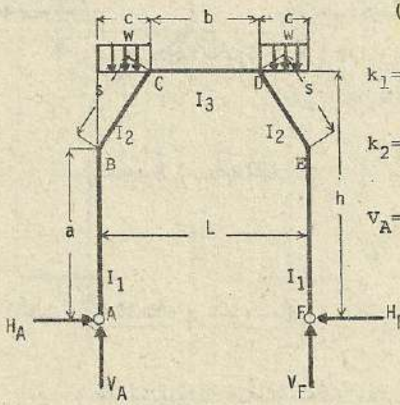
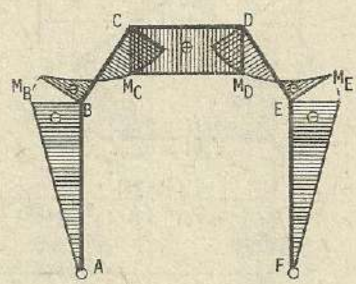
(298)



$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

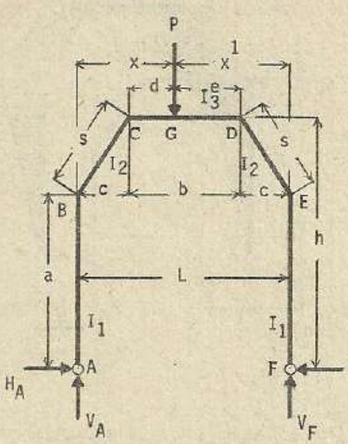
$$V_A = V_F = wc$$



$$H_A = H_F = H = \frac{wc^2}{4}\cdot\frac{6hk_2 + 5h + 3a}{2a^2\,(k_1+1) + 2ah + h^2(2+3k_2)}$$

$$M_{BA} = M_{BC} = M_{EF} = M_{ED} = -\,Ha \quad ; \quad M_{CB} = M_{CD} = M_{DC} = M_{DE} = -\,Hh + \frac{wc^2}{2}$$

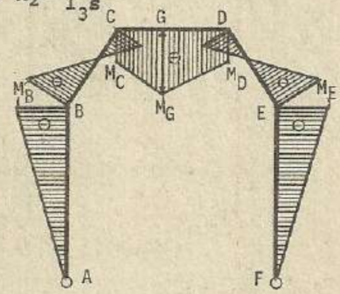
(299)



$$k_1 = \frac{I_2 a}{I_1 s} \quad ; \quad k_2 = \frac{I_2 b}{I_3 s}$$

$$V_A = \frac{Px^1}{L}$$

$$V_F = \frac{Px}{L}$$



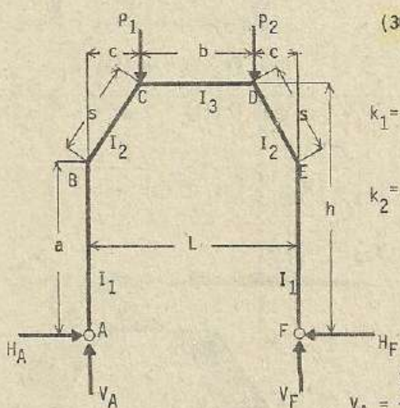
$$H_A = H_F = H = \frac{P}{2b} \cdot \frac{bc\ (2h+a) + 3hk_2\ (bx-d^2)}{2a^2\ (k_1+1) + 2ah + h^2\ (3k_2+2)}$$

En el caso donde $d = e = \frac{b}{2}$; $H = \frac{P}{8} \cdot \frac{4c\ (2h+a) + 3hk_2\ (4c+b)}{2a^2(k_1+1) + 2\ ah + h^2(3k_2+2)}$

$M_{BA} = M_{BC} = M_{EF} = M_{ED} = -Ha$; $M_{DC} = M_{DE} = -Hh + V_F c$

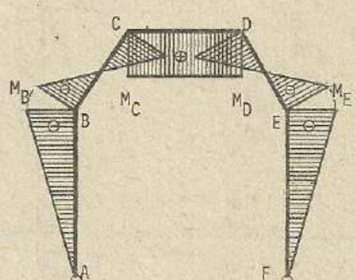
$M_{CB} = M_{CD} = -Hh + V_A c$; $M_G = -Hh + V_A x$

(300)



$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

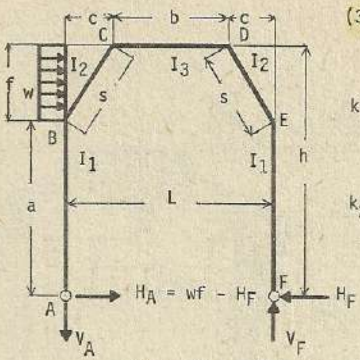


$$V_A = \frac{P_1 (L-c) + P_2 c}{L} \quad ; \quad V_F = \frac{P_2\ (L-c) + P_1 c}{L}$$

$$H_A = H_B = H = \frac{(P_1 + P_2)\ c\ \left[h\ (3k_2+2) + a\right]}{2a^2\ (k_1+1) + 2ah + h^2\ (3k_2+2)} \ ; \quad M_{BA} = M_{BC} = M_{ED} = M_{EF} = -Hh$$

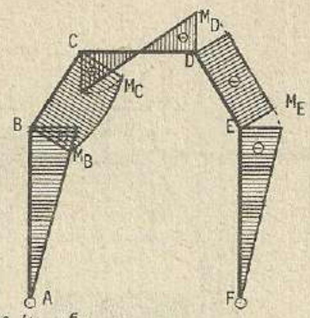
$M_{CB} = M_{CD} = -Hh + V_A c$; $M_{DC} = M_{DE} = -Hh + V_F c$

(301)



$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$



$$-V_A = V_F = \frac{wf\,(h - \frac{f}{2})}{L} \quad ; \quad H_A = wf - H_F$$

$$H_F = \frac{wf}{8} \cdot \frac{a^2(8k_1+9) + h\left[2a\,(3k_2+5) + h\,(6k_2+5)\right]}{2a^2(k_1+1) + 2ah + h^2(3k_2+2)}$$

$$M_{ED} = M_{EF} = -H_F a$$

$$M_{BA} = M_{BC} = (wf - H_F)a \quad ; \quad M_{CB} = M_{CD} = V_F\,(L-c) - H_F h \quad ; \quad M_{DC} = M_{DE} = -H_F h - V_F c$$

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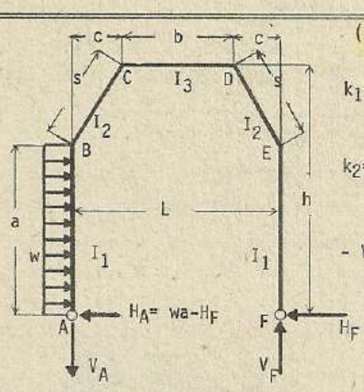
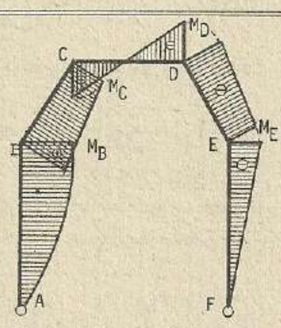
(302)



$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$-V_A = V_F = \frac{wa^2}{2L}$$



$$H_F = \frac{wa^2}{8} \cdot \frac{6h\,(k_2+1) + a\,(5k_1+6)}{2a^2\,(k_1+1) + 2\,ah + h^2(3k_2+2)}$$

$$H_A = wa - H_F$$

$$M_{BA} = M_{BC} = (wa - H_F)\,a - \frac{wa^2}{2} \quad ; \quad M_{CB} = M_{CD} = V_F\,(L-c) - H_F h$$

$$M_{DC} = M_{DE} = -H_F h + V_F c \quad ; \quad M_{ED} = M_{EF} = -H_F a$$

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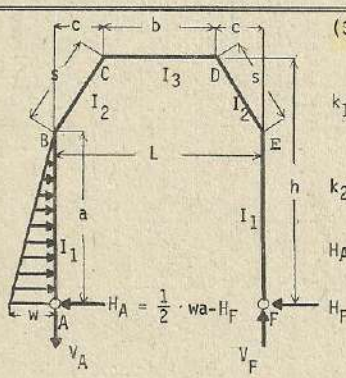
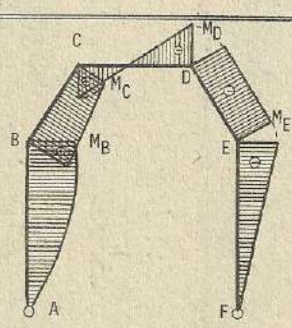
(303)



$$k_1 = \frac{I_2 a}{I_1 s}$$

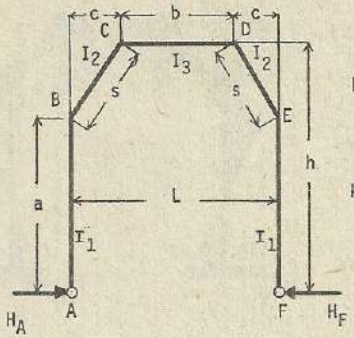
$$k_2 = \frac{I_2 b}{I_3 s}$$

$$H_A = \frac{1}{2}\,wa - H_F$$



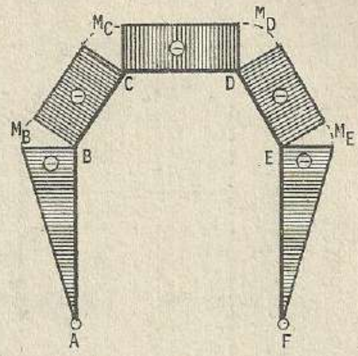
$$H_F = \frac{wa^2}{40} \cdot \frac{10h\,(k_2+1) + a\,(9k_1+10)}{2\,(1+k_1)\,a^2 + 2ha + h^2\,(3k_2+2)}$$

Esfuerzos por cambio de temperatura: (304)



$$k_1 = \frac{I_2 a}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$



$$H_A = \frac{3E\varepsilon t I_2 L}{s\left[2\,(k_1+1)\,a^2 + 2ha + h^2\,(3k_2+2)\right]}$$

$$M_B = M_E = -\,Ha \qquad\qquad M_C = M_D = -\,Hh$$

Nota : El presente corresponde al problema de incremento de temperatura. En caso de descenso, los esfuerzos se anotan con los signos contrarios.

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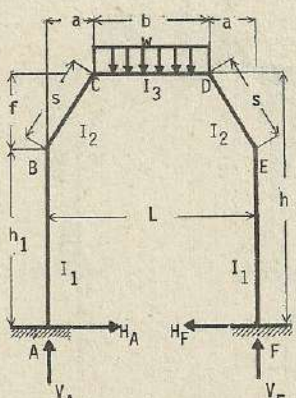
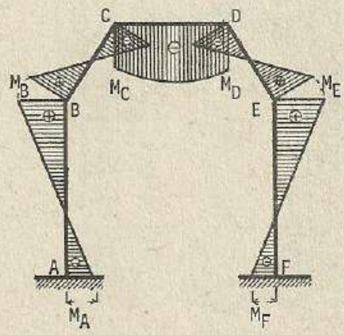
(305)



$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$x = \alpha\, a$$



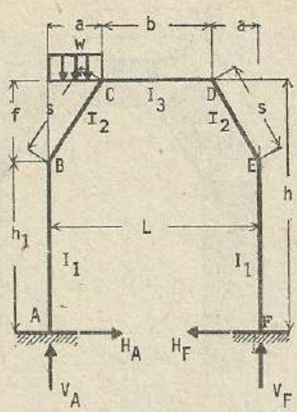
$$\beta = k_1\,(k_1+2k_2+4)\,h_1^2 + 6k_1\,(k_2+1)\,h_1 f + (6k_1k_2+4k_1+2k_2+1) \quad ; \quad V_A = V_F = \frac{wb}{2}$$

$$\gamma = (6k_1+k_2+6)\,L^2 - 4\,(k_2+3)\,.\,La + 4\,(k_2+2)\,a^2$$

$$H_A = H_F = H = \frac{wb}{4\beta}\left[k_1\left\{k_2L + 2\,(2k_2+3)\,a\right\}h_1 + k_2(2k_2+1)L + 2\,(k_2+1)\,(4k_1+1)a\right]f$$

$$M_A = M_F = -\,\frac{(k_1+k_2+2)h_1 + (k_2+1)f}{2k_1+k_2+2}\,H + \frac{wb\left[k_2L + 2\,(k_2+3)\,a\right]}{12\,(2k_1+k_2+2)}$$

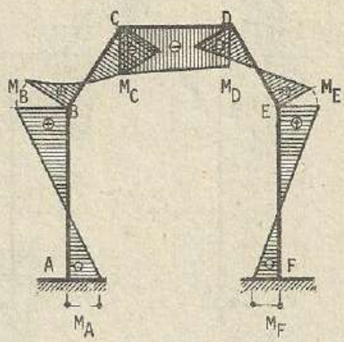
(306)



$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

Otros datos refencia (305)



$$H_A = H_F = H = \frac{wa^2}{8} \cdot \frac{k_1(6k_2+8)h_1 + (12k_1k_2+10k_1+3k_2+2)f}{\beta}$$

$$v = \frac{wa^2}{2L} \cdot \frac{(k_2+4)L^2 - (4k_2+11)\,La + 4_2(k_2+2)a^2}{\gamma} \; ; \quad V_A = \frac{wa\,(L-\frac{a}{2})}{L} + v \; ; \quad V_F = \frac{wa^2}{2L} - v$$

$$M_A = -\frac{(k_1+k_2+2)h_1+(k_2+1)\,f}{2k_1+k_2+2}\,H + \frac{wa^2\,(3k_2+4)}{12\,(2k_1+k_2+2)} + \frac{L}{2}\,v \; ; \quad M_F = -M_A - vL$$

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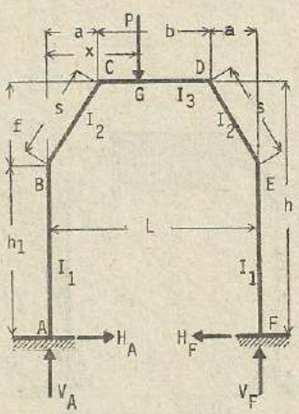
(307)



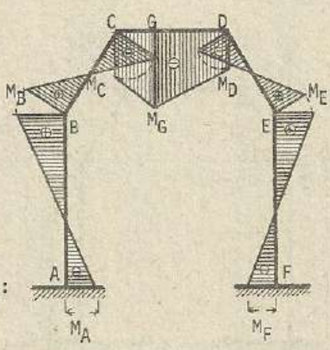
$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

$$x = \alpha L$$

$$a = nb$$

otros datos : (305)



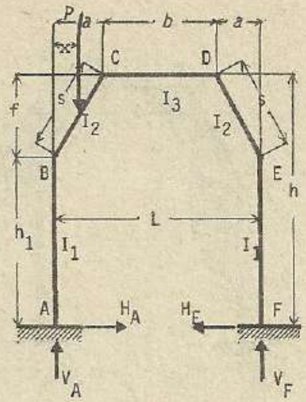
$$H_A = H_F = H = \frac{P}{2\beta}\Big[3k_1 \{ k_2\alpha(L-x) + (1-k_2 n)\,a \} h_1 + \{ 3k_2(2k_1+1)(\alpha L-\alpha x-na) + (4k_1-k_2+1)\,a \} f\Big]$$

$$v = \frac{P}{L\gamma}\Big[k_2\,\alpha L^3 + 3\,\{ (1-k_2 n)\,a - k_2\alpha x \} L^2 + 2\,\{ (3k_2\alpha + 2k_2 n-2)\,a^2 + k_2\,\alpha x^2 - 3xa \}\,L - 8\,(k_2\alpha\,a-x)a^2\Big]$$

$$V_A = \frac{P\,(L-x)}{L} + v \; ; \quad V_F = \frac{Px}{L} - v$$

$$M_A = -\frac{(k_1+k_2+2)h_1 + (k_2+1)\,f}{2k_1+k_2+2}\,H + \frac{P\left[k_2\alpha(L-x) + (1-k_2 n)\,a\right]}{2\,(2k_1+k_2+2)} + \frac{L}{2}\,v$$

$$M_F = -M_A - vL$$

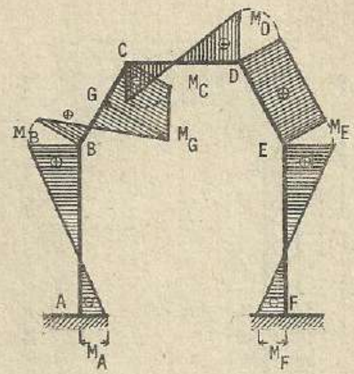


(308)

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

Otros datos : (305)

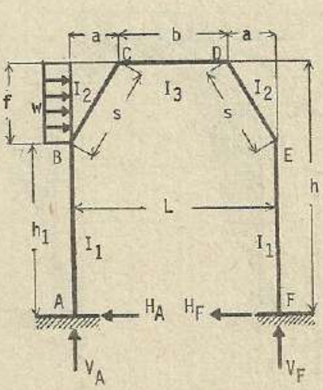


$$H_A = H_F = H = \frac{Px}{2\beta}\left[3k_1\,(2k_2+2-\alpha)h_1 + \left\{ 3\,(k_2+1)\,(2k_2+\alpha) - (2k_1+k_2+2)\alpha^2 \right\} f\right]$$

$$v = \frac{Px}{L}\cdot\frac{(k_2+6-3\alpha)L^2 - 2\,(2k_2+6-\alpha^2)\,La + 4\,(k_2+2)\,a^2}{\gamma}$$

$$V_A = \frac{P\,(L-x)}{L} + v \qquad ; \qquad V_F = \frac{Px}{L} - v$$

$$M_A = -\frac{(k_1+k_2+2)h_1 + (k_2+1)f}{(2k_1+k_2+2)}H + \frac{Px\,(k_2+2-\alpha)}{2\,(2k_1+k_2+2)} + \frac{L}{2}v \qquad ; \qquad M_F = -M_A - vL$$

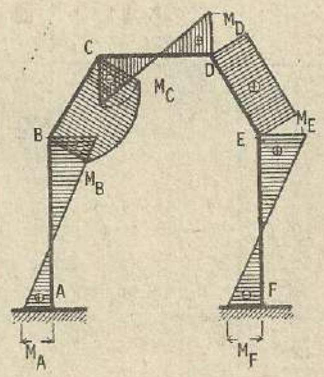


(309)

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

Otros datos (305)



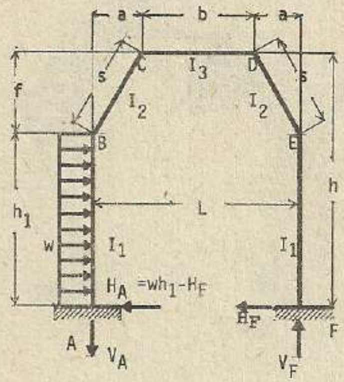
$$H_A = H_F = H = \frac{wf}{8\beta}\left[4k_1(k_1+2k_2+4)\,h_1^2 + 2k_1\,(9k_2+10)h_1 f + \left\{3k_2(4k_1+1)+2(5k_1+1)\right\}f^2\right]$$

$$V_A = V_F = V = \frac{2\left[3k_1h_1 + (3k_1+1)\,f\right]L - fa}{\gamma}\,w$$

$$M_A = -\frac{(k_1+k_2+2)h_1 + (k_2+1)\,f}{2k_1+k_2+2}H + \frac{wf}{6}\cdot\frac{(9k_1+6k_2+12)h_1 + (3k_1+3k_2+5)\,f}{2k_1+k_2+2} - \frac{L}{2}V$$

$$M_F = -\frac{1}{2}\,wf\,(2h_1+f) + M_A + VL$$

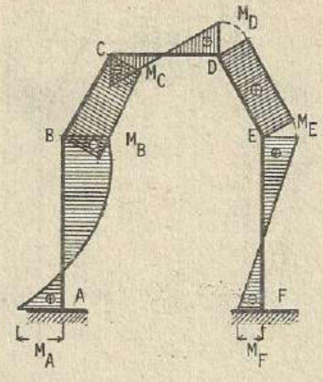
(310)



$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

Otros datos (305)



$$H_D = \frac{wh_1^2}{8} \cdot \frac{k_1(2k_1+3k_2+6)h_1 + 4k_1(k_2+1)\,f}{\beta} \quad ; \quad H_A = wh_1 - H_F$$

$$V_A = V_F = V = \frac{wh_1^2 k_1 L}{\gamma}$$

$$M_A = -\frac{(k_1+k_2+2)h_1 + (k_2+1)\,f}{2k_1+k_2+2} \cdot H + \frac{wh_1^2\,(5k_1+3k_2+6)}{6\,(2k_1+k_2+2)} - \frac{L}{2}V$$

$$M_F = -\frac{1}{2}\cdot wh_1^2 + M_A + VL$$

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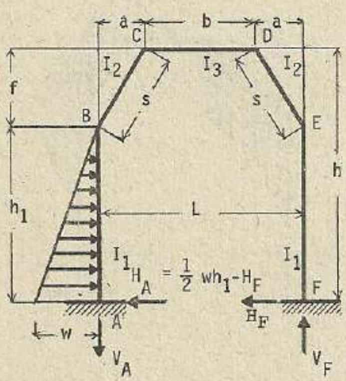
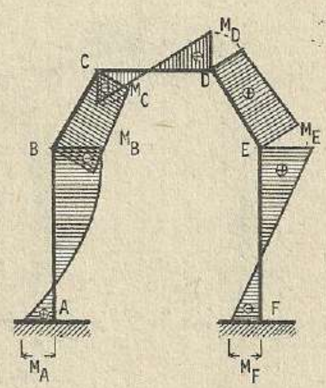
(311)



$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 h}{I_3 s}$$

otros datos (305)

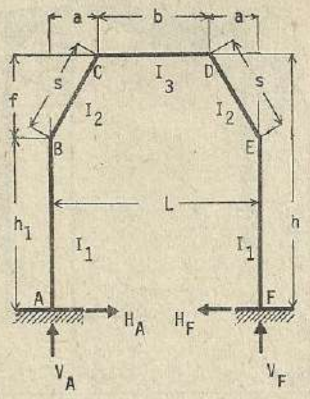


$$H_F = \frac{wh_1^2}{40} \cdot \frac{k_1\,(3k_1+4k_2+8)h_1 + 5k_1(k_2+1)\,f}{\beta} \quad ; \quad H_A = \frac{1}{2}wh_1 - H_F$$

$$V_A = V_F = V = \frac{wh_1^2 k_1\,L}{4\gamma} \quad ; \quad M_F = -\frac{1}{6}\,wh_1^2 + M_A + VL$$

$$M_A = -\frac{(k_1+k_2+2)h_1 + (k_2+1)\,f}{2k_1+k_2+2}H + \frac{wh_1^2\,(7k_1+4k_2+8)}{24\,(2k_1+k_2+2)} - \frac{L}{2}V$$

(312)

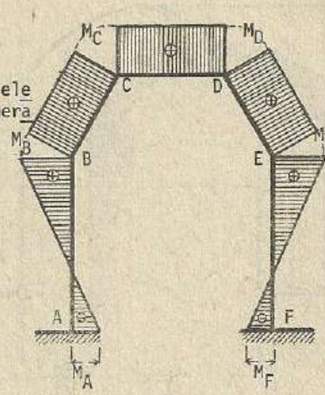


Esfuerzos por elevación de Temperatura

$$k_1 = \frac{I_2 h_1}{I_1 s}$$

$$k_2 = \frac{I_2 b}{I_3 s}$$

otros datos (305)



$$H_A = H_F = H = \frac{3\,(2k_1 + k_2 + 2)E\epsilon tLI_2}{s\beta} \quad ; \quad M_A = M_F = -\frac{3\left[(k_1+k_2+2)h_1 + (k_2+1)f\right]E\epsilon tLI_2}{s\beta}$$

Nota : En caso de descenso de temperatura, los esfuerzos se anotan con los signos contrarios.

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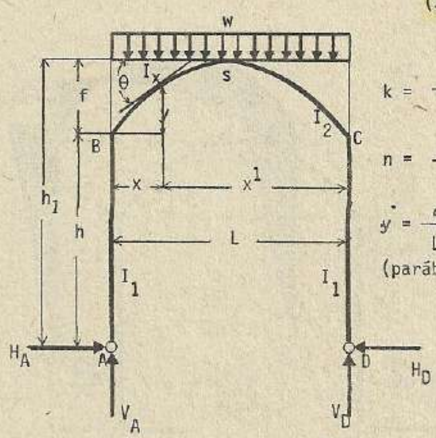
(313)

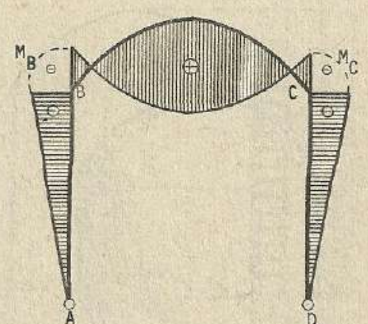


$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

$$y = \frac{4f}{L^2}\,xx'$$

(parábola)



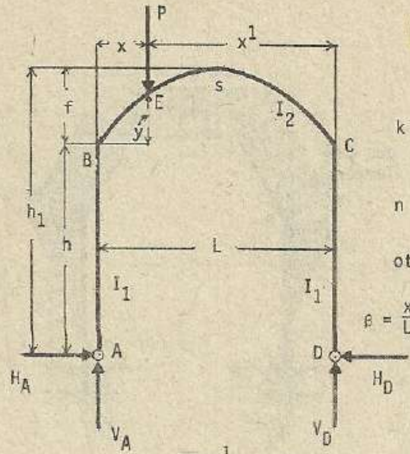
$$\alpha = 5\,(2k+3) + 4n\,(2n+5) \quad ; \quad I_x = I_2 \sec\theta$$

$$H_A = H_D = H = \frac{wL^2}{4\alpha h}\,(4n+5) = \frac{wL^2}{4}\cdot\frac{(5h + 4f)}{\left[5h^2\,(2k+3) + 4f\,(5h+2f)\right]}$$

$$V_A = V_D = V = \frac{wL}{2}$$

$$M_B = M_C = -Hh \quad ; \quad M_s = \frac{wL^2}{8} - Hh_1 \quad ; \quad M_x = \frac{wxx'}{2} - H\,(h+y)$$

(314)

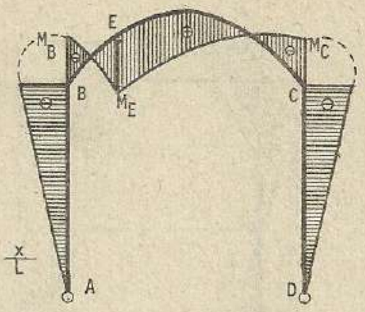


$k = \frac{I_2 h}{I_1 L}$

$n = \frac{f}{h}$

otros datos (313)

$\beta = \frac{x^1}{L}$ $\gamma = \frac{x}{L}$



$$H_A = H_D = H = \frac{5Pxx^1}{2\alpha Lh}\left[3 + 2n\,(1+\beta\gamma)\right] = \frac{5Px\,(3x^1hL^2 + 2\,fL^2 - 4fx^2L + 2fx^3)}{2L^3\left[5h^2\,(2k+3) + 4f(5h + 2f)\right]}$$

$M_B = M_C = -Hh$; $V_A = \beta P$; $V_D = \gamma P$; $M_s = \frac{Px}{2} - Hh_1$

$M_E = \frac{pxx^1}{L} - H\,(h+4\beta\gamma f)$; $M_x = M - H\,(h+y)$; M : esfuerzo de flexión de la viga BC

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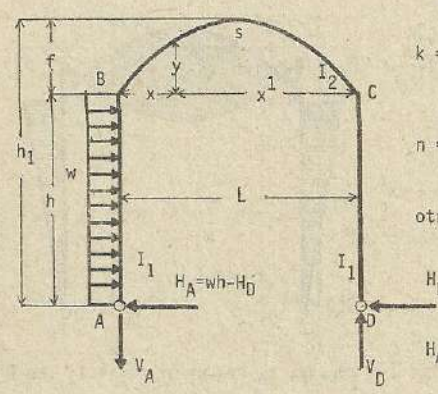
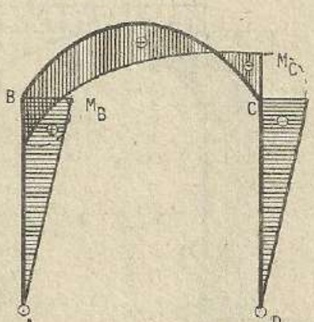
(315)



$k = \frac{I_2 h}{I_1 L}$

$n = \frac{f}{h}$

otros datos (313)



$H_A = wh - H_D$; $V_A = V_D = V = \frac{wh^2}{2L}$

$$H_D = \frac{5wh}{8\alpha}(6 + 5k + 4n) = \frac{5wh^2}{8}\cdot\frac{\left[h\,(5k+6) + 4f\right]}{\left[5h^2\,(2k+3) + 4f\,(5h+2f)\right]}$$

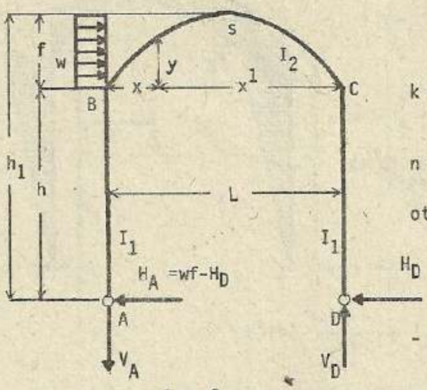
$M_B = \frac{wh^2}{2} - H_D h$; $M_C = -H_D h$: $M_s = \frac{wh^2}{4} - H_D h_1$

Esfuerzo de flexión en cada miembro : AB ⟶ $M = wy\,(h - \frac{y}{2}) - H_D y$

BC ⟶ $M = \frac{wh^2}{2}\cdot\frac{L-x}{L} - M_D y$

CD ⟶ $M = -H_D y$

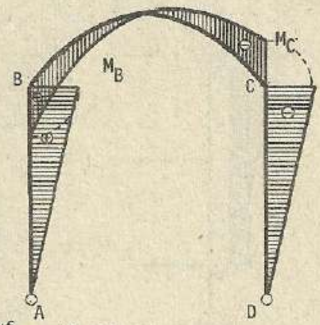
(316)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos (313)

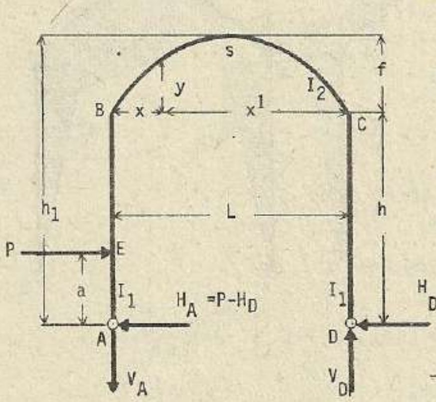


$$-V_A = V_D = \frac{wf}{2L}\,(2h+f)$$

$$H_D = \frac{wf}{28} \cdot \frac{[70h^2 (2k+3) + f (224h + 64f)]}{[5h^2 (2k+3) + 4f (5h+2f)]} \quad ; \quad H_A = wf - H_D$$

$$M_B = H_A h \quad ; \quad M_C = -H_D h \quad ; \quad M_s = \frac{V_D L}{2} - H_D (h+f)$$

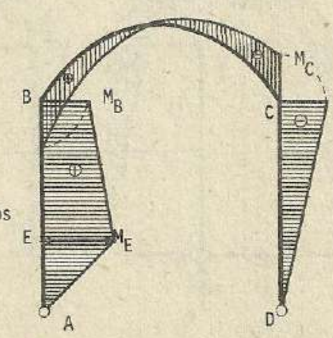
(317)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos (313)

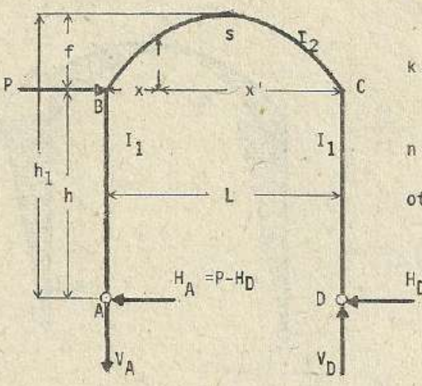


$$-V_A = V_D = \frac{Pa}{L} \quad ; \quad H_A = P - H_D$$

$$H_D = \frac{5Pa}{2h} \cdot \frac{[3h^2 (k+1) + 2fh - a^2 k]}{[5h^2 (2k+3) + 4f (5h+2f)]} \quad ; \quad M_E = H_A a \quad ; \quad M_B = Pa - H_D h$$

$$M_C = -H_D h$$

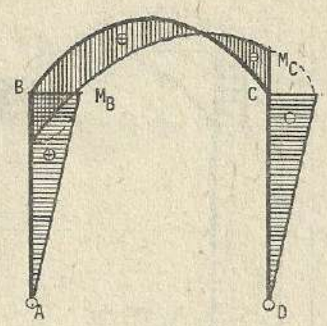
(318)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos
(313)

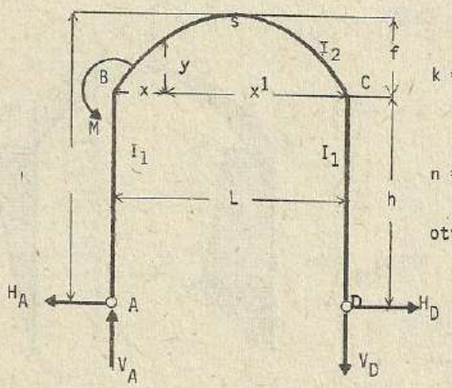


$$H_D = \frac{5P}{2\alpha} \cdot (3+2k + 2n)$$

$$M_B = (P - H_D)\, h \quad ; \quad M_s = \frac{Ph}{2} - H_D h_1 \quad ; \quad M_D = - H_D h$$

$$- V_A = V_D = \frac{Ph}{L}$$

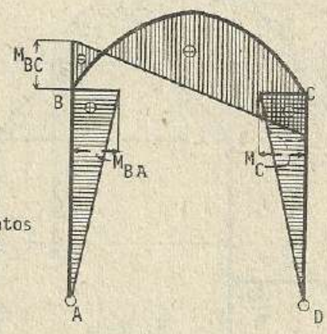
(319)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{f}{h}$$

otros datos
(313)

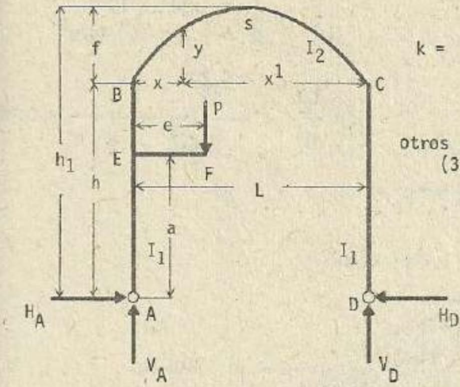


$$H_A = H_D = H = \frac{5\,(3+2n)\,M}{2\alpha h}$$

$$M_{BA} = M_C = Hh \quad ; \quad M_{BC} = Hh - M \quad ; \quad M_s = Hh_1 - \frac{M}{2}$$

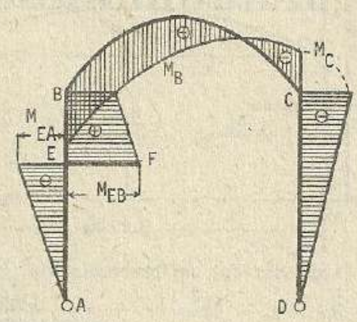
$$V_A = V_D = V = \frac{M}{L}$$

(320)



$$k = \frac{I_2 h}{I_1 L}$$

otros datos (313)

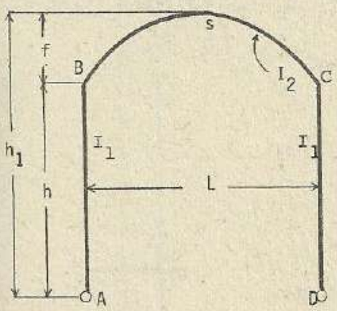


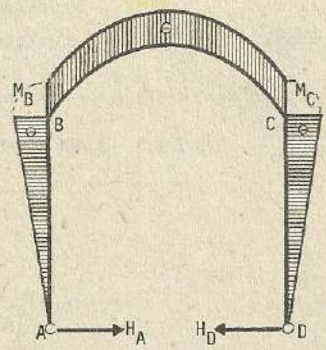
$$V_A = \frac{P\,(L-e)}{L} \quad ; \quad V_D = \frac{Pa}{L}$$

$$H_A = H_D = H = \frac{5Pe}{2h}\,\frac{[2k\,(h^2-a^2) + h\,(3h+2f)]}{[5h^2\,(2k+3) + 4f\,(5h+2f)]} \quad ; \quad M_{EA} = -Ha \quad ; \quad M_{EB} = Pe - H_D$$

$$M_B = Pe - Hh; \quad M_C = -Hh$$

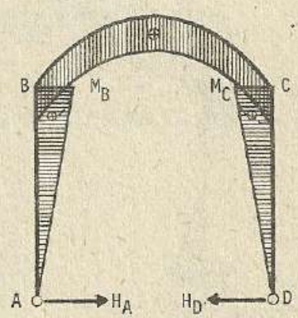
Esfuerzos por cambio de temperatura : (321)





Caso (A)

incremento de temperatura



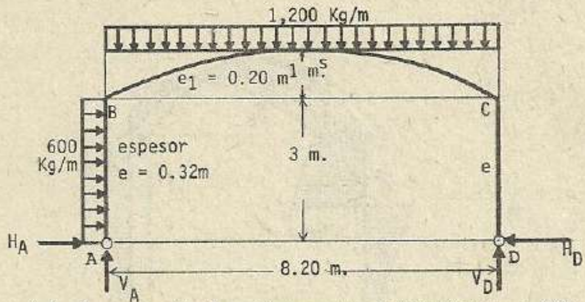
Caso (B)

descenso de temperatura

$$k = \frac{I_2 h}{I_1 L}$$

$$H_A = H_D = H = \frac{15E\varepsilon I_2 t}{\alpha h^2} \cdot \frac{15E\varepsilon I_2 t}{5h^2\,(2k+3) + 4f\,(5h+2f)} \quad ; \quad V = 0$$

$$M_B = M_C = -Hh \quad ; \quad M_s = -Hh_1$$



Datos : $t = +20\ °C$

$\varepsilon = 1/10^5 = 10^{-5}$

$E = 2.1 \times 10^9\ Kg/m^2$

$$k = \frac{I_2 h}{I_1 L} = \frac{\frac{0.20^3}{12} \times 3.0}{\frac{0.32^3}{12} \times 8.2} = 0.09$$

(a) Carga vertical uniformemente distribuida : 1,200 Kg/m.

$$H_A = H_D = H = \frac{wL^2}{4} - \frac{(5h+4f)}{5h^2(2k+3) + 4f\,(5h+2f)}$$

$$= \frac{1,200 \times 8.20^2}{4} \times \frac{5 \times 3 + 4 \times 1}{5x3x3\ (2x0.09+3) + 4x1\ (5x3+2x1)} = 1,816\ Kg.$$

$$M_B = M_C = -Hh = -1,816 \times 3 = -5,448\ \ Kg - m$$

$$M_s = \frac{wL^2}{8} - Hh_1 = \frac{1,200}{8} \times 8.2 \times 8.2 - 1,816x4 = 2,822\ Kg - m$$

(b) Carga lateral uniformemente distribuida : w = 600 Kg/m.

$$H_D = \frac{5wh^2}{8}\ \frac{[h\ (5k+6) + 4f\ ]}{[5h^2\ (2k+3) + 4f\ (5h+2f)]}$$

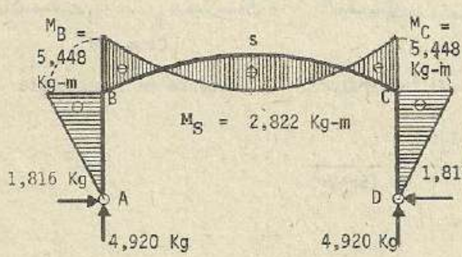
$$= \frac{5 \times 600x3^2}{8}\ \frac{[3\ (5x0.09+6) + 4x1]}{[5x3^2\ (2x0.09+3) + 4x1\ (5x3+2x1)]} = 373\ Kg.$$

$$H_A = wh - H_D = 600 \times 3 - 373 = 1,427\ Kg. \quad ; \quad V_A = V_D = \frac{wh^2}{2L} = \frac{600x3^2}{2x8.2} = 329\ Kg.$$

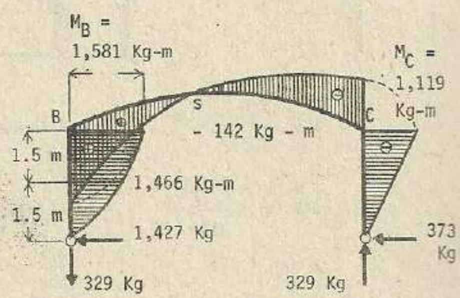
$$M_B = \frac{wh^2}{4} - H_D h = \frac{600x3^2}{2} - 373 \times 3 = 1,581\ Kg\text{-}m\ ; \quad M_C = -H_D h = -373x3 = -1,119\ Kg\text{-}m$$

$$M_s = \frac{wh^2}{4} - H_D h_1 = \frac{600x3^2}{4} - 373 \times 4 = -142\ Kg\text{-}m.$$

$$M_E = 600 \times 1.5\ (3 - \frac{1.5}{2}) - 373 \times 1.5 = 1,466\ \ Kg\text{-}m$$



Caso de carga vertical



Caso de carga lateral

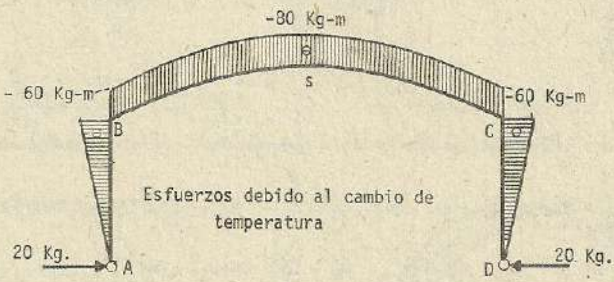
(c): Esfuerzos debido a cambio de temperatura

$$H_A = H_D = H = \frac{15EI_2\varepsilon t}{5h^2(2k+3) + 4f\ (5h+2f)}$$

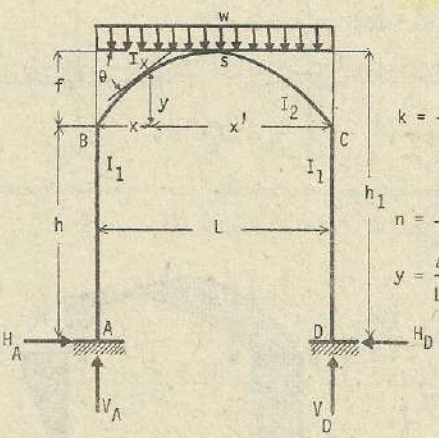
$$= \frac{15x2.1x10^9 x \ \frac{0.2^3}{12} \ x\ 10^{-5}\ x\ 20}{5x3^2\ (2x0.09+3) + 4x1\ (5x3+2x1)} = 20\ Kg.$$

$M_B = -\ M_C = -Hh = -\ 20x3 = -60$ Kg-m

$M_s = -\ Hh_1 = -\ 20x4 = -\ 80$ Kg - m



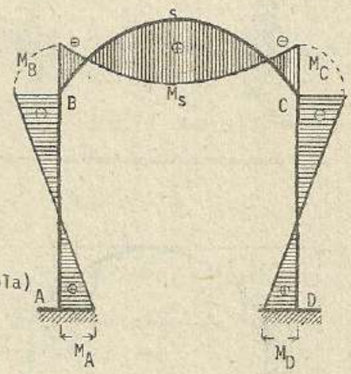
(322)



$k = \dfrac{I_2h}{I_1L}$

$n = \dfrac{f}{h}$

$y = \dfrac{4f}{L^2}\ xx$ (parábola)



$\lambda_1 = k\ (2+k+2n)$; $\lambda_2 = k\ (3+4n)$; $\mu_1 = 5k+12nk+2n$; $\mu_2 = 15k+30nk-4n^2$

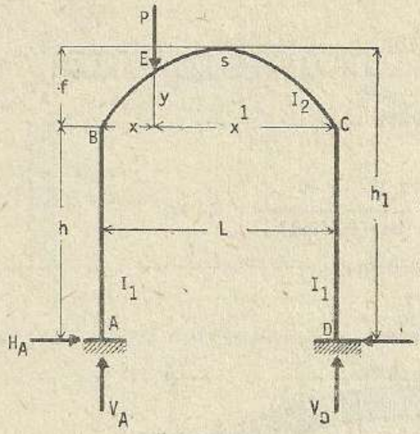
$m = 3+3k+2n$; $\alpha_1 = 15k\ (2+k) + 12\ nk\ (5+4n) + 4n^2$; $\alpha_2 = 1 + 6k$

$H_A = H_D = H = \dfrac{wL^2}{4\alpha_1 h}\ (15k+24nk+2n)$; $V_A = V_D = V = \dfrac{wL}{2}$

$M_A = M_D = -\ \dfrac{3\ (k+1)h+2f}{3\ (2k+1)}\ H + \dfrac{wL^2}{12\ (2k+1)}$; $M_x = \dfrac{wxx'}{2} - H\ (h+y) + M_A$

$M_B = M_C = M_A - Hh$; $M_s = \dfrac{wL^2}{8} + M_A - Hh_1$

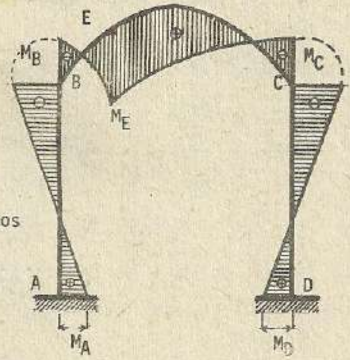
(323)



$k = \frac{I_2 h}{I_1 L}$

$n = \frac{f}{h}$

otros datos (322)



$$\alpha^1 = \frac{x}{L} \quad ; \quad \beta^1 = \frac{x^1}{L}$$

$$H_A = H_D = H = \frac{15xx^1}{2\alpha_1 hL}\left[\lambda_2 + 2n\alpha^1\beta^1 (2k+1)\right] \; ; \quad M_s = \frac{1}{2}(Px + M_A + M_D) - Hh_1$$

$$M_A = \frac{Pxx^1}{2\alpha_1 L}\left[\mu_2 + 10n\alpha^1\beta^1 m\right] - \frac{\alpha^1\beta^1 P(x^1 - x)}{2\alpha_2} \; : \quad M_D = \frac{Pxx^1}{2\alpha_1}\left[\mu_2 + 10n\alpha^1\beta^1 m\right] + \frac{\beta^1\alpha^1 P(x^1 - x)}{2\alpha_2}$$

$$M_B = M_A - Hh \; ; \quad M_C = M_D - Hh \; ; \quad M_E = \frac{Pxx^1}{L} + \beta^1 M_A + \alpha^1 M_D + H(h + 4\alpha^1\beta^1 f)$$

$$V_A = \beta^1 P - M_A + M_D \; ; \quad V_D = \alpha^1 P + M_A - M_D$$

Otra forma de solución : $k = \frac{I_2 h}{I_1 L}$; $x = \alpha^1 L$

$$\gamma = 6k + 1 \qquad \beta = 15k(k+2)h^2 + 60Kfh + 4(12k+1)f^2$$

$$H_A = H_D = H = \frac{15Px(L-x)}{2L} \cdot \frac{k(3h+4f) + 2(2k+1)f\alpha^1(1-\alpha^1)}{\beta} \; ; \quad v = \frac{P(L-x)}{L} \cdot \frac{\alpha^1(1-2\alpha^1)}{\gamma}$$

$$V_A = \frac{Px^1}{L} + v \; ; \quad V_D = \frac{Px}{L} - v$$

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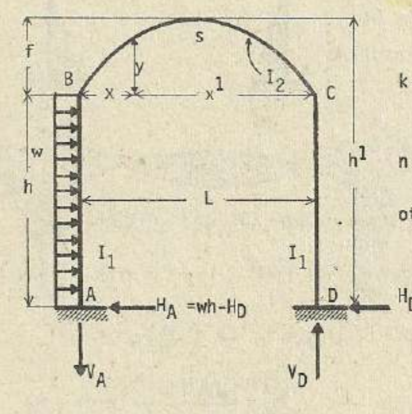
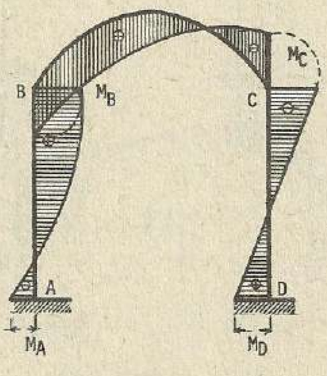
(324)



$k = \frac{I_2 h}{I_1 L}$

$n = \frac{f}{h}$

otros datos (322)



$$H_D = \frac{5(6\lambda_1 - \lambda_2)wh}{8\alpha_1} \; ; \quad H_A = wh - H_D \; ; \quad V_A = V_D = V = \frac{kwh^2}{\alpha_2 L}$$

$$M_A = -\frac{wh^2}{24\alpha_1}(\alpha_1 + 12n\,\mu_1 + \mu_2) - \frac{(1+4k)wh^2}{4\alpha_2} \; ; \qquad M_B = \frac{wh^2}{2} + M_A - H_D h$$

$$M_D = -\frac{wh^2}{24\alpha_1}(\alpha_1 + 12n\mu_1 + \mu_2) + \frac{(1+4k)wh^2}{4\alpha_2} \; ; \qquad M_C = M_D - H_D h$$

$$M_S = \frac{wh^2}{4} + \frac{M_A + M_D}{2} - H_D\, h_1$$

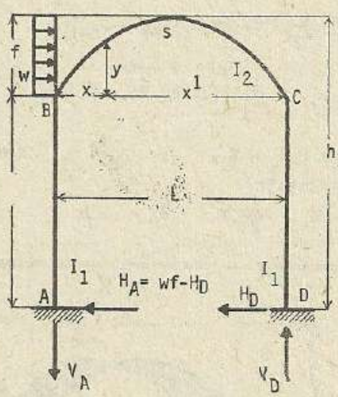
Uso de fórmula diferente : $H_D = \frac{5wh^3}{8} \cdot \frac{3k\,(2k+3)h + 8kf}{\beta}$

donde : $\beta = 15k\,(k+2)h^2 + 60kfh + 4\,(12k+1)f^2$

$v = \frac{wh^2}{L}\,\frac{k}{\gamma}$ siendo $\gamma = 6k+1$

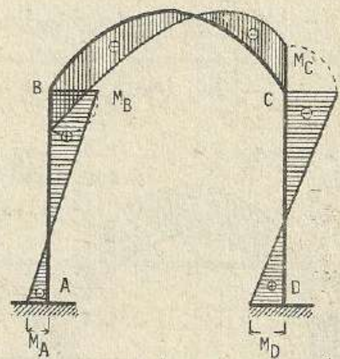
$$M_A = -\frac{3\,(k+1)h + 2f}{3\,(2k+1)} H_D + \frac{wh^2(3k+3)}{6\,(2k+1)} - \frac{L}{2} v \; ; \quad M_D = -\frac{1}{2} wh^2 + M_A + vL$$

(325)



$k = \frac{I_2 h}{I_1 L}$

$\gamma = 6\,k + 1$



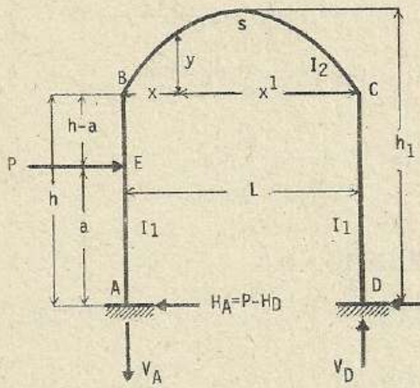
$\beta = \left[15k\,(k+2)\,h^2 + 60kfh + 4\,(12k+1)\,f^2\right]$

$$H_D = \frac{3wf}{14} \cdot \frac{35k\,(k+2)h^2 + 112kfh + 4(16k+1)\,f^2}{\beta} \; ; \qquad H_A = wf - H_D$$

$$v = \frac{wf}{8L} \cdot \frac{12kh + (12k+1)\,f}{\gamma} \; ; \qquad M_D = -\frac{1}{2}\,wf\,(2h+f) + M_A + vL$$

$$M_A = -\frac{3\,(k+1)h + 2f}{3\,(2k+1)} + \frac{wf\,\{10(3k+2)h + (10k+9)\,f\}}{20\,(2k+1)} - \frac{L}{2} v$$

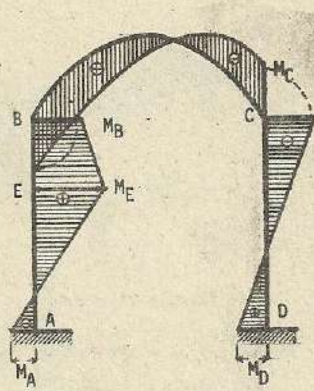
(326)



$k = \frac{I_2 h}{I_1 L}$

$n = \frac{f}{h}$

otros datos (322)



$$C_{AB} = -\frac{Pa\,(h-a)^2}{h^2} \quad ; \quad C_{BA} = -\frac{Pa^2\,(h-a)}{h^2}$$

$$H_D = \frac{15}{2\alpha_1\,h}\left[\lambda_1(Pa + C_{AB} - C_{BA}) + \lambda_2 C_{BA}\right] \quad ; \quad H_A = P - H_D$$

$$M_A = -\frac{1}{2\,\alpha_1}\left[2n\,\mu_1(Pa + C_{AB} - C_{BA}) - \alpha_1 C_{AB} - \mu_2\,C_{BA}\right] - \frac{1}{2\alpha_2}\left[(1+2k)Pa - 3k(C_{AB} + C_{BA})\right]$$

$$M_D = -\frac{1}{2\alpha_1}\left[2n\mu_1\,(Pa + C_{AB} - C_{BA}) - \alpha_1\,C_{AB} - \mu_2\,C_{BA}\right] + \frac{1}{2\alpha_2}\left[(1+2k)\,Pa - 3k\,(C_{AB} + C_{BA})\right]$$

$$M_B = Pa + M_A - H_D h \quad ; \quad V_A = V_D = V = \frac{1}{L}\,(Pa + M_A - M_D) \quad ; \quad M_C = M_D - H_D h$$

$$M_x = (Pa + M_A)\,\frac{x^1}{L} + M_D\,\frac{x}{L} - H_D\,(h+y) \quad ; \quad M_s = \frac{1}{2}\,VL \pm M_D - H_D h$$

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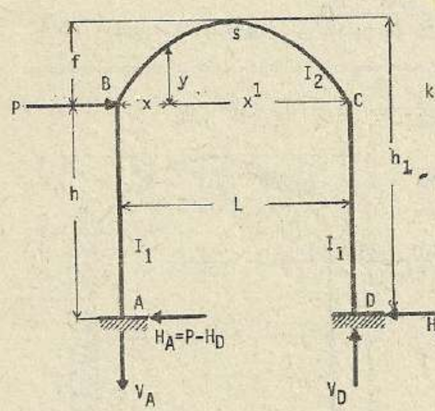
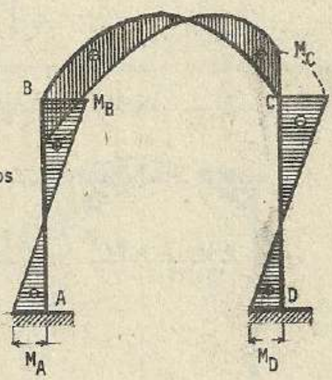
(327)



$k = \frac{I_2 h}{I_1 L}$

otros datos (322)

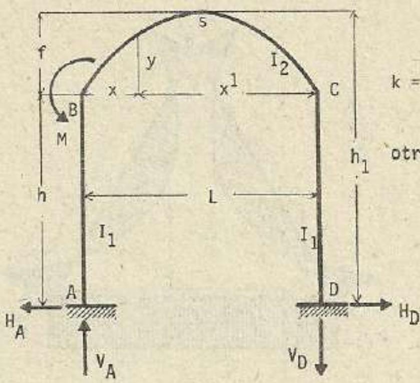


$$H_D = \frac{15\lambda_1 P}{2\alpha_1} \quad ; \quad H_A = P - H_D \quad ; \quad V_A = V_D = V = \frac{3kPh}{\alpha_2 L}$$

$$M_A = -\frac{\mu_1 Pf}{\alpha_1} - (1+3k)\,\frac{Ph}{2\,\alpha_2} \quad ; \quad M_D = -\frac{\mu_1 Pf}{\alpha_1} + (1+3k)\,\frac{Ph}{2\,\alpha_2} \quad ; \quad M_B = (P - H_D)h + M_A$$

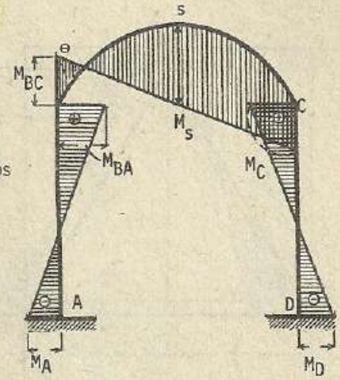
$$M_C = M_D - H_D h \quad ; \quad M_s = M_D + \frac{1}{2}\,VL - H_D\,h_1$$

(328)



$$k = \frac{I_2 h}{I_1 L}$$

otros datos (322)



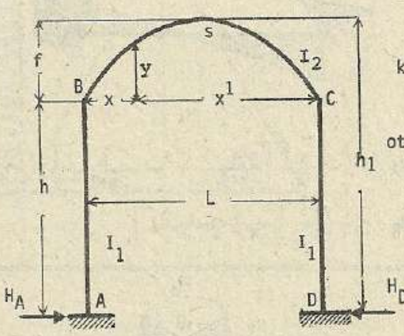
$$H_A = H_D = H = \frac{15\lambda_2 M}{2\alpha_1 h} \quad ; \quad V_A = V_D = V = \frac{6kM}{\alpha_2 L} \quad ; \quad M_A = -\frac{\mu_2 M}{2\alpha_1} + \frac{M}{2\alpha_2}$$

$$M_D = -\frac{\mu_2 M}{2\alpha_1} + \frac{M}{2\alpha_2} \quad ; \quad M_{BA} = M_A + Hh \quad ; \quad M_{BC} = M_{BA} - M$$

$$M_s = M_D + Hh_1 - \frac{VL}{2} \quad ; \quad M_C = M_D + Hh$$

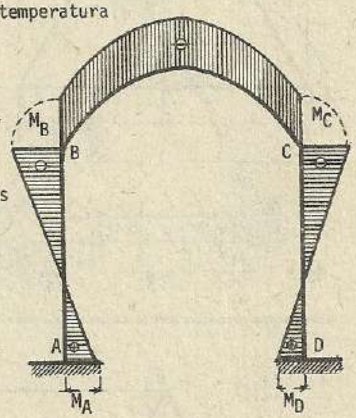
(329)

Esfuerzos por cambio de temperatura



$$k = \frac{I_2 h}{I_1 L}$$

otros datos (322)



$$H_A = H_D = H = \frac{3\,(1+2k)}{h} \cdot \frac{15EI_2\varepsilon t}{\alpha_1 h}$$

$$M_A = M_D = m \cdot \frac{15EI_2\varepsilon t}{\alpha_1 h} \qquad M_B = M_C = M_A - Hh \quad ; \quad M_s = M_A - Hh_1$$

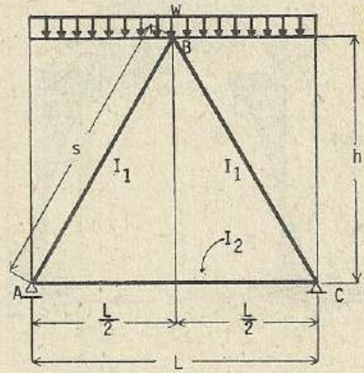
Otra forma de solución :

$$H_A = H_D = H = \frac{45\,(2k+1)\,E\varepsilon I_2 t}{\beta}$$

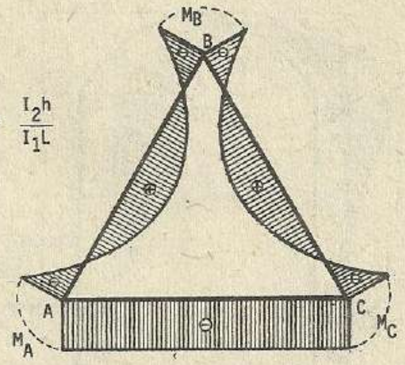
$$M_A = M_D = \frac{-15\,[3\,(k+1)h + 2f]\,EI_2\varepsilon t}{\beta}$$

Nota : El presente problema corresponde al incremento de temperatura, en caso de descenso, los esfuerzos se anotan con los signos contrarios.

(330)

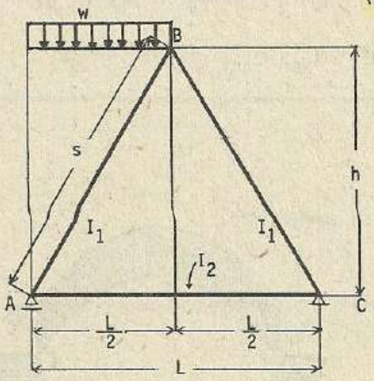


$k = \frac{I_2 h}{I_1 L}$

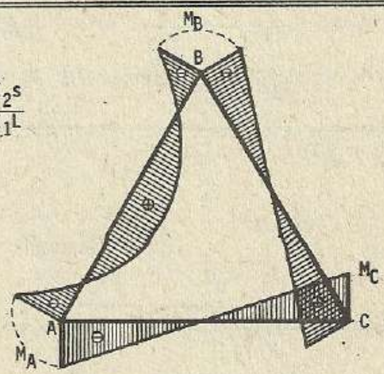


$H = \frac{wL}{16} \cdot \frac{L}{h} \cdot \frac{2k+5}{k+2}$; $M_A = M_C = -\frac{wL^2}{48} k \frac{1}{(k+2)}$; $M_B = -\frac{wL^2}{48} k \frac{(3+k)}{(k+2)}$

(331)



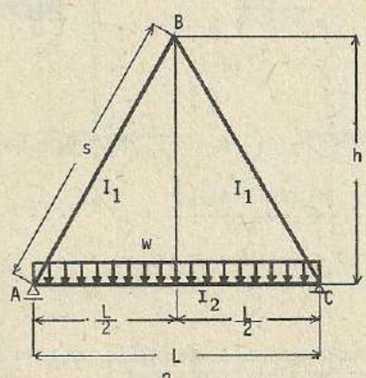
$k = \frac{I_2 s}{I_1 L}$



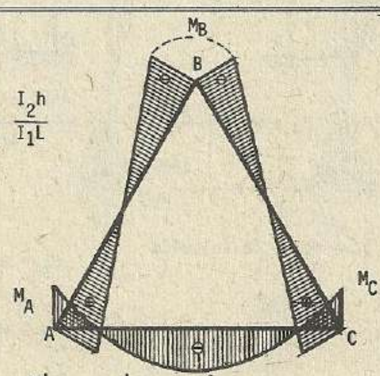
$H = \frac{wL}{32} \cdot \frac{L}{h} \cdot \frac{(2k+5)}{(k+2)}$; $M_A = -\frac{wL^2}{96}\left[\frac{1}{k+2} + \frac{3}{2k+1}\right]k$

$M_C = -\frac{wL^2}{96}\left[\frac{1}{(k+2)} - \frac{3}{2k+1}\right]k$; $M_B = -\frac{wL^2}{96}\left[\frac{1}{k+2} + 1\right]$

(332)



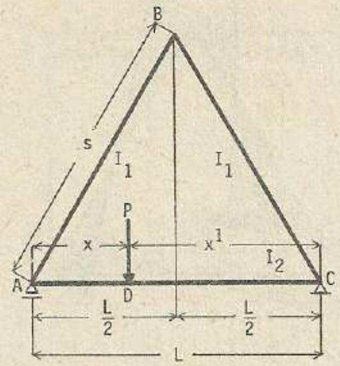
$k = \frac{I_2 h}{I_1 L}$



$H = \frac{wL}{4} \cdot \frac{L}{h} \cdot \frac{1}{k+2}$

$M_A = M_C = \frac{wL^2}{6} \cdot \frac{1}{k+2}$; $M_B = -\frac{wL^2}{12(k+2)}$

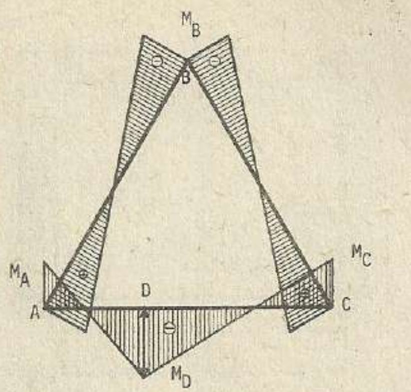
(333)



$$k = \frac{I_2 h}{I_1 L}$$

$$\alpha = \frac{x}{L}$$

$$\beta = \frac{x^1}{L}$$



$$H = \frac{3}{2} P \frac{L}{h} \cdot \frac{\alpha\beta}{k+2} \; ; \qquad M_A = \frac{PL}{2} \alpha\beta \left[\frac{2}{k+2} + \frac{\beta - \alpha}{2k+1}\right] ; \quad M_C = \frac{PL}{2} \left[\frac{2}{k+2} - \frac{\beta - \alpha}{2k+1}\right] \alpha\beta$$

$$M_B = - \frac{Pxx^1}{L^2} \left[\frac{x+x^1}{2\,(k+2)}\right]$$

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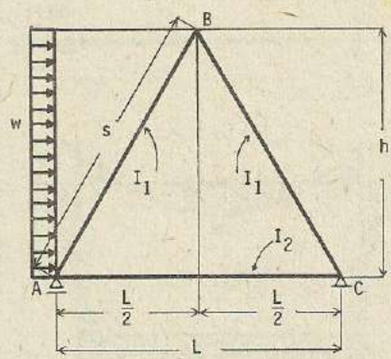
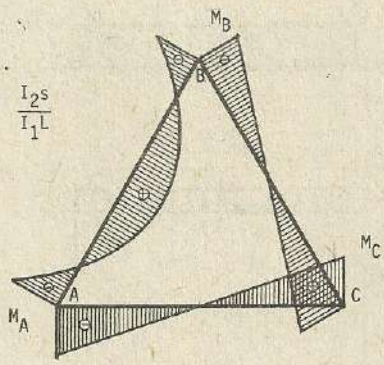
(334)



$$k = \frac{I_2 s}{I_1 L}$$

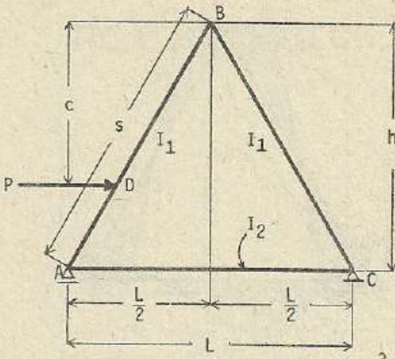


$$H_A = - \frac{wh}{2} \left[1 + \frac{2k+3}{4\,(k+2)}\right] \; ; \qquad H_C = - \frac{wh}{2} \left[- 1 + \frac{2k+3}{4\,(k+2)}\right]$$

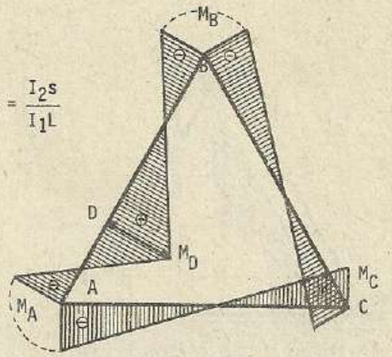
$$M_A = - \frac{kwh^2}{24} \left[\frac{1}{k+2} + \frac{3}{2k+1}\right] \; ; \qquad M_C = - \frac{kwh^2}{24} \left[\frac{1}{k+2} - \frac{3}{2k + 1}\right]$$

$$M_B = - \frac{(k+3)\; wh^2}{24\;(k+2)}$$

(335)



$$k = \frac{I_2 s}{I_1 L}$$



$$H_A = -\frac{P}{2}\left[1 + \left(\frac{c}{h}\right)^2 \frac{3(k+1) - \frac{c}{h}(2k+1)}{k+2}\right]$$

$$H_C = -\frac{P}{2}\left[-1 + \left(\frac{c}{h}\right)^2 \frac{3(k+1) - \frac{c}{h}(2k+1)}{k+2}\right]$$

$$M_A = -\frac{P}{2}\, k\left(1 - \frac{c}{h}\right)\left[\frac{\frac{c}{h}}{k+2} + \frac{\frac{c}{h}+1}{2k+1}\right]$$

$$M_B = -\frac{P}{2}\,\frac{(h-c)c}{h^2}\left[\frac{c}{k+2} + (h-c)\right] ; \qquad M_C = -\frac{P}{2}\, k\left(1 - \frac{c}{h}\right)\left[\frac{\frac{c}{h}}{k+2} - \frac{\frac{c}{h}+1}{2k+1}\right]$$

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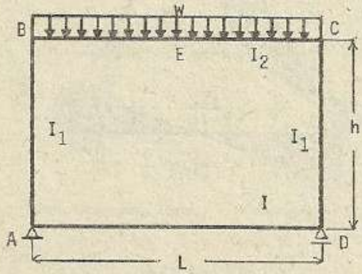
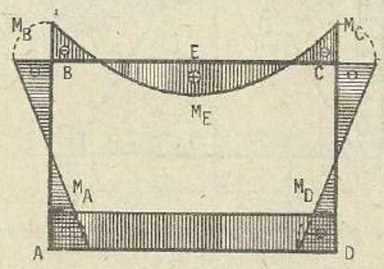
(336)





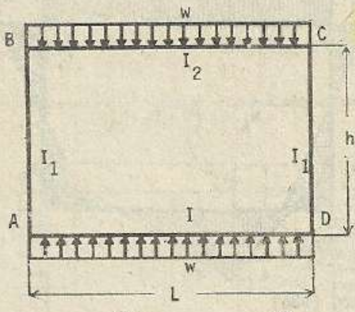
$$k = \frac{I_2 h}{I_1 L} \qquad k^1 = \frac{I_2}{I} \qquad \mu_1 = k\beta + k^1\alpha \qquad \mu_2 = \gamma + \gamma^1$$

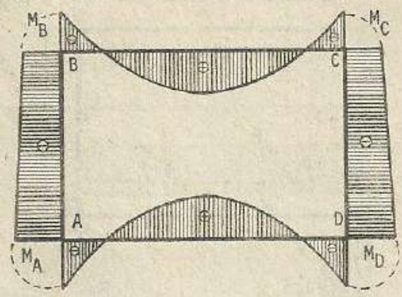
$$\alpha = 2k+3 \qquad \beta = k+2 \qquad \gamma = 3k+1$$

$$\alpha^1 = 2k+3k^1 \qquad \beta^1 = k+2k^1 \qquad \gamma^1 = 3k+k^1$$

$$M_A = M_D = \frac{kwL^2}{12\mu_1} \quad ; \qquad M_B = M_C = -\frac{\alpha^1 wL^2}{12\mu_1} \quad ; \quad M_E = \frac{wL^2}{8} - M_B$$

(337)





$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; otros datos, referencia (336)

$M_A = M_D = -\frac{wL^2}{12\mu_1}(3k^1 + 2kk^1 - K)$; $M_B = M_C = -\frac{wL^2}{12\mu_1}(3k^1 - kk^1 + 2k)$

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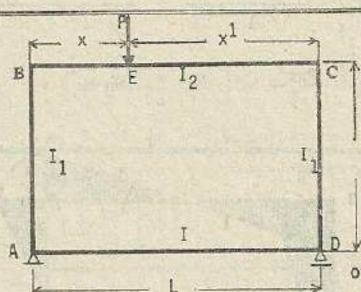
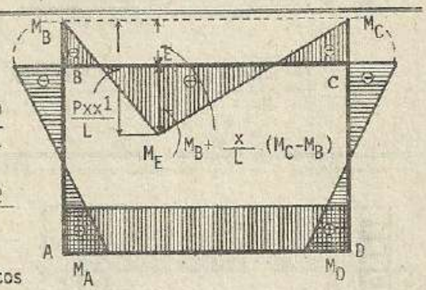
(338)



$k = \frac{I_2 h}{I_1 L}$

$k^1 = \frac{I_2}{I}$

otros datos (336)



$C_{BC} = \frac{-Px(x^1)^2}{L^2}$; $C_{CB} = \frac{Px^2x^1}{L^2}$; $M_A = -\frac{k(C_{BC} + C_{CB})}{2\mu_1} + \frac{C_{BC} - C_{CB}}{2\mu_2}$

$M_D = -\frac{k(C_{BC} + C_{CB})}{2\mu_1} - \frac{C_{BC} + C_{CB}}{2\mu_2}$; $M_B = \frac{\alpha^1(C_{BC} + C_{CB})}{2\mu_1} + \frac{C_{BC} - C_{CB}}{2\mu_2}$

$M_C = \frac{\alpha^1(C_{BC} - C_{CB})}{2\mu_1} - \frac{C_{BC} - C_{CB}}{2\mu_2}$; $M_E = \frac{Pxx^1}{L} - \left[M_B + \frac{x}{L}(M_C - M_B)\right]$

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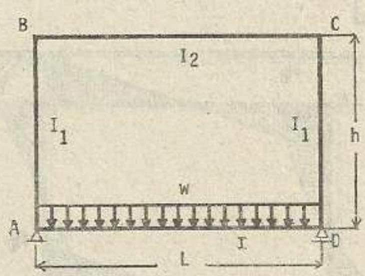
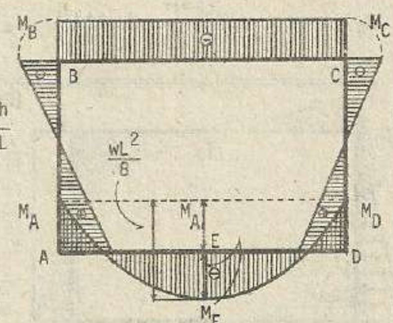
(339)



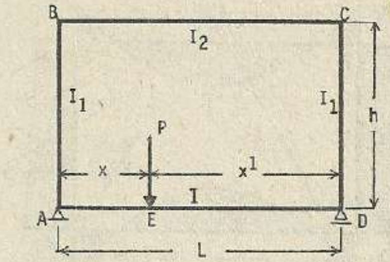
$k = \frac{I_2 h}{I_1 L}$

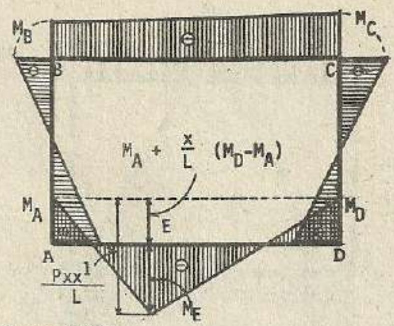


$C_{AD} = C_{DA} = \frac{wL^2}{12}$; $M_B = -C_{AD}\frac{k}{(k+1)(k+3)} = M_C$

$M_A = C_{AD}\frac{2k+3}{(k+1)(k+3)} = M_D$; $M_E = \frac{wL^2}{8} - M_A$

(340)





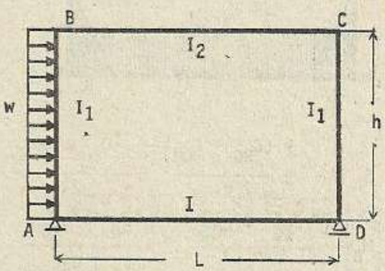
$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; otros datos (336)

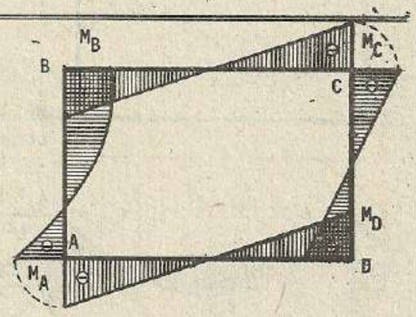
$C_{AD} = \frac{Px\,(x^1)^2}{L^2}$; $C_{DA} = \frac{Px^2 x^1}{L^2}$; $M_A = -\frac{\alpha k^1 (C_{AD}+C_{DA})}{2\mu_1} - \frac{k^1(C_{AD} - C_{DA})}{2\mu_2}$

$M_D = -\frac{\alpha k^1(C_{AD}+C_{DA})}{2\mu_1} + \frac{k^1(C_{AD} - C_{DA})}{2\mu_2}$; $M_B = \frac{kk^1(C_{AD}+C_{DA})}{2\mu_1} - \frac{k^1(C_{AD} - C_{DA})}{2\mu_2}$

$M_C = \frac{kk^1(C_{AD}+C_{DA})}{2\mu_1} + \frac{k^1\,(C_{AD}-C_{DA})}{2\mu_2}$; $M_E = \frac{Pxx^1}{L} - \left[M_A + \frac{x}{L}\cdot(M_D - M_A) \right]$

(341)



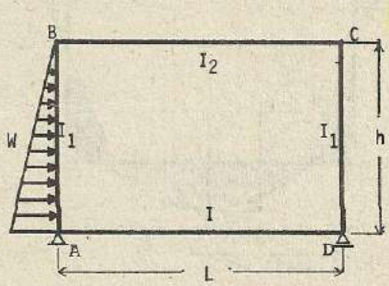


$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$ otros datos (336)

$M_A = \frac{wh^2}{4}\left[- \frac{k\text{-}(k+3)}{6\mu_1} - \frac{4k+1}{\mu_2}\right]$; $M_D = \frac{wh^2}{4}\left[- \frac{k\,(k+3)}{6\mu_1} + \frac{4k+1}{\mu_2}\right]$

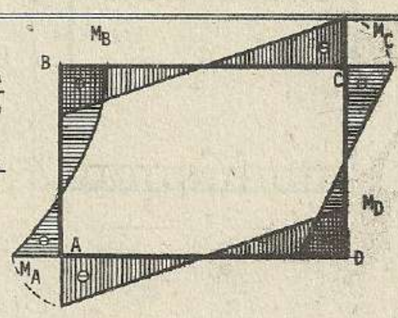
$M_B = M_C = \frac{wh^2}{4}\left[- \frac{k\,(k+3k^1)}{6\mu_1} \pm \frac{2k+k^1}{\mu_2}\right]$

(342)



$k = \frac{I_2 h}{I_1 L}$

$k^1 = \frac{I_2}{I}$



$m = \frac{1}{1+6k+k^1}$ $n = \frac{1}{2+k+\frac{k^1}{k}\,(2k+3)}$

$$M_A = -\frac{Wh}{3}\left[\frac{3k+8}{20}\,n + \frac{9k+2}{4}\,m\right] \quad ; \quad M_D = -\frac{Wh}{3}\left[\frac{3k+8}{20}\,n - \frac{9k+2}{4}\,m\right]$$

$$M_B = -\frac{wh}{2}\left[\frac{2k+7k^1}{20}\,n - \frac{3k+2k^1}{4}\,m\right] \quad ; \quad M_C = -\frac{Wh}{2}\left[\frac{2k+7k^1}{20}\,n + \frac{3k+2k^1}{4}\,m\right]$$

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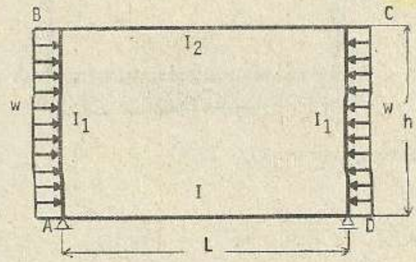
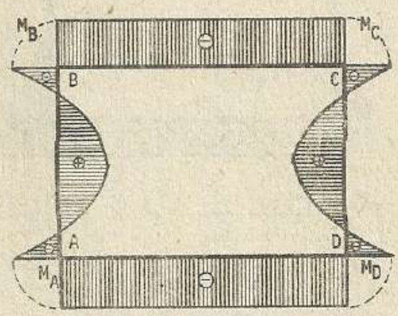
(343)





$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k^1 = \frac{I_2}{I} \quad ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = M_D = -\frac{wh^2}{12}(k+3)\,n \quad ; \quad M_B = M_C = -\frac{wh^2}{12}(3k^1+k)\,n$$

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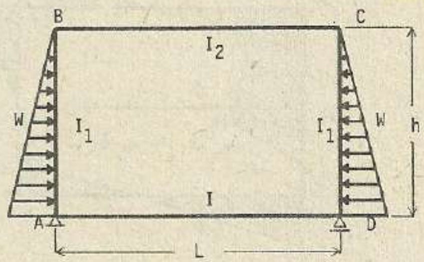
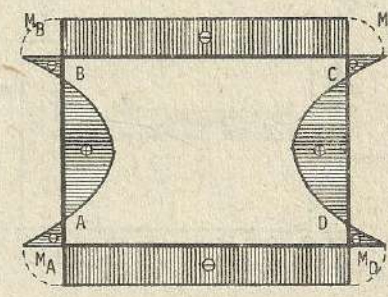
(344)





$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k^1 = \frac{I_2}{I} \quad ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = M_D = -\frac{Wh}{30}(3k+8)\,n \quad ; \quad M_B = M_C = -\frac{Wh}{30}\cdot(2k+7k^1)\,n$$

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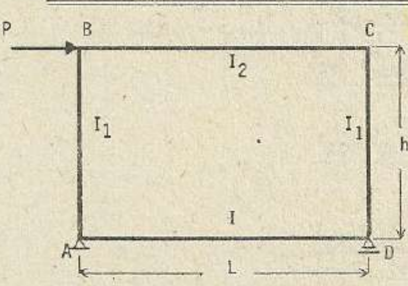
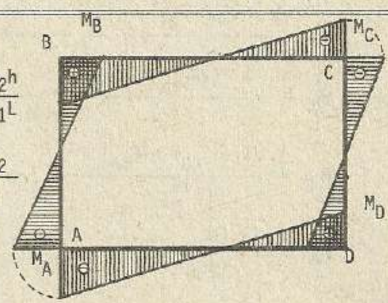
(345)



$$k = \frac{I_2 h}{I_1 L}$$

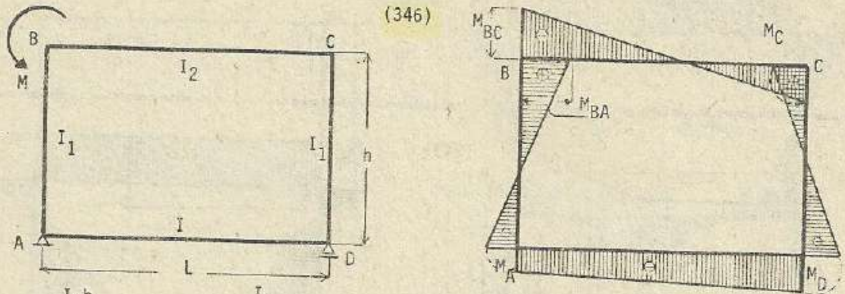
$$k^1 = \frac{I_2}{I}$$

otros datos (336)



$$M_A = -\frac{\gamma Ph}{2\mu_2} \quad ; \quad M_B = \frac{\gamma^1 Ph}{2\mu_2} \quad ; \quad M_C = -\frac{\gamma^1 Ph}{2\mu_2} \quad ; \quad M_D = \frac{\gamma Ph}{2\mu_2}$$

(346)

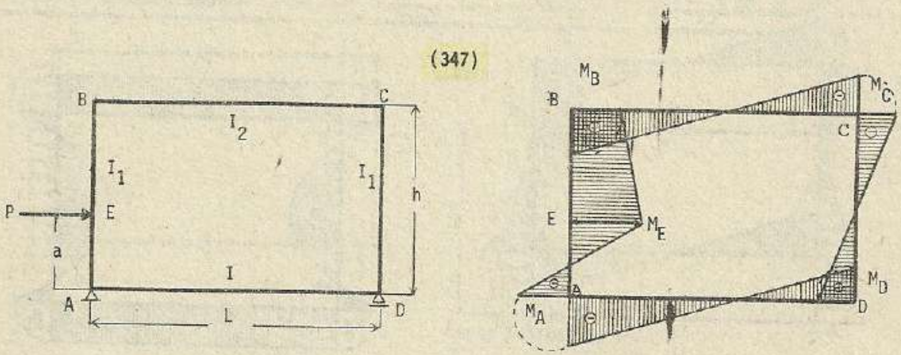


$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; otros datos, referencia (336) .

$$M_A = -\frac{M}{2}\left[\frac{k}{\mu_1} - \frac{1}{\mu_2}\right] \quad ; \quad M_D = -\frac{M}{2}\left[\frac{k}{\mu_1} + \frac{k}{\mu_2}\right]$$

$$M_{BA} = \frac{M}{2}\left[\frac{\alpha^1}{\mu_1} - \frac{1}{\mu_2}\right] \quad ; \quad M_C = \frac{M}{2}\left[\frac{\alpha^1}{\mu_1} + \frac{1}{\mu_2}\right] ; \quad M_{BC} = M_{BA} - M$$

(347)



$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; otros datos (336)

$$C_{AB} = -\frac{Pa(h-a)^2}{h^2} \quad ; \quad C_{BA} = -\frac{Pa^2\,(h-a)}{h^2}$$

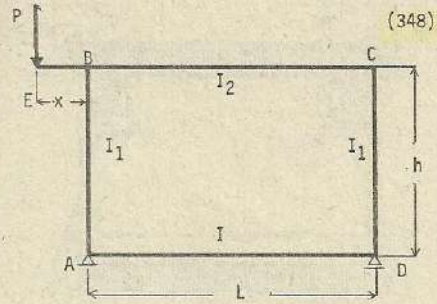
$$M_D = \frac{k\,(\beta C_{AB} + C_{BA})}{2\mu_1} + \frac{\gamma Pa - 3k\,(C_{AB} + C_{BA})}{2\mu_2}$$

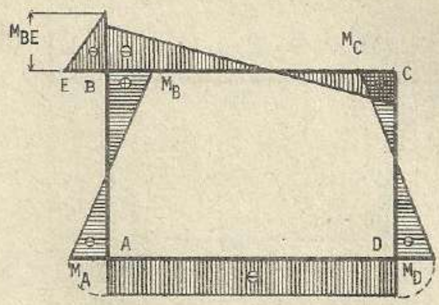
$$M_B = \frac{k\,(k^1 C_{AB} + \beta^1 C_{BA})}{2\mu_1} + \frac{\gamma^1 Pa + 3k\,(C_{AB} + C_{BA})}{2\mu_2}$$

$$M_C = \frac{k(k^1 C_{AB} + \beta^1 C_{BA})}{2\mu_1} - \frac{\gamma^1 Pa + 3k\,(C_{AB} + C_{BA})}{2\mu_2}$$

$$M_A = \frac{k\,(\beta C_{AB} + C_{BA})}{2\,\mu_1} - \frac{\gamma\,Pa - 3\,k\,(C_{AB} + C_{BA})}{2\,\mu_2}$$

(348)





$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k^1 = \frac{I_2}{I} \quad ; \quad m = \frac{1}{1+6k+k^1} \quad ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = -\frac{Px}{2}(n-m) \quad ; \quad M_D = -\frac{Px}{2}(n+m)$$

$$M_{BA} = \frac{Px}{2}\left[\left(2+3\frac{k^1}{k}\right) n + m\right] \quad ; \quad M_C = \frac{Px}{2}\left[\left(2+3\frac{k^1}{k}\right) n - m\right]$$

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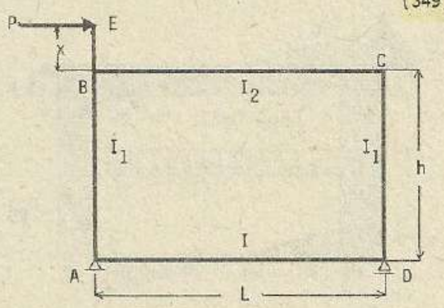
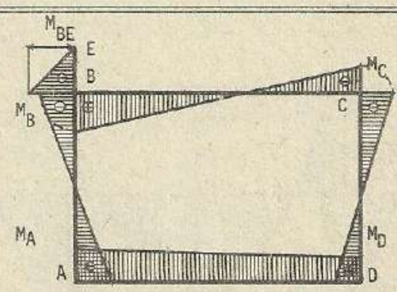
(349)





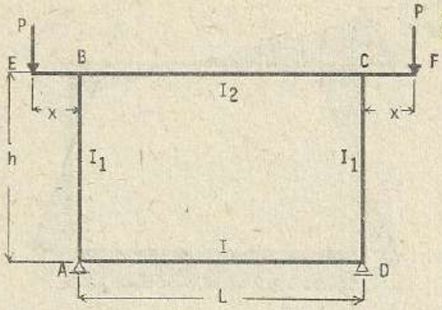
$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k^1 = \frac{I_2}{I} \quad ; \quad m = \frac{1}{1+6k+k^1} \quad ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

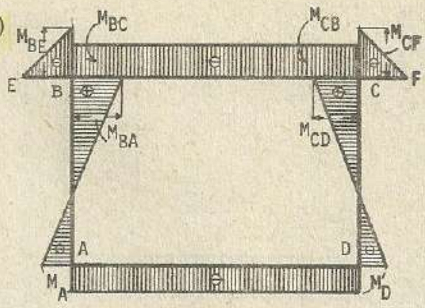
$$M_A = \frac{Ph}{2}\left[\frac{x}{h} n - \left(1+3k+\frac{x}{h}\right) m\right] \quad ; \quad M_D = \frac{Ph}{2}\left[\frac{x}{h} n + \left(1+3k+\frac{x}{h}\right) m\right]$$

$$M_{BA} = -\frac{Ph}{2}\left[\left(2+3\frac{k^1}{k}\right)\frac{x}{h} n - \left(k^1+3k-\frac{x}{h}\right) m\right]$$

$$M_C = -\frac{Ph}{2}\left[\left(2+3\frac{k^1}{k}\right)\frac{x}{h} n + \left(k^1+3k-\frac{x}{h}\right) m\right]$$

(350)





$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; $m = \frac{1}{1+6k+k^1}$; $n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$

$M_A = M_D - Pxn$; $M_{BA} = Px\left(2+3\frac{k^1}{k}\right) n$; $M_{BC} = M_{BA} - Px$

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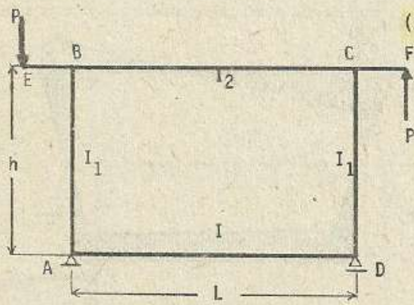
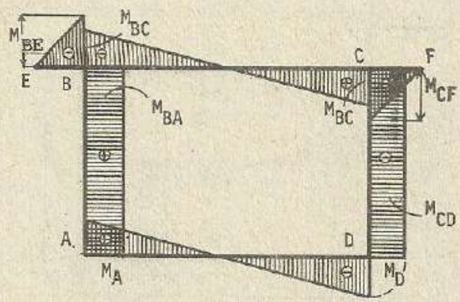
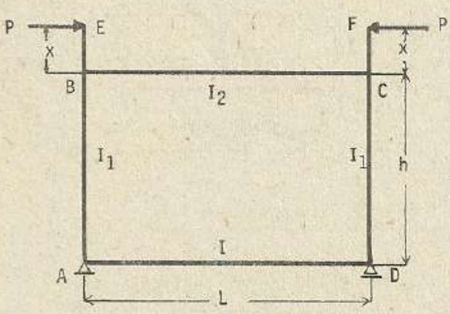
(351)

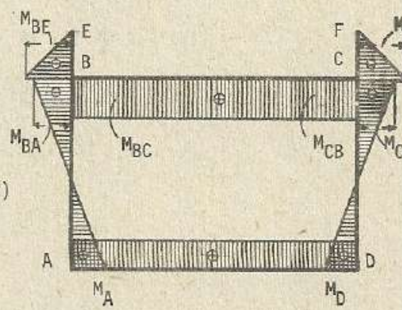




$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; $m = \frac{1}{1+6k+k^1}$; $n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$

$M_A = Pxm = -M_D = M_{BA} = -M_{CD}$ $-M_{BC} = M_{CB} = M_{BA} - Px$

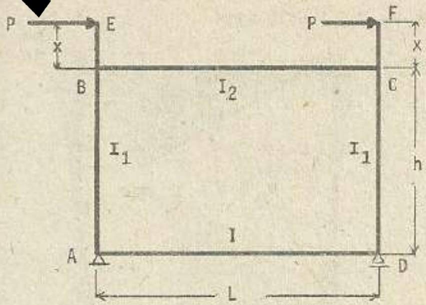




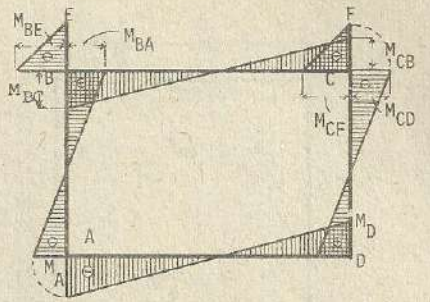
(352)

$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; $m = \frac{1}{1+6k+k^1}$; $n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$

$M_A = Pxn = M_D$; $M_{BA} = -Px\left(2+3\frac{k^1}{k}\right) n = M_{BC} = M_{CB} = M_{CD}$



(353)

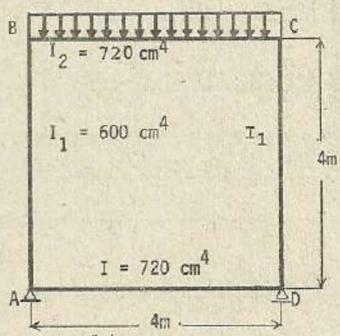


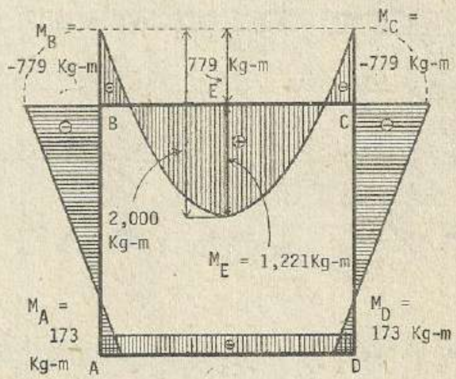
$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k^1 = \frac{I_2}{I} \quad ; \quad m = \frac{1}{1+6k+k^1} \quad ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)}$$

$$M_A = -M_D = -P\left[h(1+3k)+x\right]m \quad ; \quad M_{BA} = -M_{CD} = P\left[h(k^1+3k)-x\right]m$$

$$M_{BC} = -M_{CB} = M_{BA} + Px.$$

<u>Solución de problemas numéricos</u> :





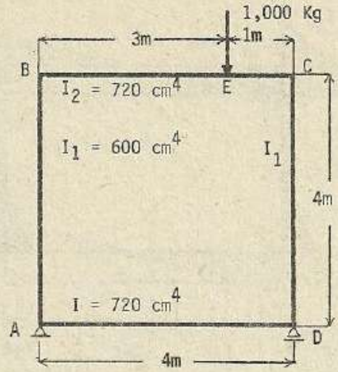
$$k = \frac{I_2 h}{I_1 L} = \frac{720 \times 4}{600 \times 4} = 1.2 \quad ; \quad k^1 = \frac{I_2}{I} = \frac{720}{720} = 1$$

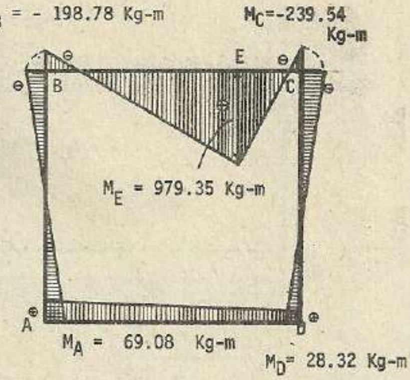
$$\alpha = 2k+3 = 2 \times 1.2 + 3 = 5.4 \quad ; \quad \alpha^1 = 2k + 3k^1 = 2 \times 2.1 + 3 \times 1 = 5.4$$

$$\beta = k+2 = 1.2 + 2 = 3.2 \quad ; \quad \mu_1 = k\beta + k^1\alpha = 1.2 \times 3.2 + 1 \times 5.4 = 9.24$$

$$M_A = M_D = \frac{kwL^2}{12\mu_1} = \frac{1.2 \times 1{,}000 \times 4 \times 4}{12 \times 9.24} = 173 \text{ Kg-m}; \quad M_B = M_C = -\frac{\alpha^1 wL^2}{12\mu_1} = -\frac{5.4 \times 1{,}000 \times 4 \times 4}{12 \times 9.24} = -779 \text{ Kg-m.}$$

$$M_E = \frac{wL^2}{8} - M_B = \frac{1{,}000 \times 4 \times 4}{8} - 779 = 1{,}221 \text{ Kg-m}$$





$$k = \frac{I_2 h}{I_1 L} = \frac{720x4}{600x4} = 1.2 \quad ; \qquad K^1 = \frac{I_2}{I} = \frac{720}{720} = 1$$

$$C_{BC} = -\frac{Px\,(x^1)^2}{L^2} = -\frac{1,000x3x1x1}{4x4} = -187.5 \text{ Kg-m} \; ; \quad \gamma = 3k+1 = 3x1.2+1 = 4.6$$

$$C_{CB} = \frac{Px^2x^1}{L^2} = -\frac{1,000x3x3x1}{4x4} = -562.5 \text{ Kg-m} \; ; \quad \gamma^1 = 3k+k^1 = 3x1.2+1 = 4.6$$

$$\alpha = 2k+3 = 2x1.2+3 = 5.4 \; ; \quad \alpha^1 = 2k+3k^1 = 2x1.2+3x1 = 5.4 \; ; \quad \beta = k+2 = 1.2+2 = 3.2$$

$$\mu_1 = k\beta + k^1\alpha = 1.2x3.2 + 1x5.4 = 9.24 \; ; \quad \mu_2 = \gamma + \gamma^1 = 4.6+4.6 = 9.2$$

$$M_A = -\frac{k(C_{BC}+C_{CB})}{2\mu_1} + \frac{(C_{BC}+C_{CB})}{2\mu_2} = \frac{1.2\left[(-187.5)+(-562.5)\right]}{2 \times 9.24} + \frac{(-187.5)-(-562.5)}{2x9.2} = 69.08 \text{ Kg - m}$$

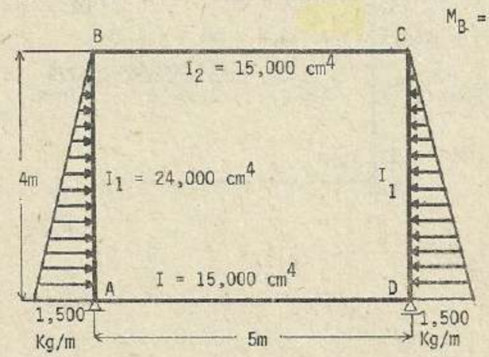
$$M_B = -\frac{\alpha^1(C_{BC}+C_{CB})}{2\mu_1} + \frac{C_{BC}-C_{CB}}{2\mu_2} = +\frac{5.4\left[(-187.5)+(-562.5)\right]}{2x9.24} + \frac{(-187.5)-(-562.5)}{2x9.2} = -198.78 \text{ Kg-m}$$

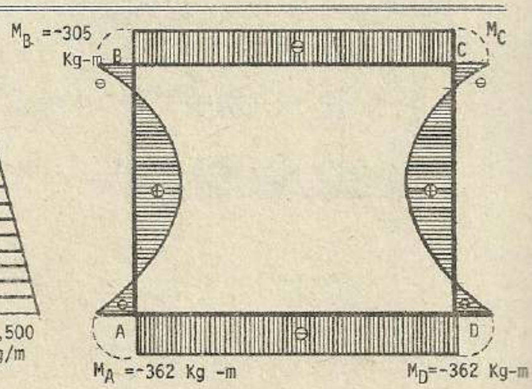
$$M_C = +\frac{\alpha^1(C_{BC}+C_{CB})}{2\mu_1} - \frac{C_{BC}-C_{CB}}{2\mu_2} = +\frac{5.4\left[(-187.5)+(-562.5)\right]}{2x9.24} - \frac{(-187.5)-(-562.5)}{2x9.2} = -239.54 \text{ Kg-m}$$

$$M_E = \frac{Pxx^1}{L} - \left[M_B + \frac{x}{L}(M_C - M_B)\right] = \frac{1,000x3x1}{4} - \left[-198.78 + \frac{3}{4}(-239.54+198.78)\right] =$$

979.35 Kg-m

$$M_D = -\frac{K(C_{BC}+C_{CB})}{2\mu_1} - \frac{C_{BC}-C_{CB}}{2\mu_2} = -\frac{1.2\{(-187.5)+(-562.5)\}}{2 \times 9.24} - \frac{(-187.5)-(-562.5)}{2 \times 9.2} = 28.32 \text{ Kg-m}$$





$$k = \frac{I_2 h}{I_1 L} = \frac{15{,}000 \times 4}{24{,}000 \times 5} = \frac{1}{2} = 0.5 \quad ; \qquad k^1 = \frac{I_2}{I} = \frac{15{,}000}{15{,}000} = 1$$

$$m = \frac{1}{1+6k+k^1} = \frac{1}{1+6\times0.5+1} = \frac{1}{5} \quad ; \quad n = \frac{1}{2+k+\frac{k^1}{k}(2k+3)} = \frac{1}{2+0.5+\frac{1}{0.5}(2\times0.5+3)} = \frac{1}{10.5}$$

$$M_A = M_D = -\frac{Wh}{30}(3k+8)\,n$$

$$= -\frac{1{,}500\times4\times\frac{1}{2}\times4}{30}(3\times0.5+8)\times\frac{1}{10.5} = -362 \text{ Kg-m}$$

$$M_B = M_C = -\frac{Wh}{30}(2k+7k^1)\,n$$

$$= \frac{1{,}500\times4\times\frac{1}{2}\times4}{30}(2\times0.5+7\times1)\times\frac{1}{10.5} = -305 \text{ Kg - m}$$

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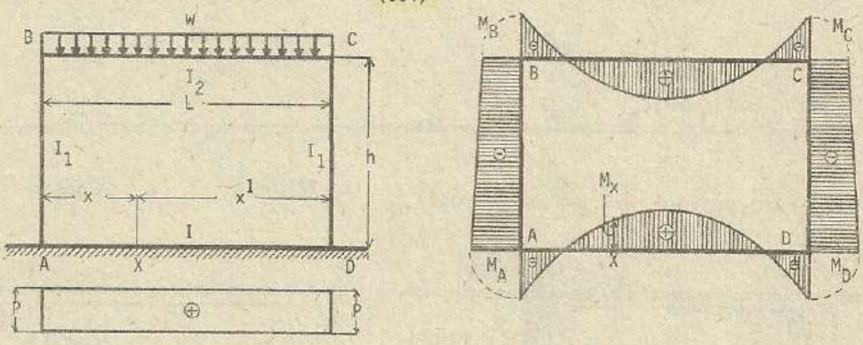
(354)



$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k^1 = \frac{I_2}{I} \quad ; \qquad \mu_1 = k\beta + k^1\alpha \quad ; \quad \mu_2 = \gamma + \gamma^1$$

$$\alpha = 2k+3 \quad ; \quad \beta = k+2 \quad ; \qquad \gamma = 3k+1 \quad ; \quad \delta = 3k+1 - \frac{k^1}{5}$$

$$\alpha^1 = 2k+3k^1 \quad ; \quad \beta^1 = k+2k^1 \quad ; \qquad \gamma^1 = 3k+k^1 \quad ; \quad \delta^1 = 3k+k^1 - \frac{k^1}{5}$$

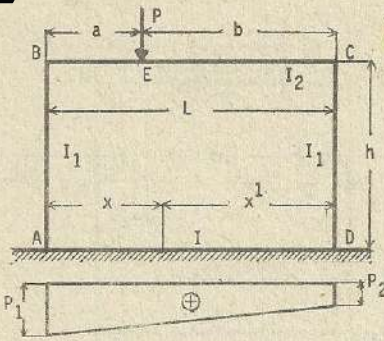
$$\lambda = \frac{x}{L} \qquad \lambda^1 = \frac{x^1}{L}$$

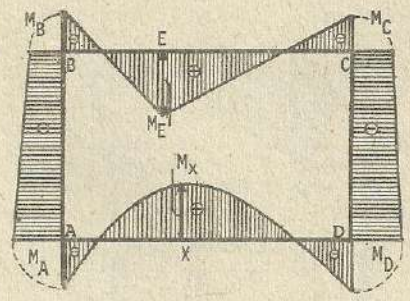
$$C_{BC} = -\frac{wL^2}{12}; \qquad C_{CB} = -\frac{wL^2}{12} \quad ; \qquad M_A = M_D = -\frac{\alpha k^1 wL + 6k(C_{BC} + C_{CB})}{12\,\mu_1}$$

$$M_B = M_C = +\frac{kk^1\,wL + 6\alpha^1(C_{BC} + C_{CB})}{12\,\mu_1}$$

$$M_x = \lambda^1 M_A + \lambda\, M_D + \frac{1}{6}(2w+p)\,xx^1 \qquad p = w$$

(355)





$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; otros datos, referencia (354)

$$C_{BC} = -\frac{Pab^2}{L^2} \quad ; \quad C_{CB} = -\frac{Pa^2b}{L^2}$$

$$M_A = -\frac{\alpha k^1 PL + 6k(C_{BC} + C_{CB})}{12\mu_2} - \frac{k^1 P(b-a) - 10(C_{BC} - C_{CB})}{20\mu_2}$$

$$M_D = -\frac{\alpha k^1 PL + 6k(C_{BC} + C_{CB})}{12\mu_1} + \frac{k^1 P(b-a) - 10(C_{BC} - C_{CB})}{20\mu_2}$$

$$M_B = +\frac{kk^1 PL + 6\alpha^1(C_{BC} + C_{CB})}{12\mu_1} - \frac{k^1 P(b-a) - 10(C_{BC} - C_{CB})}{20\mu_2}$$

$$M_C = +\frac{kk^1 PL + 6\alpha^1(C_{BC} + C_{CB})}{12\mu_1} + \frac{k^1 P(b-a) - 10(C_{BC} - C_{CB})}{20\mu_2}$$

$$M_x = \lambda^1 M_A + \lambda M_D + \frac{1}{6}\left(\frac{2P}{L} + PX\right)xx^1 \quad ; \quad P_1 = \frac{2P(2b-a)}{L^2} \quad : \quad P_2 = \frac{2P(2a-b)}{L^2}$$

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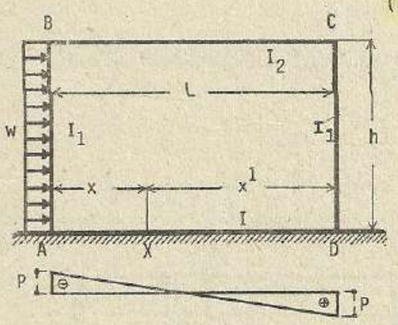
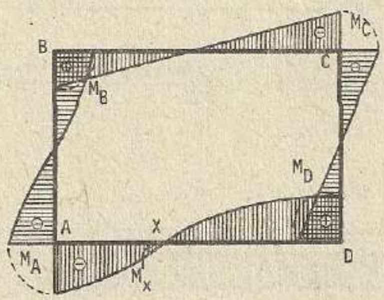
(356)





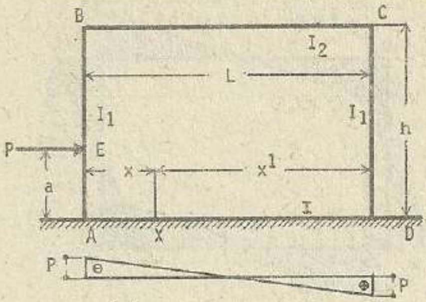
$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; otros datos (354)

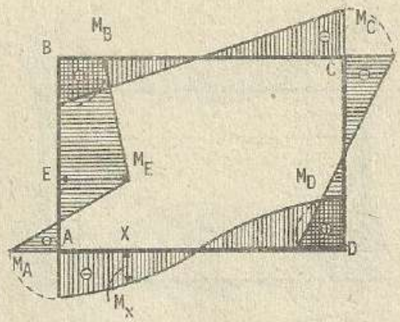
$$M_A = \frac{wh^2}{4}\left[-\frac{k(k+3)}{6\mu_1} - \frac{5+20k-k^1}{5\mu_2}\right] \quad ; \quad M_D = \frac{wh^2}{4}\left[-\frac{k(k+3)}{6\mu_1} + \frac{5+20k-k^1}{5\mu_2}\right]$$

$$M_B = \frac{wh^2}{4}\left[-\frac{k(k+3k^1)}{6\mu_1} + \frac{2(5k+3k^1)}{5\mu_2}\right] \quad ; \quad M_C = \frac{wh^2}{4}\left[-\frac{k(k+3k^1)}{6\mu_1} - \frac{2(5k+3k^1)}{5\mu_2}\right]$$

$$M_x = \lambda^1 M_A + \lambda M_D - n\frac{wh^2}{2} \quad ; \quad p = \frac{3wh^2}{L^2}$$

(357)





$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$ otros datos : (354)

$$C_{AB} = -\frac{Pa(h-a)^2}{h^2} \quad ; \quad C_{BA} = -\frac{Pa^2(h-a)}{h^2}$$

$$M_A = \frac{k(\beta C_{AB} + C_{BA})}{2\mu_1} + \frac{\delta Pa - 3k(C_{AB} + C_{BA})}{2\mu_2} \quad ; \quad M_D = \frac{k(\beta C_{AB} + C_{BA})}{2\mu_1} - \frac{\delta Pa - 3k(C_{AB} + C_{BA})}{2\mu_2}$$

$$M_B = \frac{k(k^1 C_{AB} + \beta^1 C_{BA})}{2\mu_1} + \frac{\delta^1 Pa - 3k(C_{AB} + C_{BA})}{2\mu_2} \quad ; \quad M_C = \frac{k(k^1 C_{AC} + \beta^1 C_{BA})}{2\mu_1} - \frac{\delta^1 Pa - 3k(C_{AB} + C_{BA})}{2\mu_2}$$

$$M_x = \lambda^1 M_A + \lambda M_D - \eta Pa \qquad p = \frac{6Pa}{L^2}$$

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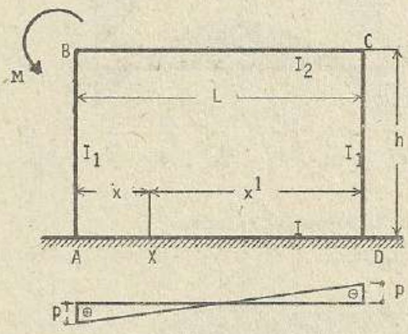
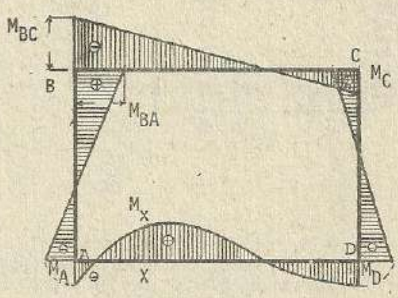
(358)





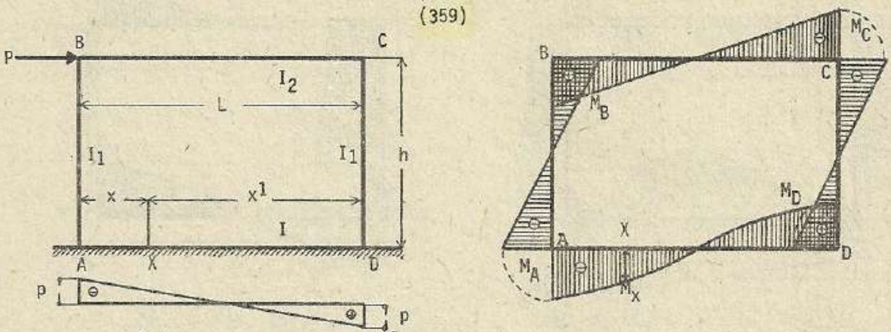
$k = \frac{I_2 h}{I_1 L}$; $k^1 = \frac{I_2}{I}$; otros datos (354)

$$M_A = -\frac{M}{2}\left[\frac{k}{\mu_1} - \frac{5-k^1}{5\mu_2}\right]; \quad M_D = -\frac{M}{2}\left[\frac{k}{\mu_1} + \frac{5-k^1}{5\mu_2}\right]; \quad M_{BA} = \frac{M}{2}\left[\frac{\alpha^1}{\mu_1} + \frac{5-k^1}{5\mu_2}\right]$$

$$M_C = \frac{M}{2}\left[\frac{\alpha^1}{\mu_1} - \frac{5-k^1}{5\mu_2}\right]; \quad M_{BC} = M_{BA} - M \quad ; \quad M_x = \lambda^1 M_A + \lambda M_D + \eta M$$

$$p = \frac{6M}{L^2}$$

(359)



$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k^1 = \frac{I_2}{I} \quad ;$$ otros datos, referencia (354)

$$M_A = -\frac{\delta P h}{2\mu_2} \quad ; \quad M_B = +\frac{\delta^1 P h}{2\mu_2} \quad ; \quad M_C = -\frac{\delta^1 P h}{2\mu_2} \quad ; \quad M_D = +\frac{\delta P h}{2\mu_2} \quad ; \quad p = \frac{6Ph}{L^2}$$

$$M_X = (\lambda - \lambda^1) M_D - \eta P h$$

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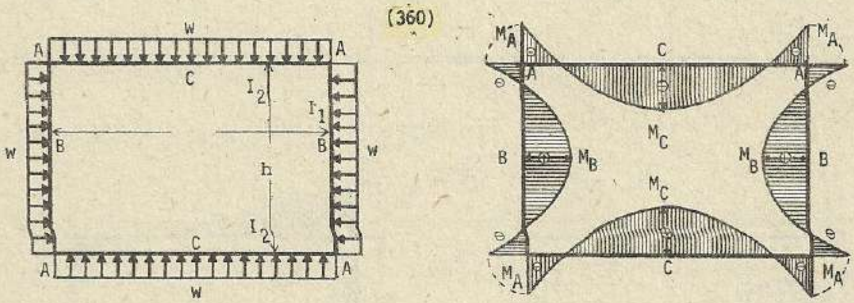
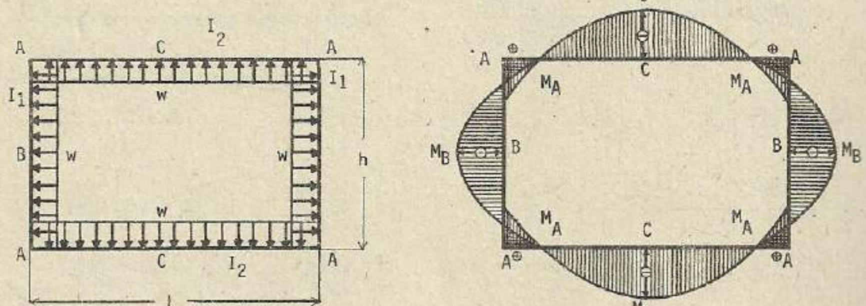
(360)

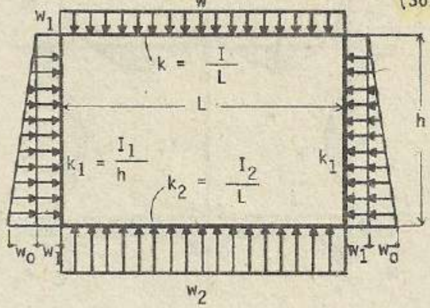


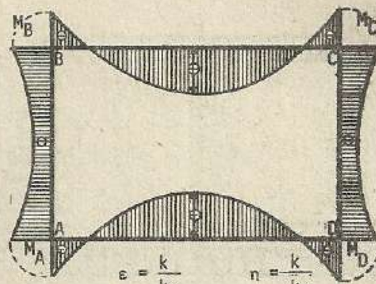
$$k = \frac{I_2 h}{I_1 L} \quad ; \quad n = \frac{h}{L} \quad ; \quad M_A = -\frac{wL^2}{12}\,\frac{1+n^2k}{1+k} \quad ; \quad M_B = M_A + \frac{wh^2}{8} \quad ; \quad M_C = M_A + \frac{wL^2}{8}$$

Nota : Se ha expuesto el problema de la acción de la carga (w) externa. En caso donde las fuerzas provengan desde el interior del marco, los esfuerzos de flexión poseen signos contrarios.



(361)





$\varepsilon = \frac{k}{k_1}$ $\quad\quad n = \frac{k}{k_2}$

$$M_{AD} = \frac{w_2L^2(3+2\varepsilon)n - \varepsilon\, wL^2}{12\{(2+\varepsilon)\varepsilon + (3+2\varepsilon)n\}} + \frac{w_1h^2(3+\varepsilon)\varepsilon}{12\{(2+\varepsilon)\varepsilon+(3+2\varepsilon)n\}} + \frac{w_0h^2(8+3\varepsilon)\varepsilon}{60\{(2+\varepsilon)\varepsilon+(3+2\varepsilon)n\}} = -M_{AB}$$

$$M_{BC} = \frac{\varepsilon\eta w_2L^2 - wL^2(3\eta + 2\varepsilon)}{12\{(2+\varepsilon)\varepsilon + (3+2\varepsilon)\eta\}} - \frac{w_1h^2(3\eta-\varepsilon)\varepsilon}{12\{(2+\varepsilon)\varepsilon + (3+2\varepsilon)n\}} - \frac{w_0h^2(7\eta+2\varepsilon)\varepsilon}{60\{(2+\varepsilon)\varepsilon + (3+2\varepsilon)n\}} = -M_{AB}$$

En caso donde $I = I_2$, entonces $\eta = 1$, y si $w_2 = w$, tenemos :

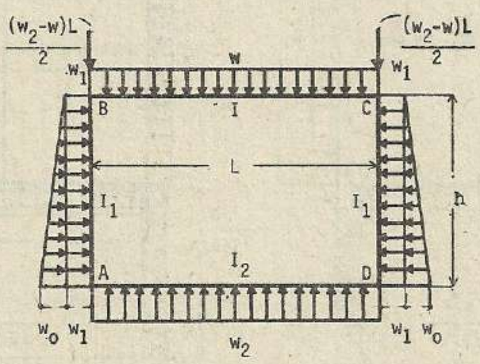
$$M_{AD} = \frac{wL^2}{12(1+\varepsilon)} + \frac{w_1h^2\varepsilon}{12(1+\varepsilon)} + \frac{w_0h^2(8+3\varepsilon)\varepsilon}{60(\varepsilon^2+4\varepsilon+3)} = -M_{AB}$$

$$M_{BC} = -\frac{wL^2}{12(1+\varepsilon)} - \frac{w_1h^2\varepsilon}{12(1+\varepsilon)} - \frac{w_0h^2(7+2\varepsilon)\varepsilon}{60(\varepsilon^2+4\varepsilon+3)\varepsilon} = -M_{BA}$$

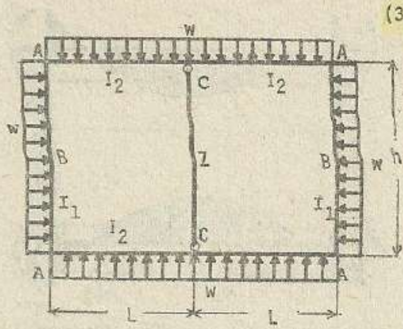
Método de solución diferente $\quad k_1 = \frac{h}{L}\,\frac{I}{I_1}$; $\quad k_2 = \frac{h}{L}\,\frac{I_2}{I_1}$; $\quad \alpha = (2+k_1)(2+k_2) - 1$

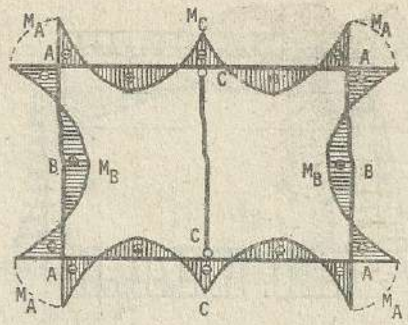
$$M_B = -\frac{(3+2k_2)}{\alpha}\cdot\frac{wL^2}{12} + \frac{k_1}{\alpha}\cdot\frac{w_2L^2}{12} - \frac{(3+k_2)k_1}{\alpha}\cdot\frac{w_1h^2}{12} - \frac{(7-2k_2)k_1}{\alpha}\cdot\frac{w_0h^2}{60}$$

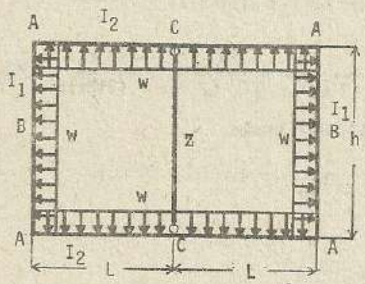
$$M_A = \frac{k_2}{\alpha}\cdot\frac{wL^2}{12} - \frac{(3+2k_1)}{\alpha}\cdot\frac{w_2L^2}{12} - \frac{(3+k_1)k_2}{\alpha}\cdot\frac{w_1h^2}{12} - \frac{(8+3k_1)k_2}{\alpha}\cdot\frac{w_0h^2}{60}$$

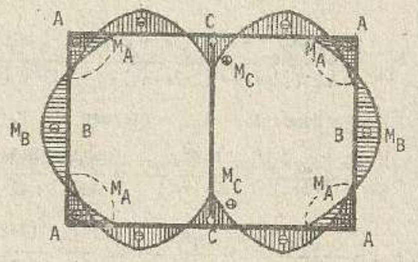


(362)





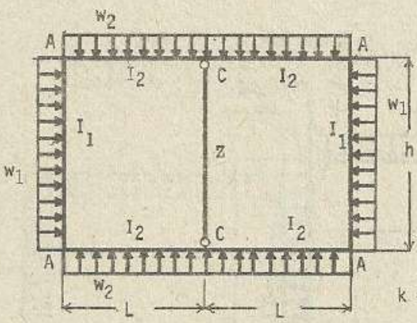




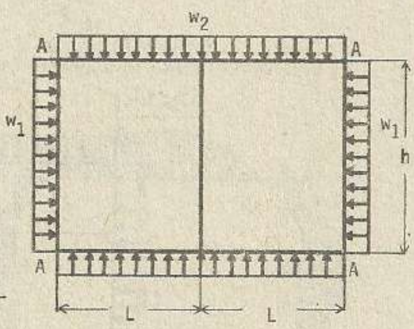
$$k = \frac{I_2 h}{I_1 L} \quad ; \quad n = \frac{h}{L} \quad ; \qquad M_A = -\frac{wL^2}{12} \cdot \frac{1+2n^2k}{1+2k} \quad ; \quad M_B = M_A + \frac{wh^2}{8}$$

$$M_C = -\frac{wL^2}{12} \cdot \frac{1+(3-n^2)\,k}{1+2k} \quad ; \qquad Z = \frac{wL^2}{2} \cdot \frac{2+(5-n^2)\,k}{1+2k}$$

Nota : Cuando la carga acciona desde la parte externa, el miembro interno trabaja como columna de soporte, en caso contrario se convierte en elemento de amarre, (Z) representa la fuerza en sentido axial.



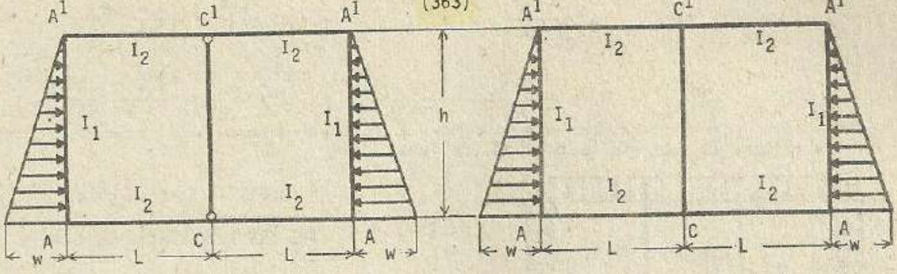
$$k = \frac{I_2 h}{I_1 L}$$



$$M_A = -\frac{2w_1h^2k + w_2L^2}{12\,(2k+1)} \quad ; \quad M_C = \frac{w_1h^2\,k - (3k+1)\,w_2L^2}{12\,(2k+1)}$$

Si $w_1 = w_2$ y $h = L$ entonces : $M_A = M_C = -\frac{wL^2}{12}$

(363)



$$k = \frac{I_2 h}{I_1 L}$$

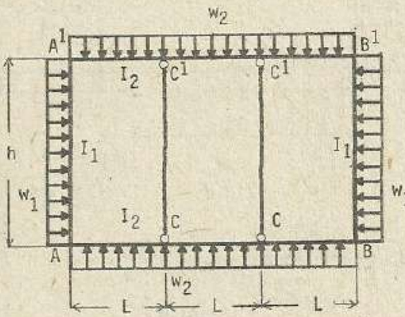
$$M_A = - \frac{wh^2(12k+61)\ k}{120\ (2k+1)\ (k+6)}$$

$$M_{A^1} = - \frac{wh^2\ (8k+59)\ k}{120\ (2k+1)\ (k+6)}$$

$$M_C = \frac{wh^2(3k+29)\ k}{120\ (2k+1)\ (k+6)}$$

$$M_{C^1} = \frac{wh^2\ (7k+31)\ k}{120\ (2k+1)\ (k+6)}$$

(364)

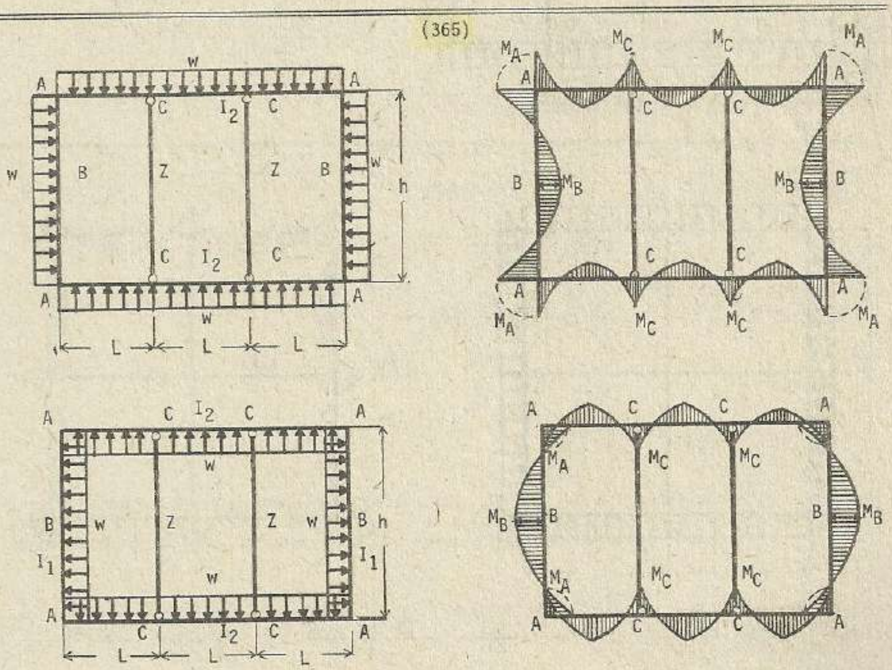


$$k = \frac{I_2 h}{I_1 L}$$

$$M_A = - \frac{2w_2L^2 + 5w_1\ h^2\ k}{12\ (5k+3)}$$

$$M_C = - \frac{3\ (2k+1)\ w_2L^2 - w_1h^2k}{12\ (5k+3)}$$

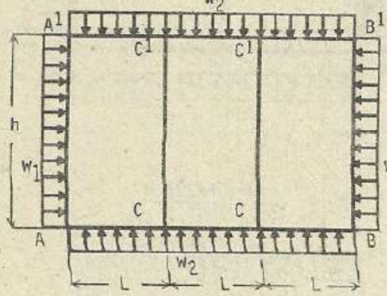
(365)



$$k = \frac{I_2 h}{I_1 L} \quad ; \quad n = \frac{h}{L} \quad ; \quad M_A = -\frac{wL^2}{12}\;\frac{3 + 5n^2 k}{3 + 5k}$$

$$M_B = M_A + \frac{wh^2}{8} \quad ; \quad M_C = -\frac{wh^2}{12}\;\frac{3 + (6-n^2)k}{3 + 5k} \quad ; \quad Z = \frac{wL^2}{2}\;\frac{6 + (11-n^2)k}{3 + 5k}$$

Se reitera la nota explicada en el problema (362)



$$M_{AA^1} = -\frac{w_2L^2\,(3k+1) + (5k+2)\,w_1h^2\,k}{12\,(5k^2 + 5k + 1)}$$

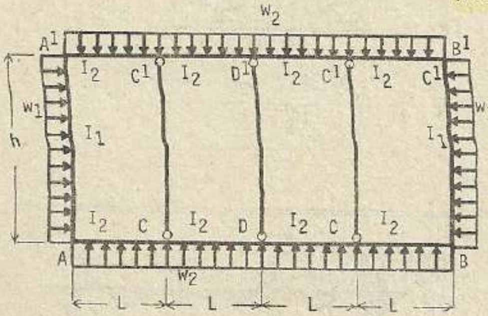
$$M_{CA} = -\frac{w_2L^2\,(6k^2+6k+1) - (k+1)\,w_1h^2k}{12\,(5k^2 + 5k+1)}$$

$$M_{CC^1} = \frac{w_2L^2k - w_1h^2k}{12\,(5k^2 + 5k+1)}$$

$$M_{CC} = -\frac{w_2L^2\,(6k^2 + 5k+1) - w_1h^2\,k}{12\,(5k^2 + 5k+1)}$$

$$k = \frac{I_2 h}{I_1 L}$$

(366)

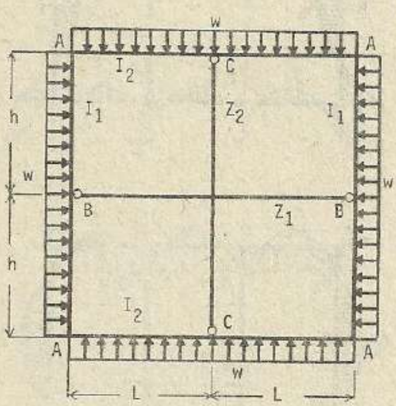


$$k = \frac{I_2 h}{I_1 L} \quad ; \quad M_A = -\frac{4w_2L + 7w_1h^2k}{12\,(7k+4)}$$

$$M_C = \frac{2w_1h^2k - (9k+4)\,w_2L^2}{12\,(7k+4)}$$

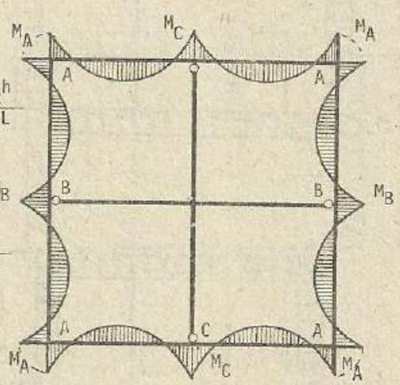
$$M_D = -\frac{w_1h^2k + (6k+4)\,w_2L^2}{12\,(7k+4)}$$

(367)



$$k = \frac{I_2 h}{I_1 L}$$

$$n = \frac{h}{L}$$

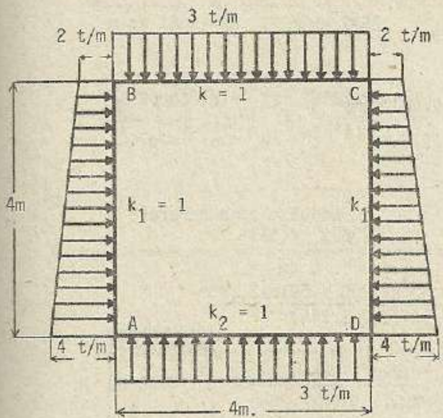


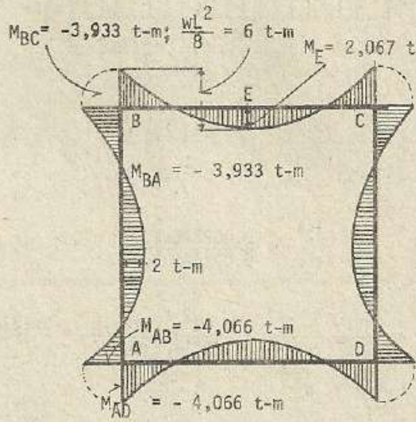
$$M_A = -\frac{wL^2}{12}\;\frac{1+n^2\,k}{1+k} \quad ; \quad M_B = -\frac{wL^2}{24}\;\frac{(3+2k)\;n^2-1}{1 + k} \quad ; \quad M_C = -\frac{wL^2}{24}\;\frac{(2+3k) - n^2k}{1 + k}$$

$$Z_1 = \frac{wL^2}{4} \cdot \frac{(4+5k) - n^2 k}{1+k} \quad ; \qquad Z_2 = \frac{wL^2}{4n} \cdot \frac{(5+4k)\, n^2 - 1}{1+k}$$

Observar la nota del problema (362)

Solución de problema numérico :





Referencia (361)

$$\varepsilon = \frac{k}{k_1} = \frac{1}{1} = 1 \quad ; \quad \eta = \frac{k}{k_2} = \frac{1}{1} = 1$$

$$M_{AD} = \frac{w_2 L^2 (3+2\varepsilon)\eta - \varepsilon w L^2}{12\ \{(2+\varepsilon)\varepsilon + (3+2\varepsilon)\eta\}} + \frac{w_1 h^2 (3+\varepsilon)\varepsilon}{12\ \{(2+\varepsilon)\varepsilon + (3+2\varepsilon)\eta\}} + \frac{w_0 h^2\ (8+3\varepsilon)\varepsilon}{60\ \{(2+\varepsilon)\varepsilon + (3+2\varepsilon)\eta\}}$$

$$= \frac{3x4x4(3+2x1)x1 - 1x3x4x4}{12\ \{(2+1)x1 + (3+2x1)x1\}} + \frac{2x4x4\ (3+1)\ x1}{12\ \{(2+1)x1 + (3+2x1)x1\}}$$

$$+ \frac{2x4x4(8+3x1)x1}{60\ \{(2+1)x1 + (3+2x1)x1\}} = 4.066 \text{ t-m} = -M_{AB}$$

$$M_{BC} = \frac{\varepsilon\eta\, w_2 L^2 - wL^2\ (3\eta+2\varepsilon)}{12\ \{(2+\varepsilon)\varepsilon + (3+2\varepsilon)\eta\}} - \frac{w_1 h^2\ (3\eta+\varepsilon)\varepsilon}{12\ \{(2+\varepsilon)\varepsilon + (3+2\varepsilon)\eta\}} - \frac{w_0 h^2\ (7\eta + 2\varepsilon)\varepsilon}{60\ \{(2+\varepsilon)\varepsilon + (3+2\varepsilon)\eta\}}$$

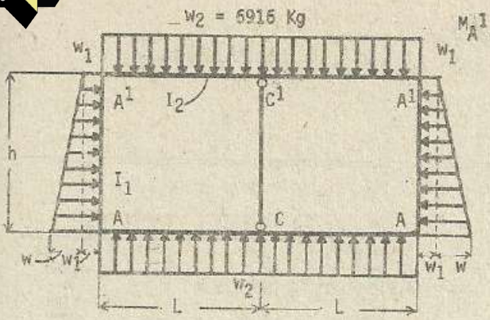
$$= \frac{1x1x3x4x4 - 3x4x4(3x1+2x1)}{12\ \{(2+1)x1 + (3+2x1)x1\}} - \frac{2x4x4(3x1+1)x1}{12\{\ (2+1)x1 + (3+2x1)\ x1\ \}}$$

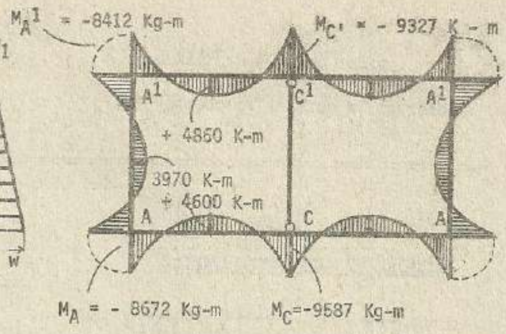
$$- \frac{2x4x4x(7x1+2x1)\ x1}{60\ \{(2+1)x1 + (3+2x1)x\}} = -3.933 \text{ t-m} = -M_{BA}$$

$$M_E = \frac{wL^2}{8} - M_{BC} = \frac{3x4x4}{8} - 3.933 = 2.067 \text{ t-m}$$

Debido a que la aplicación del método exacto, para la solución del esfuerzo de momento positivo de los muros extremos, es sumamente fatigoso, se recurre a un sistema abreviado, considerando una carga uniformemente distribuida

$$\frac{1}{2}\ (2+4) = 3 \text{ t/m} \quad ; \quad \frac{3x4x4}{8} - \frac{1}{2}\cdot(4.066+3.933) = 6 - 4 = 2 \text{ t-m.}$$





$L = 3.985$ m ; $h = 4.620$ m ; $I_2 = 0.003275 m^2$; $I_1 = 0.002995 m^4$

$w_1 = 2574$ Kg. ; $w = 4186$ Kg. ; $w_2 = 6916$ Kg ; $w + w_1 = 6760$ Kg.

$$k = \frac{I_2 h}{I_1 L} = \frac{0.003275 x 4.620}{0.002995 x 3.985} = 1.268$$

Se adoptan como referencia (362) y (363)

$$M_A = -\frac{2w_1h^2k+w_2L^2}{12\ (1+2k)} - \frac{wh^2(12k+61)k}{120(1+2k)(6+k)} = \frac{2x2574x4.62^2x1.268 + 6916x3.985^2}{12\ (1+2x1.268)}$$

$$- \frac{4186x4.62^2(12x1.268+61)x1.268}{120\ (1+2x1.268)(6+1.268)} = -8672 \text{ Kg - m}$$

$$M_A^1 = -\frac{2w_1h^2k+w_2L^2}{12\ (1+2k)} - \frac{wh^2(8k+59)k}{120(1+2k)(6+k)} = -\frac{2x2574x4.62^2x1.268 + 6916x3.985^2}{12(1+2x1.268)}$$

$$- \frac{4186x4.62^2(8x1.268+59)x1.268}{120(1+2x1.268)(6+1.268)} = -8412 \text{ Kg - m}$$

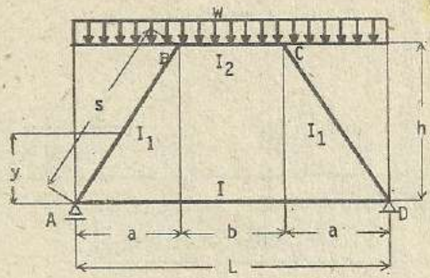
$$M_C = \frac{w_1h^2k-(1+3k)w_2L^2}{12\ (1+2k)} + \frac{wh^2(3k+29)k}{120(1+2k)(6+k)} = \frac{2574x4.62^2x1.268 - (1+3x1.268)x6916x3.985^2}{12\ (1+2x1.268)}$$

$$+ \frac{4186x4.62^2\ (3x1.268+29)x1.268}{120(1+2x1.268)\ (6+1.268)} = -9587 \text{ Kg - m}$$

$$M_{C1} = \frac{w_1h^2k-(1+3k)w_2L^2}{12\ (1+2k)} + \frac{wh^2(7k+31)k}{120(1+2k)(6+k)} = \frac{2574x4.62^2x1,268-(1+3x1.268)x6916x3.985^2}{12\ (1+2x1.268)}$$

$$+ \frac{4186x4.62^2(7x1.268+31)x1.268}{120(1+2x1.268)\ (6+1.268)} = -9327 \text{ Kg - m}$$

(368)



$$k_1 = \frac{I_2 h}{I_1 b} \quad ; \quad k_2 = \frac{I_s}{I_1 L} \quad ; \quad \alpha = \frac{x}{L}$$

$$\beta = \frac{x^1}{L} \quad ; \quad \lambda = \frac{a}{L}$$

$$\mu = \frac{1}{k_2}(3+2k_1) + 2 + k_1 \quad ; \quad \mu_2 = \frac{1}{1+k_1}\mu$$

$$\nu = 1 + \frac{k_1}{k_2} + 6k_1 - 4\lambda\left[(1+3k_1) - \lambda(1+2k_1)\right]$$

$$H = \frac{wL}{4}\left[\frac{L}{h}\lambda\left(2\frac{b}{L} + \lambda\frac{6+5k_1}{3+2k_1}\right) + \frac{1}{\mu_2}\left(\frac{b}{h}\,\frac{b}{L}\,\frac{1+k_2}{k_2(1+k_1)} - \lambda^2\frac{L}{h}\cdot\frac{3+k_1}{3+2K_1}\right)\right]$$

$$M_A = M_D = \frac{wL^2}{12}\cdot\frac{\left(\frac{b}{L}\right)^2 - \lambda^2(3+k_1)}{\mu}$$

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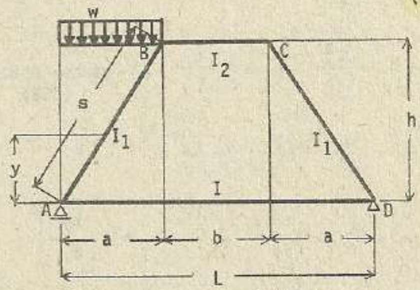
(369)



$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I_s}{I_1 L}$$

otros datos, referencia (368)

$$H = \frac{wa}{8}\lambda\frac{L}{h}\,\frac{1}{3+2k_1}\left[6+5k_1 - \frac{3+k_1}{\mu_2}\right]$$

$$M_A = -\frac{wa^2}{4}\left[\frac{3+k_1}{6\mu} + \frac{\frac{b}{L}\left\{1+3k_1 - 2\lambda(1+2k_1)\right\} + k_1(1-\lambda)}{\nu}\right]$$

$$M_D = -\frac{wa^2}{4}\left[\frac{3+k_1}{6\mu} - \frac{\frac{b}{L}\left\{1+3k_1 - 2\lambda(1+2k_1)\right\} + k_1(1-\lambda)}{\nu}\right]$$

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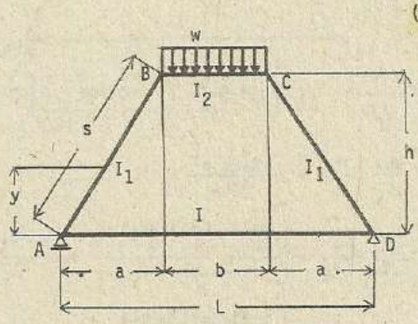
(370)

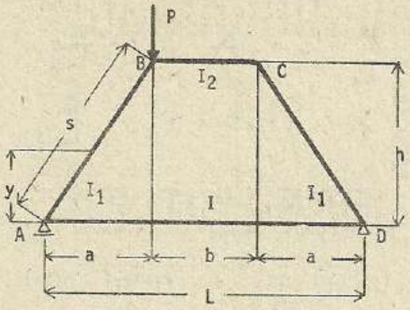


$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I\ s}{I_1 L}$$

otros datos (368)

$$H = \frac{wb}{4}\left[2\lambda\frac{L}{h} + \frac{b}{h}\cdot\frac{1}{k_2\mu_2}\;\frac{1+k_2}{1+k_1}\right]$$

$$M_A = M_D = +\frac{wb^2}{12}\cdot\frac{1}{\mu}$$



(371)

$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I\, s}{I_1 L} \quad ; \quad \text{otros datos (368)}$$

$$H = \frac{P}{2} \cdot \lambda \cdot \frac{L}{h}$$

$$M_A = -\frac{Pa}{2} \cdot \frac{\frac{b}{L}\left[(1+3k_1) - 2\lambda(1+2k_1)\right]}{v}$$

$$M = +\frac{Pa}{2} \cdot \frac{\frac{b}{L}\left[(1+3k_1) - 2\lambda(1+2k_1)\right]}{v}$$

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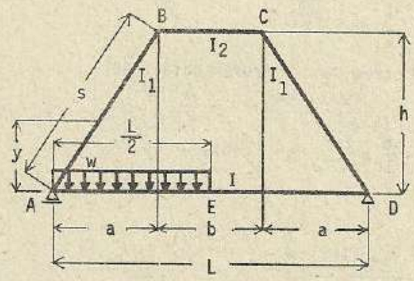
(372)



$$k_1 = \frac{I_2 s}{I_1 b} \quad k_2 = \frac{I\, s}{I_1 L} \quad ; \quad \text{otros datos (368)}$$

$$H = \frac{wL}{8} \cdot \frac{1}{h} \cdot \frac{1}{k_2 \mu_2}$$

$$M_A = \frac{wL}{16} \cdot \frac{1}{k_2}\left[\frac{\frac{2}{3}\cdot(3+2k_1)}{\mu} + \frac{k_1}{4v}\right]$$

$$M_D = \frac{wL}{16} \cdot \frac{1}{k_2}\left[\frac{\frac{2}{3}\cdot(3+2k_1)}{\mu} - \frac{k_1}{4v}\right]$$

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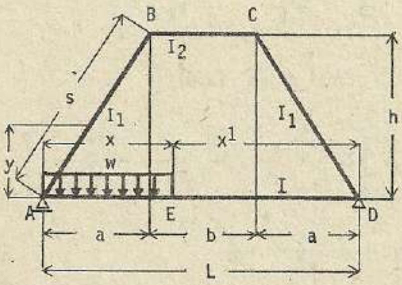
(373)



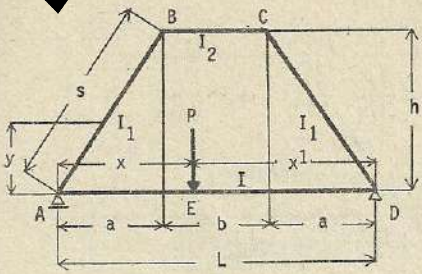
$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I\, s}{I_1 L} \quad ; \quad \text{otros datos (368)}$$

$$H = \frac{wL}{4} \cdot \frac{L}{h} \cdot \frac{(3-2\alpha)\alpha^2}{k_2 \mu_2}$$

$$M_A = \frac{wL^2}{4} \cdot \frac{1}{k_2}\, \alpha^2 \left[\frac{\frac{1}{3}(3-2\alpha)(3+2k_1)}{\mu} + \frac{\beta k_1}{v}\right]$$

$$M_D = \frac{wL^2}{4} \cdot \frac{1}{k_2}\, \alpha^2 \left[\frac{\frac{1}{3}(3-2\alpha)(3+2k_1)}{\mu} - \frac{\beta k_1}{v}\right]$$

(374)



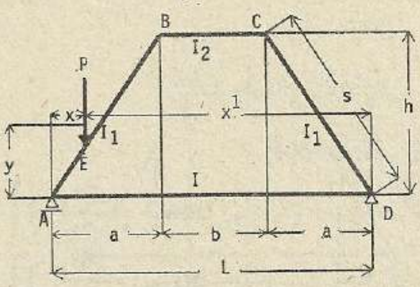
$$k_1 = \frac{I_2 s}{I_1 b} \; ; \quad k_2 = \frac{I\, s}{I_1 L} \; ; \quad \text{otros datos (368)}$$

$$H = \frac{3}{2} \cdot P \cdot \frac{L}{h} \cdot \frac{\alpha\beta}{k_2 \mu_2}$$

$$M_A = \frac{PL}{2} \alpha\beta \cdot \frac{1}{k_2} \cdot \left[\frac{3+2k_1}{\mu} + \frac{(\beta-\alpha)\, k_1}{v} \right]$$

$$M_D = \frac{PL}{2} \cdot \alpha\beta \cdot \frac{1}{k_2} \cdot \left[\frac{3+2k_1}{\mu} - \frac{(\beta-\alpha)\, k_1}{v} \right]$$

(375)



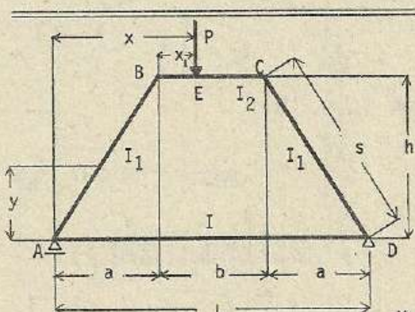
$$k_1 = \frac{I_2 s}{I_1 b} \; ; \quad k_2 = \frac{I\, s}{I_1 L} \; ; \quad \text{otros datos (368)}$$

$$H = \frac{3}{2} p\alpha \cdot \frac{L}{h} \left[\frac{1}{3} \left(\frac{x}{a}\right)^2 - \frac{1}{\mu_2} \left\{ \frac{2+k_1}{3+2k_1} - \frac{x}{a} + \left(\frac{x}{a}\right)^2 \frac{1+k_1}{3+2k_1} \right\} \right]$$

$$M_A = -\frac{PL}{2} \cdot \alpha \left[\frac{3+2k_1}{\mu} \left\{ \frac{2+k_1}{3+2k_1} - \frac{x}{a} + \left(\frac{x}{a}\right)^2 \frac{1+k_1}{3+2k_1} \right\} + \frac{1}{v} \left\{ \frac{b}{L} \left[(1+3k_1) - 2\lambda(1+2k_1) \right] + k_1 \left[(3-2\lambda) - 3\frac{x}{a} + 2\lambda \left(\frac{x}{a}\right)^2 \right] \right\} \right]$$

$$M_D = -\frac{PL}{2} \alpha \left[\frac{3+2k_1}{\mu} \left\{ \frac{2+k_1}{3+2k_1} - \frac{x}{a} + \left(\frac{x}{a}\right)^2 \frac{1+k_1}{3+2k_1} \right\} - \frac{1}{v} \left\{ \frac{b}{L} \left[(1+3k_1) - 2\lambda(1+2k_1) \right] + k_1 \left[(3-2\lambda) - 3\frac{x}{a} + 2\lambda \left(\frac{x}{a}\right)^2 \right] \right\} \right]$$

(376)



$$k_1 = \frac{I_2 s}{I_1 b} \; ; \quad k_2 = \frac{I\, s}{I_1 L}$$

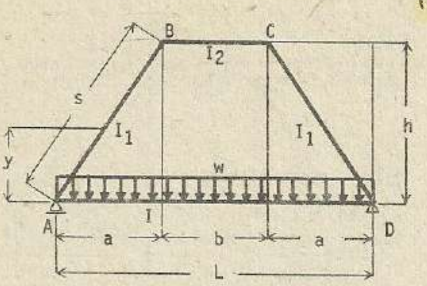
Otros datos (368)

$$H = \frac{P}{2} \left[\lambda \frac{L}{h} + \frac{1+k_2}{1+k_1} \cdot 3 \cdot \frac{b}{h} \cdot \frac{\frac{x_1}{b}\left(1 - \frac{x_1}{b}\right)}{k_2 \mu_2} \right]$$

$$M_A = \frac{Pb}{2} \left[\frac{\frac{x_1}{b}\left(1 - \frac{x_1}{b}\right)}{\mu} - \frac{\left(1 - 2\frac{x_1}{b}\right)}{v} \left\{ (1-2\lambda) \frac{x_1}{b} \left(1 - \frac{x_1}{b}\right) + \lambda(1+3k_1) - 2\lambda^2(1+2k_1) \right\} \right]$$

$$M_D = \frac{Pb}{2} \left[\frac{\frac{x_1}{b}\left(1 - \frac{x_1}{b}\right)}{\mu} + \frac{\left(1 - 2\frac{x_1}{b}\right)}{v} \left\{ (1 - 2\lambda) \frac{x_1}{b} \left(1 - \frac{x_1}{b}\right) + \lambda(1+3k_1) - 2\lambda^2 (1+2k_1) \right\} \right]$$

(377)



$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I\,s}{I_1 L}$$

otros datos (368)

$$H = \frac{PL}{4} \cdot \frac{L}{h} \cdot \frac{1}{k_2 \mu_2}$$

$$M_A = M_D = \frac{PL^2}{12} \cdot \frac{3+2v}{k_2 \mu}$$

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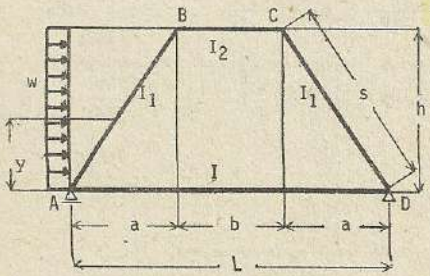
(378)



$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I\,s}{I_1 L}$$

otros datos (368)

$$H_A = -\frac{wh}{2}\left[1 + \frac{1}{4(3+2k_1)}\left\{3(2+k_1) + \frac{3+k_1}{\mu_2}\right\}\right]$$

$$H_D = -\frac{wh}{2}\left[-1 + \frac{1}{4(3+2k_1)}\left\{3(2+k_1) + \frac{3+k_1}{\mu_2}\right\}\right]$$

$$M_A = -\frac{wh^2}{4}\left[\frac{3+k_1}{6\mu} + \frac{\frac{b}{L}\left\{(1+3k_1) - 2\lambda\,(1+2k_1)\right\} + k_1\,(1-\lambda)}{v}\right]$$

$$M_D = -\frac{wh^2}{4}\left[\frac{3+k_1}{6\mu} - \frac{\frac{b}{L}\left\{(1+3k_1) - 2\lambda\,(1+2k_1)\right\} + k_1\,(1-\lambda)}{v}\right]$$

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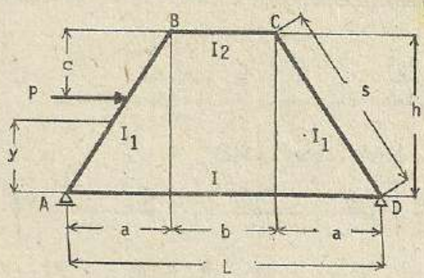
(379)



$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I\,s}{I_1 L}$$

otros datos (368)

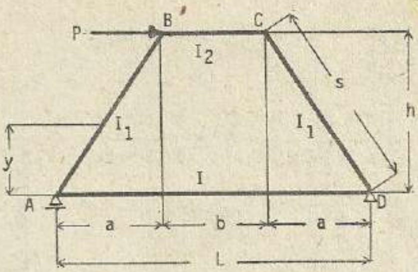
$$H_A = -\frac{P}{2}\left[1 + \frac{c}{h}\,\frac{1}{3+2k_1}\left\{k_1\,\frac{c}{h}\left(3 - \frac{c}{h}\right) + 3 + 3\,\frac{1 - \frac{c}{h}\left[1 + \frac{c}{h}\,(1+k_1)\right]}{\mu 2}\right\}\right]$$

$$H_D = -\frac{P}{2}\left[-1 + \frac{c}{h}\cdot\frac{1}{3+2k_1}\left\{k_1 \cdot \frac{c}{h}\left(3 - \frac{c}{h}\right) + 3 + 3\,\frac{\left(1 - \frac{c}{h}\right)\left[1 + \frac{c}{h}\cdot(1+k_1)\right]}{\mu 2}\right\}\right]$$

$$M_A = -\frac{P}{2}\,(h-c)\left[\frac{c}{h}\cdot\frac{1 + \frac{c}{h}\,(1+k_1)}{\mu} + \frac{\frac{b}{L}\left[(1+3k_1) - 2\lambda\,(1+2k_1)\right] + k_1\,\frac{c}{h}\left[3 + 2\lambda\left(\frac{c}{h} - 2\right)\right]}{v}\right]$$

$$M_D = -\frac{P}{2}\,(h-c)\left[\frac{c}{h}\cdot\frac{1 + \frac{c}{h}\,(1+k_1)}{\mu} - \frac{\frac{b}{L}\left[(1+3k_1) - 2\lambda(1+2k_1)\right] + k_1\,\frac{c}{h}\left[3 + 2\lambda\left(\frac{c}{h} - 2\right)\right]}{v}\right]$$

(380)



$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I\ s}{I_1 L}$$

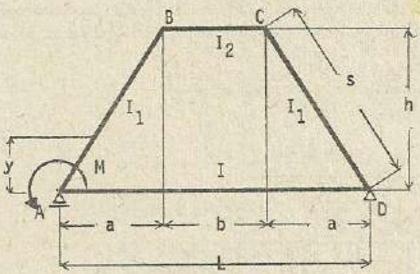
otros datos (368)

$$H_A = -\frac{P}{2} \quad ; \quad H_D = +\frac{P}{2}$$

$$M_A = -\frac{Ph}{2} \cdot \frac{b}{L} \cdot \frac{1}{\mu}\left[(1+3k_1) - 2\lambda\ (1+2k_1)\right]$$

$$M_D = +\frac{Ph}{2} \cdot \frac{b}{L} \cdot \frac{1}{\mu}\left[(1+3k_1) - 2\lambda\ (1+2k_1)\right]$$

(381)



$$k_1 = \frac{I_2 s}{I_1 b} \quad ; \quad k_2 = \frac{I\ s}{I_1 L}$$

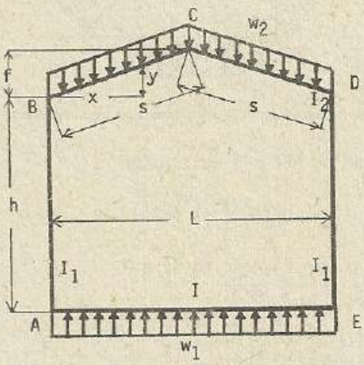
otros datos (368)

$$H = \frac{3}{2} \cdot \frac{M}{h} \cdot \frac{1}{k_2 \mu_2}$$

$$M_A = -\frac{M}{2} \cdot \frac{1}{k_2}\left(\frac{3-2k_1}{\mu} + \frac{k_1}{v}\right)$$

$$M_D = -\frac{M}{2} \cdot \frac{1}{k_2}\left(\frac{3-2k_1}{\mu} - \frac{k_1}{v}\right)$$

(382)



$$k_1 = \frac{I_2 h}{I_1 s} \quad ; \quad k_2 = \frac{I_2 L}{I s} \quad ; \quad n = \frac{k_1\ h+2h+f}{2 + 2k_1 + k_2}$$

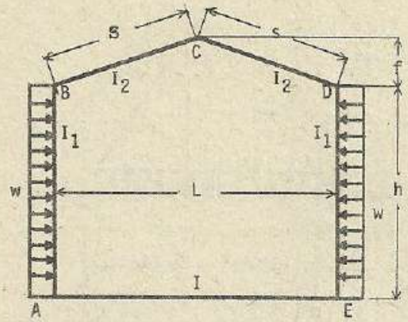
$$M' = -\frac{L^2\ (2w_2 + w_1 k_2)}{12\ (2 + 2k_1 + k_2)} \qquad M_A = M' + H'n$$

$$M_B = M' - H'\ (h - n) \quad ; \quad M_D = M' - H'\ (h - n)$$

$$M_C = M' - H\ (h - n + f) + \frac{w_2 L^2}{8} \quad ; \quad M_E = M' + H'\ n$$

$$H' = \frac{w_2 L^2\left[(2 + 2k_1 + k_2)\ 5f + 8\ (k_1 h + k_2 h - f)\right] - 4\ (k_1 h + 2h + f)\ k_2\ wL^2}{16\left[h^2\ (k_1^2 + 2k_1 k_2 + 4k_1 + 6k_2) + 6hf\ (k_1 + k_2) + f^2\ (4k_1 + 2k_2 + 1)\right]}$$

(383)



$$k_1 = \frac{I_2 h}{I_1 s} \quad ; \quad k_2 = \frac{I_2 L}{I\ s}$$

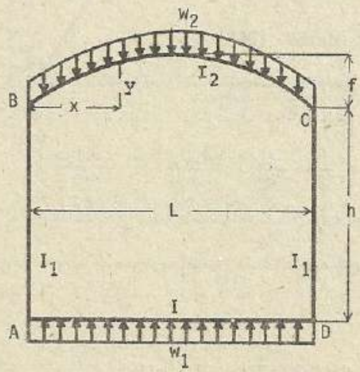
$$n = \frac{k_1 h + 2h + f}{2 + 2k_1 + k_2} \quad ; \quad M' = \frac{wh^2 (2k_1 + 3k_2)}{6\ (2 + 2k_1 + k_2)}$$

$$M_A = M' - H'\ n - \frac{wh^2}{2} = M_E$$

$$M_B = M' + H'\ (h - n) = M_D \quad ; \quad M_C = M' + H'\ (h - n + f)$$

$$H' = - \frac{wh^2 \left[k_1 h\ (2k_1 + 5k_2 - 2) + (4h + 2f)\ (2k_1 + 3k_2) \right]}{4 \left[h_2\ (k_1^2 + 2k_1k_2 + 4k_1 + 6k_2) + 6hf\ (k_1 + k_2) + f^2 (4k_1 + 2k_2 + 1) \right]}$$

(384)



$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2}{I}$$

$$n = \frac{3h + 3k_1 h + 2f}{3\ (1 + k_2 + 2k_1)}$$

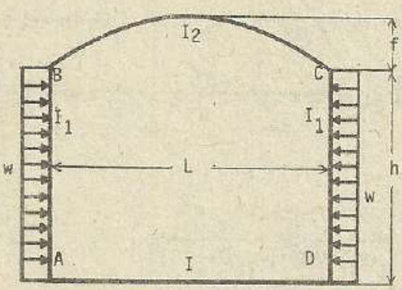
$$M' = - \frac{(w_2 + k_2 w_1)\ L^2}{12\ (1 + k_2 + 2k_1)}$$

$$M_A = M_D = - \frac{(w_2 + k_2 w_1)\ L^2}{12\ (1 + k_2 + 2k_1)} + H' n$$

$$M_B = M_C = M' - H\ (h - n)$$

$$H' = \frac{w_2 L^2 \left(12f\ (1 + k_2 + 2k_1) + 5\ (3k_2 h + 3k_1 h - 2f) \right) - 5w_1 L^2 (3h + 3k_1 h + 2f)\ k_2}{4 \left[15h^2 \left((2 + k_1) k_1 + (2k_1 + 3)\ k_2 \right) + 60\ hf\ (k_1 + k_2) + 4f^2\ (1 + 6k_2 + 12k_1) \right]}$$

(385)



$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2}{I}$$

$$n' = \frac{3h + 3k_1 h + 2f}{3\ (1 + k_2 + 2k_1)}$$

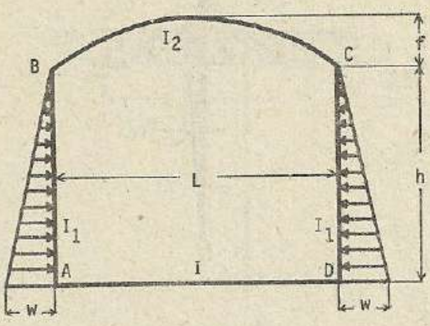
$$M' = \frac{wh^2\ (2k_1 + 3k_2)}{6\ (1 + k_2 + 2k_1)}$$

$$M_A = M_D = M' - \frac{wh^2}{2} + H'\ n$$

$$M_B = M_C = M' - H' n$$

$$H = - \frac{5wh^2 \left[3k_1 h\ (3 + 2k) + 3k_2 h\ (6 + 5k_1) + 4f\ (2k_1 + 3k_2) \right]}{4 \left[15h^2 \left\{ (2 + k_1) k_1 + (2k_2 + 3) k_2 \right\} + 60\ hf\ (k_2 + k_1) + 4f^2\ (1 + 6k_2 + 12k_1) \right]}$$

(386)



$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2}{I}$$

$$n = \frac{3h + 3k_1 h + 2f}{3\,(1 + k_2 + 2k_1)}$$

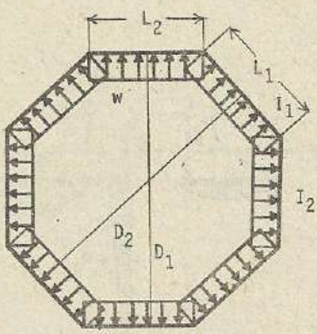
$$M' = \frac{wh^2 (k_1 + 2k_2)}{12\,(1 + k_2 + 2k_1)}$$

$$M_A = M_D = M' - \frac{wh^2}{6} + H'\,n$$

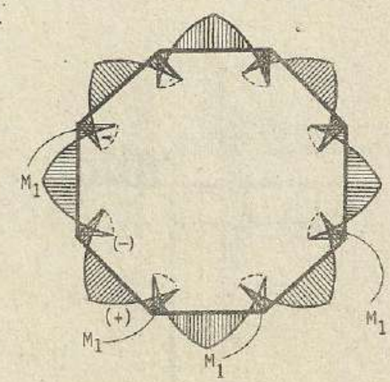
$$M_B = M_C = M' - H'n$$

$$H = - \frac{wh^2 \left[3k_1 h\,(3k_1 + 4) + 3k_2 h\,(9k_1 + 10) + 10f\,(k_1 + 2k_2)\right]}{4\left[15h^2 \left\{ (2 + k_1)\,k_1 + (2k_1 + 3)\,k_2 \right\} + 60\,hf\,(k_2 + k_1) + 4f^2 (1 + 6k_2 + 12k_1)\right]}$$

(387)



$$k = \frac{I_1 L_2}{I_2 L_1}$$



M_1 : Momento de flexión de cada punto extremo

Z_1 : Fuerza de tensión axial del lado L_1

Z_2 : Fuerza de tensión axial del lado L_2

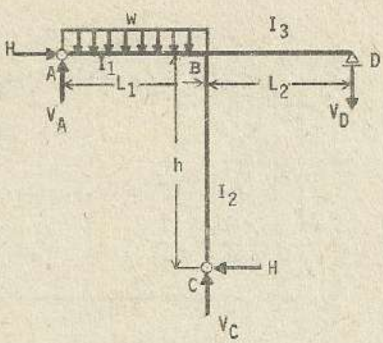
$$M_1 = - \frac{w}{12} \cdot \frac{L_1^2 + kL_2^2}{1 + k} \quad ; \quad Z_1 = \frac{wD_1}{2} \quad ; \quad Z_2 = \frac{wD_2}{2}$$

Tanque rectangular $D_1 = L_1$: $D_2 = L_2$

Tanque cuadrado $L_1 = L_2 = L$ $M_1 = -\frac{wL^2}{12}$ $Z = + \frac{wL}{2}$

$k = 1$

(388)

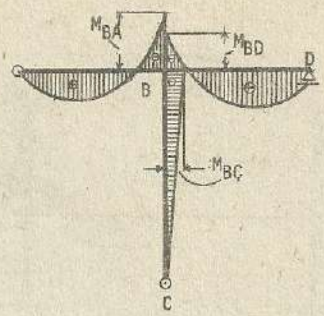


$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

$$\lambda_1 = \frac{L_1}{L_2}$$

$$\lambda_2 = \frac{L_2}{L_1}$$



$$\mu = k_1 + k_1 k_2 + k_2 \qquad M_{BC} = -\frac{wL_1^2}{8} \cdot \frac{k_1 k_2}{\mu}$$

$$M_{BD} = -\frac{wL_1^2}{8} \cdot \frac{k_1}{\mu} \quad ; \quad M_{BA} = M_{BC} + M_{BD} \quad ; \quad H = -\frac{M_{BC}}{h}$$

$$V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} \quad ; \quad V_C = \frac{wL_1}{2} - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2} \quad ; \quad V_D = -\frac{M_{BC}}{L_2}$$

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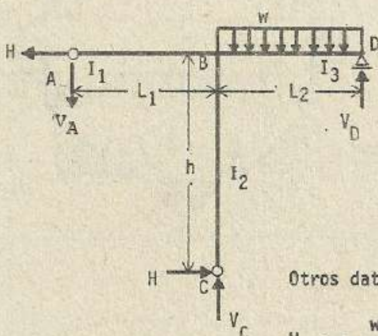
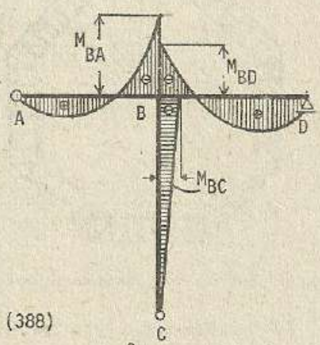
(389)



$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$



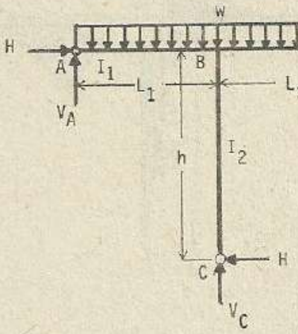
Otros datos, referencia (388)

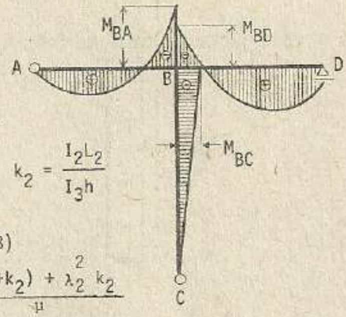
$$M_{BA} = -\frac{wL_2^2}{8} \cdot \frac{k_2}{\mu} \quad ; \quad M_{BC} = -\frac{wL_2^2}{8} \cdot \frac{k_1 k_2}{\mu}$$

$$M_{BD} = M_{BA} - M_{BC} \quad ; \quad H = \frac{M_{BC}}{h} \quad ; \quad V_A = -\frac{M_{BA}}{L_1}$$

$$V_C = -\frac{wL_2}{2} - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2} \quad ; \quad V_D = \frac{wL_2}{2} + \frac{M_{BD}}{L_2}$$

(390)





$$k_1 = \frac{I_2 L_1}{I_1 h} \quad ; \quad k_2 = \frac{I_2 L_2}{I_3 h}$$

Otros datos (388)

$$M_{BA} = -\frac{wL_1^2}{8} \cdot \frac{k_1(1+k_2) + \lambda_2^2\, k_2}{\mu}$$

$$M_{BD} = -\frac{wL_1^2}{8} \cdot \frac{k_1 + \lambda_2^2\, k_2\,(1+k_1)}{\mu}$$

$$M_{BC} = M_{BA} - M_{BD} \quad ; \quad H = -\frac{M_{BC}}{h} \; ; \quad V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} \quad ; \quad V_D = \frac{wL_2}{2} + \frac{M_{BD}}{L_2}$$

$$V_C = \frac{w\,(L_1 + L_2)}{2} - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2}$$

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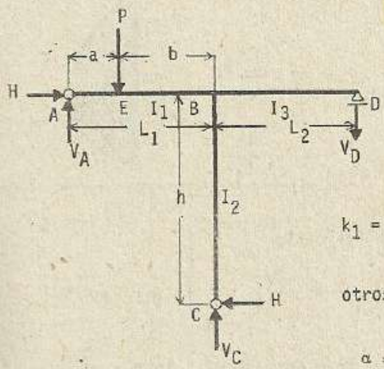
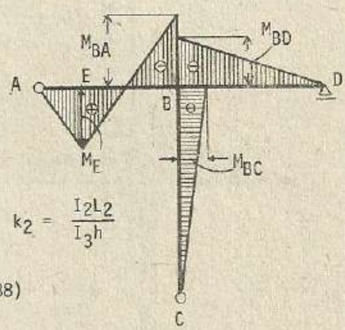
(391)





$$k_1 = \frac{I_2 L_1}{I_1 h} \quad ; \quad k_2 = \frac{I_2 L_2}{I_3 h}$$

otros datos (388)

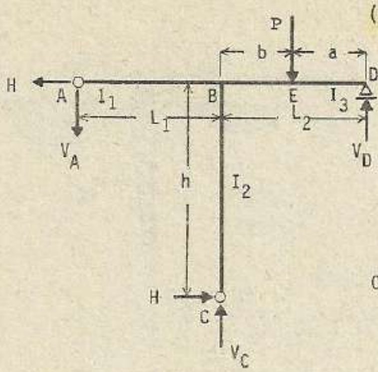
$$\alpha = \frac{a}{L_1}$$

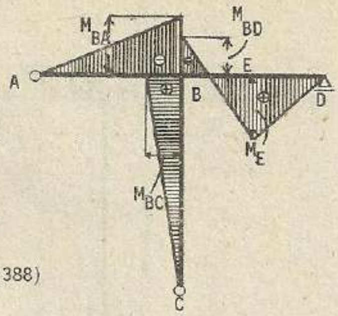
$$M_{BC} = -\frac{pa\,(1-\alpha^2)}{2} \cdot \frac{k_1 k_2}{\mu} \quad ; \quad M_{BD} = -\frac{Pa\,(1-\alpha^2)}{2} \cdot \frac{k_1}{\mu}$$

$$M_{BA} = M_{BC} + M_{BD} \quad ; \quad H = -\frac{M_{BC}}{h} \quad : \quad V_D = -\frac{M_{BD}}{L_2}$$

$$V_A = \frac{Pb}{L_1} + \frac{M_{BA}}{L_1} \quad ; \quad V_C = \frac{Pa}{L_1} - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2}$$

(392)





$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

Otros datos (388)

$$\beta = \frac{a}{L_2}$$

$$H = \frac{M_{BC}}{h} \quad ; \quad M_{BA} = -\frac{Pa\,(1-\beta^2)}{2} \cdot \frac{k_2}{\mu} \quad ; \quad M_{BC} = +\frac{Pa\,(1-\beta^2)}{2} \cdot \frac{k_1 k_2}{\mu}$$

$$M_{BD} = M_{BA} - M_{BC} \quad ; \quad V_D = \frac{Pb}{L_2} + \frac{M_{BD}}{L_2} \quad ; \quad V_A = -\frac{M_{BA}}{L_1} \quad ; \quad V_C = \frac{Pa}{L_2} - \frac{M_{BD}}{L_2} - \frac{M_{BA}}{L_2}$$

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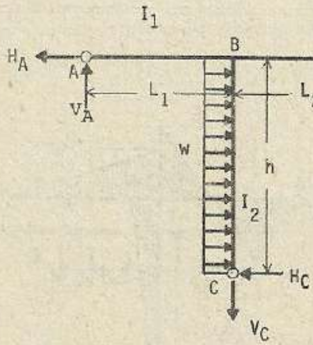
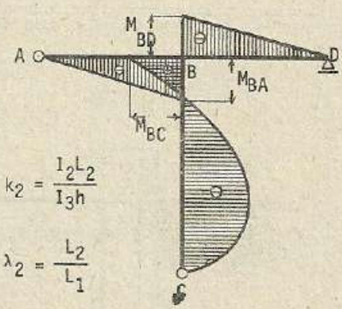
(393)





$$k_1 = \frac{I_2 L_1}{I_1 h} \quad ; \quad k_2 = \frac{I_2 L_2}{I_3 h}$$

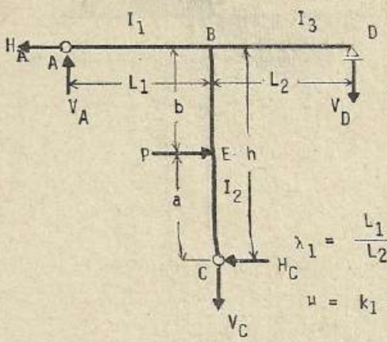
$$\lambda_1 = \frac{L_1}{L_2} \qquad \lambda_2 = \frac{L_2}{L_1}$$

$$\mu = k_1 + k_1 k_2 + k_2$$

$$M_{BA} = +\frac{wL^2}{8} \cdot \frac{k_2}{\mu} \quad ; \quad H_A = \frac{wh}{2} + \frac{M_{BC}}{h} \quad ; \quad V = \frac{M_{BA}}{L_1} \quad ; \quad V_C = V_A - V_D$$

$$M_{BD} = -\frac{wh}{8} \cdot \frac{k_1}{\mu} \quad ; \quad H_C = \frac{wh}{2} - \frac{M_{BC}}{h} \quad ; \quad M_{BC} = M_{BA} - M_{BD} \quad ; \quad V_D = -\frac{M_{BD}}{L_2}$$

(394)

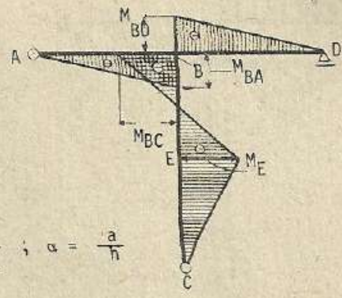


$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

$$\lambda_1 = \frac{L_1}{L_2} \; ; \; \lambda_2 = \frac{L_2}{L_1} \; ; \; \alpha = \frac{a}{h}$$

$$\mu = k_1 + k_1 k_2 + k_2$$



$$M_{BA} = + \frac{Pa\,(1-\alpha^2)}{2} \cdot \frac{k_2}{\mu} \; ; \quad H_A = P\,\alpha + \frac{M_{BC}}{h} \; ; \quad V_D = - \frac{M_{BC}}{L_2}$$

$$M_{BD} = - \frac{Pa\,(1-\alpha^2)}{2} \cdot \frac{k_1}{\mu} \; ; \quad H_C = P\,(1-\alpha) - \frac{M_{BC}}{h} \; ; \quad V_A = \frac{M_{BA}}{L_1}$$

$$M_{BC} = M_{BA} - M_{BD} \quad : \quad V_C = V_A - V_D$$

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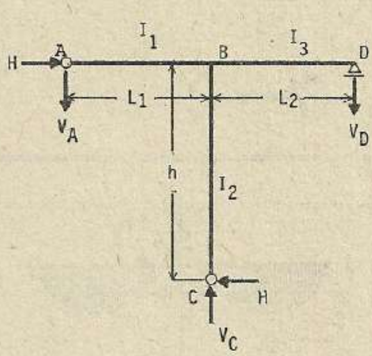
(395)

Esfuerzos debido al cambio de temperatura

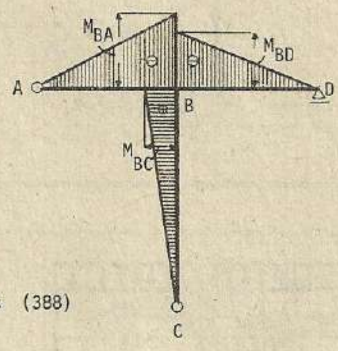


$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

otros datos (388)

$$\lambda_3 = \frac{L_1 L_2}{h^2}$$

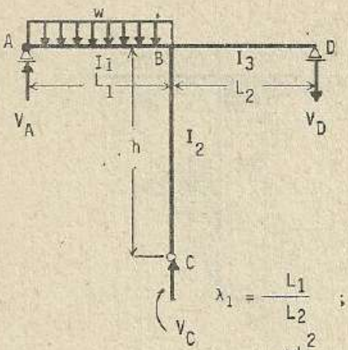


$$H = - \frac{M_{BC}}{h} \; ; \quad M_{BA} = - \frac{3E\varepsilon I_2 t}{L_2} \cdot \frac{1 + \lambda_2 + \lambda_2 k_2 + \lambda_3 k_2}{\mu} ; \quad M_{BC} = M_{BA} - M_{BD}$$

$$M_{BD} = - \frac{3E\varepsilon I_2 t}{L_2} \cdot \frac{1 + \lambda_2 + k_1 - \lambda_3 k_1}{\mu} \; ; \quad V_A = - \frac{M_{BA}}{L_1} \; ; \quad V_C = - \frac{M_{BA}}{L_1} - \frac{M_{BD}}{L_2}$$

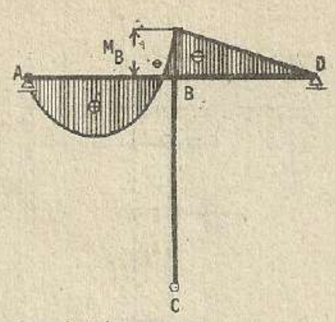
$$V_D = - \frac{M_{BD}}{L_2}$$

(396)



$$k_1 = \frac{I_2 L_1}{I_2 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$



$$\lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1} \quad ; \quad \mu = k_1 + k_2$$

$$M_B = -\frac{wL_1^2}{4} \cdot \frac{k_1}{2\mu} \quad ; \quad V_A = \frac{wL_1}{2} + \frac{M_B}{L_1}$$

$$V_C = \frac{wL_1}{2} - \frac{M_B}{L_1} - \frac{M_B}{L_2} \quad ; \quad V_D = -\frac{M_B}{L_2}$$

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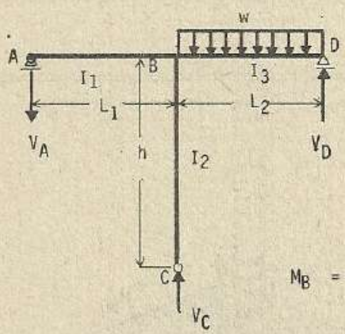
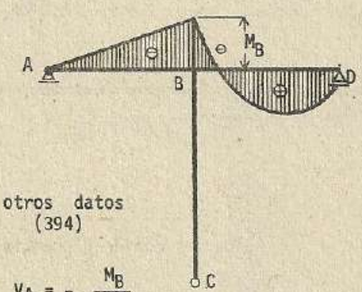
(397)



$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

otros datos (394)



$$M_B = -\frac{wL_2^2}{8} \cdot \frac{k_2}{2\mu} \quad ; \quad V_A = -\frac{M_B}{L_1}$$

$$V_C = \frac{wL_2}{2} - \frac{M_B}{L_1} - \frac{M_B}{L_2} \quad ; \quad V_D = \frac{wL_2}{2} + \frac{M_B}{L_2}$$

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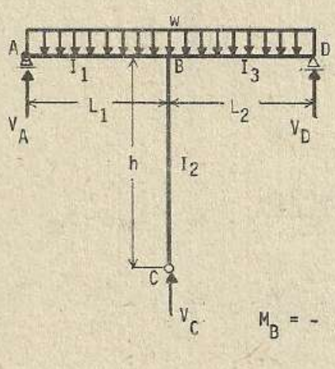
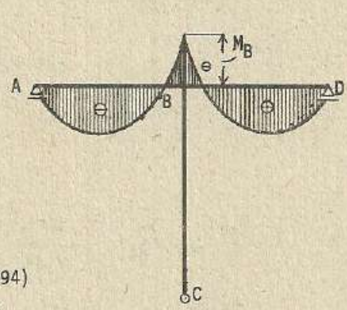
(398)



$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$

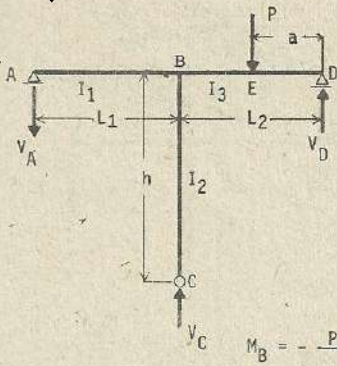
otros datos (394)



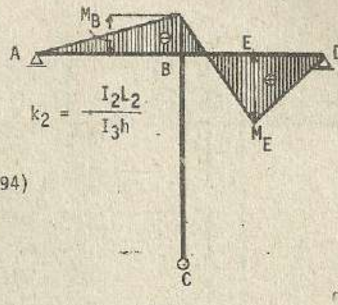
$$M_B = -\frac{wL_1^2}{8} \cdot \frac{k_1 + \lambda_2^2 k_2}{\mu}$$

$$V_A = \frac{wL_1}{2} + \frac{M_B}{L_1} \quad ; \quad V_C = \frac{w(L_1+L_2)}{2} - \frac{M_B}{L_1} - \frac{M_B}{L_2} \quad ; \quad V_D = \frac{wL_2}{2} + \frac{M_B}{L_2}$$

(399)



$$k_1 = \frac{I_2 L_1}{I_1 h} \quad ; \quad k_2 = \frac{I_2 L_2}{I_3 h}$$



otros datos (394)

$$\alpha = \frac{a}{L_1}$$

$$M_B = -\frac{Pa\,(1-\alpha^2)}{2} \cdot \frac{k_1}{\mu} \quad ; \quad V_A = P\,(1-\alpha) + \frac{M_B}{L_1}$$

$$V_C = P\,\alpha - \frac{M_B}{L_1} - \frac{M_B}{L_2} \quad ; \quad V_D = -\frac{M_B}{L_2}$$

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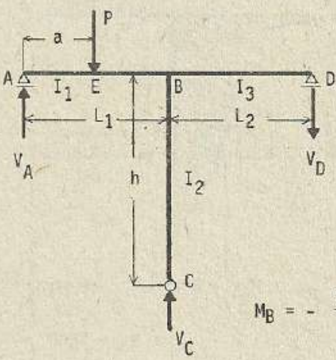
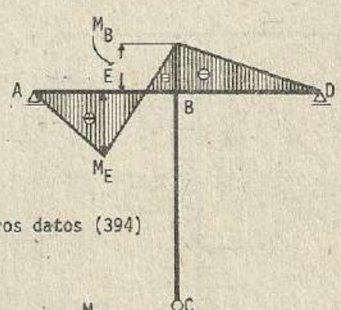
(400)



$$k_1 = \frac{I_2 L_1}{I_1 h}$$

$$k_2 = \frac{I_2 L_2}{I_3 h}$$



otros datos (394)

$$\beta = \frac{a}{L_2}$$

$$M_B = -\frac{Pa\,(1-\beta^2)}{2} \cdot \frac{k_2}{\mu} \quad ; \quad V_A = -\frac{M_B}{L_1}$$

$$V_C = P\,\beta - \frac{M_B}{L_1} - \frac{M_B}{L_2} \quad ; \quad V_D = P\,(1-\beta) + \frac{M_B}{L_2}$$

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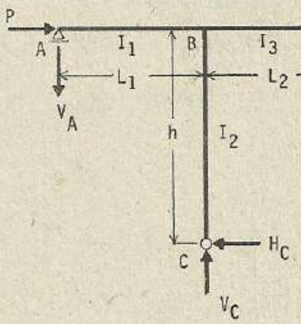
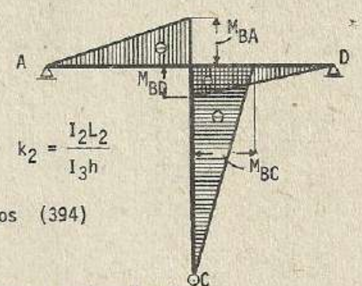
(401)



$$k_1 = \frac{I_2 L_1}{I_1 h} \quad ; \quad k_2 = \frac{I_2 L_2}{I_3 h}$$



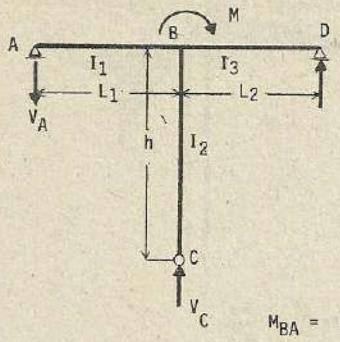
otros datos (394)

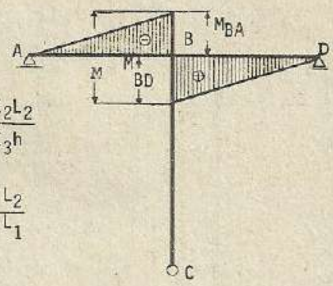
$$M_{BA} = -Ph\,\frac{k_2}{\mu} \quad ; \quad M_{BC} = -Ph = M_{BA} - M_{BD}$$

$$M_{BD} = -Ph\,\frac{k_1}{\mu} \quad ; \quad V_C = V_A - V_D \quad ; \quad V_D = \frac{M_{BD}}{L_2}$$

$$H_C = P$$

(402)





$$k_1 = \frac{I_2 L_1}{I_1 h} \quad ; \quad k_2 = \frac{I_2 L_2}{I_3 h}$$

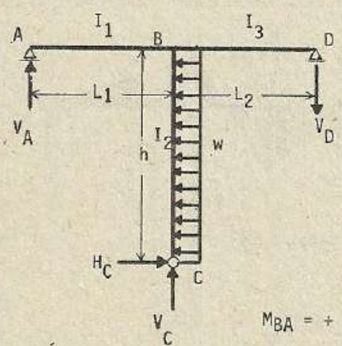
$$\lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1}$$

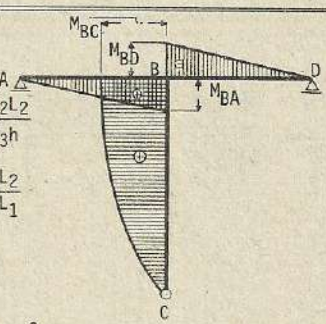
$$\mu = k_1 + k_2$$

$$M_{BA} = -M\frac{k_2}{\mu} \quad ; \quad M + M_{BA} - M_{BD} = 0 \quad ; \quad V_A = -\frac{M_{BA}}{L_1}$$

$$M_{BD} = +M\frac{k_1}{\mu} \quad ; \quad V = \frac{M_{BD}}{L_2} \quad ; \quad V_C = V_A - V_D$$

(403)





$$k_1 = \frac{I_2 L_1}{I_1 h} \quad ; \quad k_2 = \frac{I_2 L_2}{I_3 h}$$

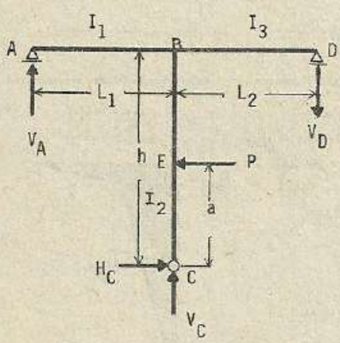
$$\lambda_1 = \frac{L_1}{L_2} \qquad \lambda_2 = \frac{L_2}{L_1}$$

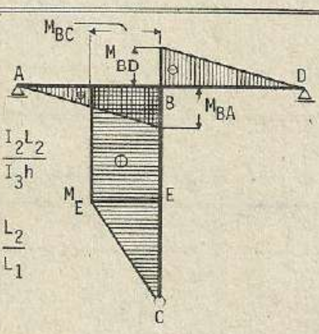
$$\mu = k_1 + k_2$$

$$M_{BA} = +\frac{wh^2}{2}\cdot\frac{k_2}{\mu} \quad ; \quad M_{BC} = +\frac{wh^2}{2} = -M_{BA} - M_{BD} \quad ; \quad H_C = wh$$

$$M_{BD} = -\frac{wh^2}{2}\cdot\frac{k_1}{\mu} \quad ; \quad V_A = \frac{M_{BA}}{L_1} \quad ; \quad V_D = -\frac{M_{BD}}{L_2} \quad ; \quad V_C = V_A - V_D$$

(404)





$$k_1 = \frac{I_2 L_1}{I_1 h} \qquad k_2 = \frac{I_2 L_2}{I_3 h}$$

$$\lambda_1 = \frac{L_1}{L_2} \qquad \lambda_2 = \frac{L_2}{L_1}$$

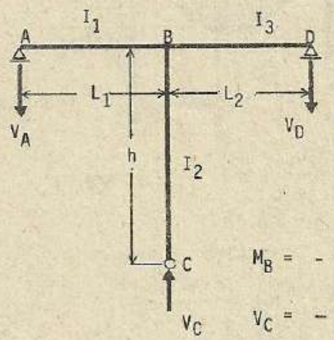
$$\mu = k_1 + k_2$$

$$M_{BA} = +Pa\frac{k_2}{\mu} \quad ; \quad M_{BC} = +Pa = M_{BA} - M_{BD} \quad ; \quad H_C = P$$

$$M_{BD} = -Pa\frac{k_1}{\mu} \quad ; \quad V_A = \frac{M_{BA}}{L_1} \quad ; \quad V_D = -\frac{M_{BD}}{L_2}$$

$$V_C = V_D - V_A$$

(405) : Esfuerzos por cambio de temperatura

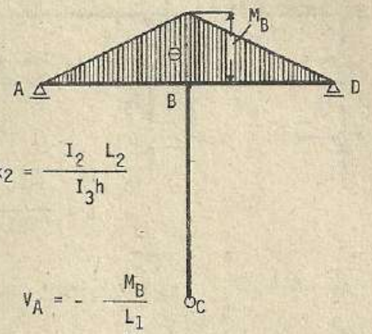


$$k_1 = \frac{I_2 L_1}{I_1 h} \quad ; \quad k_2 = \frac{I_2 \; L_2}{I_3 h}$$

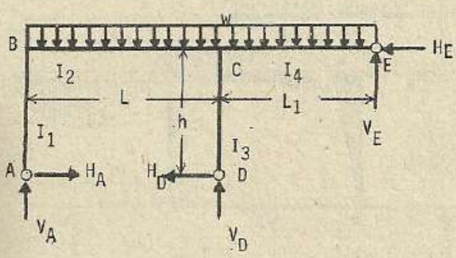
otros datos (394)

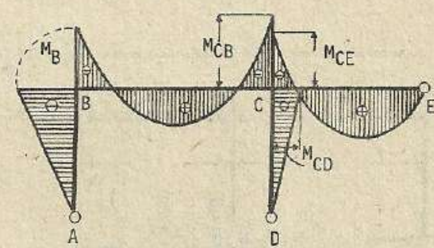
$$M_B = -\frac{3E\varepsilon I_2 t}{L_2} \cdot \frac{1+\lambda_2}{\mu} \quad ; \quad V_A = -\frac{M_B}{L_1}$$

$$V_C = -\frac{M_B}{L_1} - \frac{M_B}{L_2} \quad ; \quad V_D = -\frac{M_B}{L_2}$$



(406)





$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k_1 = \frac{I_2 h}{I_3 L} \quad ; \quad k_2 = \frac{I_2 L_1}{I_4 L} \quad ; \quad \lambda = \frac{L}{L_1} \quad ; \quad \lambda_1 = \frac{L_1}{L}$$

$$s_1 = k+kk_1+k_1 \quad ; \quad s_2 = k+kk_2+k_2 \quad ; \quad s = k_1+k_1k_2+k_2 \quad ; \quad \mu = s\,(3+4k) + k_1k_2$$

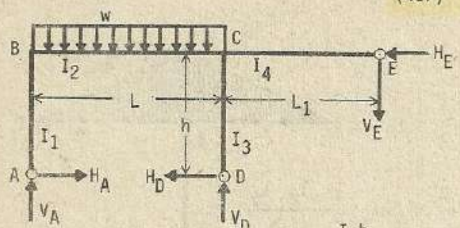
$$M_B = -\frac{wL^2}{4} \cdot \frac{s + k_1k_2\,(1-\lambda_1^2)}{\mu} \quad ; \quad M_{CB} = -\frac{wL^2}{4} \cdot \frac{(1+2k)(k_1+k_2) + 2\lambda_1^2 k_1 k_2 \;\; (1+k)}{\mu}$$

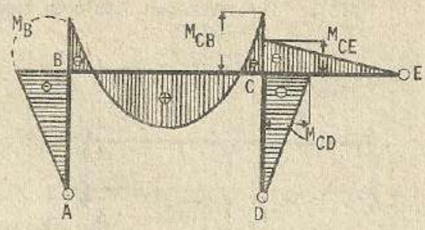
$$M_{CD} = -\frac{wL^2 k_2}{8} \cdot \frac{2(1+2k) - \lambda_1^2(3+4k)}{\mu} \quad ; \quad M_{CE} = -\frac{wL^2}{8} \cdot \frac{2k_1(1+2k) + \lambda_1^2\, k_2(3+4s_1)}{\mu}$$

$$H_D = -\frac{M_{CD}}{h} \quad ; \quad H_A = -\frac{M_B}{h} \quad ; \quad H_E = H_A - H_D \quad ; \quad V_A = \frac{wL}{2} + \frac{M_{CB} - M_B}{L}$$

$$V_D = \frac{w\,(L+L_1)}{2} + \frac{M_B - M_{CB}}{L} - \frac{M_{CE}}{L_1} \quad ; \quad V_E = \frac{wL_1}{2} + \frac{M_{CE}}{L}$$

(407)





$$k = \frac{I_2 h}{I_1 L} \; ; \; k_1 = \frac{I_2 h}{I_3 L} \; ; \; k_2 = \frac{I_2 L_1}{I_4 L} \; ; \; \lambda = \frac{L}{L_1} \; ; \; \lambda_1 = \frac{L_1}{L}$$

$$s_1 = k + kk_1 + k_1 \; ; \; s_2 = k + k_2 k + k_2 \; ; \; s = k_1 + k_2 k_1 + k_2 \; ; \; \mu = s\,(3+4k) + k_1 k_2$$

$$M_B = -\frac{wL^2}{4} \cdot \frac{s + k_1 k_2}{\mu} \; ; \; M_{CD} = -\frac{wL^2 \; k_2}{4} \cdot \frac{1+2k}{\mu} \; ; \; M_{CB} = -\frac{wL^2 (k_1 + k_2)}{4} \cdot \frac{1+2k}{\mu}$$

$$M_{CE} = -\frac{wL^2 \; k_1}{4} \cdot \frac{1+2k}{\mu} \; ; \; H_D = -\frac{M_{CD}}{h} \; ; \; H_A = -\frac{M_B}{h} \; ; \; H_E = -\frac{M_B - M_{CD}}{h}$$

$$V_A = \frac{wL}{2} + \frac{M_{CB} - M_B}{L} \; ; \; V_D = \frac{wL}{2} + \frac{M_B - M_{CB}}{L} - \frac{M_{CE}}{L_1} \; ; \; V_E = -\frac{M_{CE}}{L_1}$$

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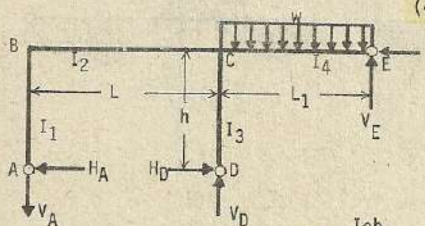
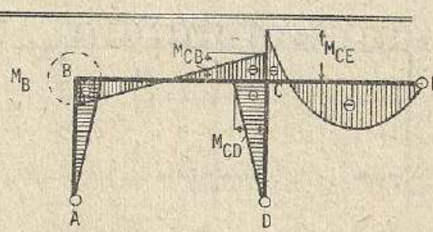
(408)





$$k = \frac{I_2 h}{I_1 L} \; ; \; k_1 = \frac{I_2 h}{I_3 L} \; ; \; k_2 = \frac{I_2 L_1}{I_4 L}$$

$$\lambda = \frac{L}{L_1} \; ; \; \lambda_1 = \frac{L_1}{L} \; ; \; s_1 = k + kk_1 + k_1$$

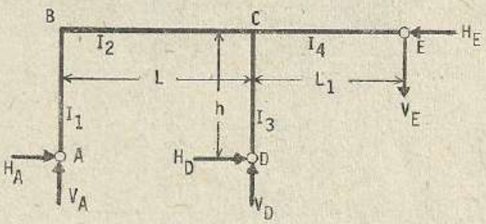
$$s_2 = k + kk_2 + k_2 \; ; \; s = k_1 + k_1 k_2 + k_2 \; ; \; \mu = s\,(3+4k) + k_1 k_2$$

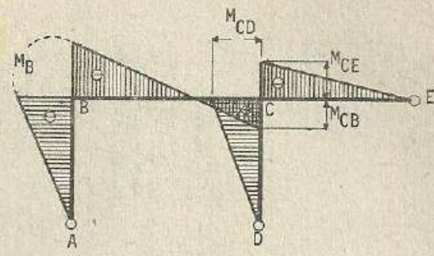
$$M_B = +\frac{wL_1^2 k_1 \; k_2}{4\mu} \; ; \; M_{CB} = -\frac{wL_1^2 k_1 k_2}{2} \cdot \frac{1+k}{\mu} \; ; \; M_{CD} = +\frac{wL_1^2 \cdot k_2}{8} \cdot \frac{3+4k}{\mu}$$

$$M_{CE} = -\frac{wL_1^2 k_2}{8} \cdot \frac{3+4s_1}{\mu} \; ; \; H_D = \frac{M_{CD}}{h} \; ; \; H_A = +\frac{M_B}{h} \; ; \; H_E = +\frac{M_{CD} - M_B}{h}$$

$$V_A = \frac{M_B - M_{CB}}{L} \; ; \; V_D = \frac{wL_1}{2} + \frac{M_B - M_{CB}}{L} - \frac{M_{CE}}{L_1} \; ; \; V_E = \frac{wL_1}{2} + \frac{M_{CE}}{L_1}$$

Esfuerzos por cambio de temperatura : (409)





$$k = \frac{I_2 h}{I_1 L} \quad ; \quad k_1 = \frac{I_2 h}{I_3 L} \; ; \quad k_2 = \frac{I_2 L_1}{I_4 L} \; ; \quad \lambda = \frac{L}{L_1} \; ; \; \lambda_1 = \frac{L_1}{L} \qquad \alpha = \frac{h^2}{L_1^2}$$

$$s_1 = k + kk_1 + k_1 \quad ; \quad s_2 = 1 + kk_2 + k_2 \quad ; \quad s = k_1 + k_1 k_2 + k_2 \; ; \quad \mu = s\,(3+4k) + k_1 k_2$$

$$M_B = -\frac{6EI_2 \varepsilon t L_1}{hL} \cdot \frac{k_2 + 2\,(\lambda + 1)s - \alpha k_1}{\mu} \; ; \qquad M_{CB} = \frac{6EI_2 \varepsilon t L_1}{hL} \cdot \frac{2(k_2 - \alpha k_1)\;(1+k) + (\lambda+1)(k_1 + k_3)}{\mu}$$

$$M_{CE} = -\frac{3EI_2 \varepsilon t L_1}{hL} \cdot \frac{3 + 4k - 2\,(\lambda+1)k_1 + \alpha\,(3+4s_1)}{\mu} \; ; \quad M_{CD} = M_{CB} - M_{CE}$$

$$H_A = -\frac{M_B}{h} \; ; \quad H_D = \frac{M_{CD}}{h} \; ; \qquad H_E = H_A + H_D \; ; \quad V_A = \frac{M_{CB} - M_B}{L} \; ; \quad V_D = V_E - V_A$$

Nota : En caso de descenso de temperatura los signos de los esfuerzos de momentos son contrarios a las presentadas $\qquad V_E = -\frac{M_{CE}}{L_1}$

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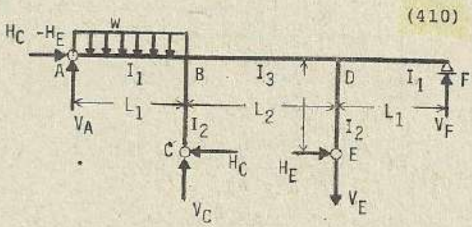
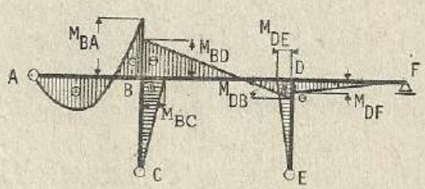
(410)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \qquad k_2 = \frac{I_1 h}{I_2 L_1} \; ; \; \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \; ; \qquad \lambda_1 = \frac{L_1}{L_2} \; ; \; \lambda_2 = \frac{L_2}{L_1}$$

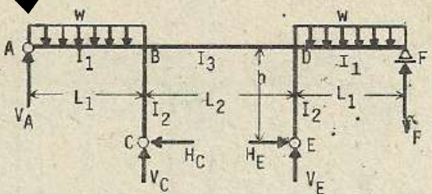
$$M_{BA} = -\frac{wL_1^2}{16}\left[\frac{1+2k_1}{\mu_2} + \frac{3+2k_1}{\mu_1}\right] \; ; \qquad M_{DF} = +\frac{wL_1^2}{16}\left[\frac{1+2k_1}{\mu_2} - \frac{3+2k_1}{\mu_1}\right]$$

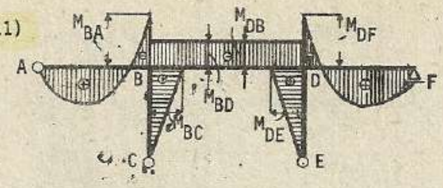
$$M_{BD} = -\frac{wL_1^2\,k_1}{8}\left[\frac{1}{\mu_2} + \frac{1}{\mu_1}\right] \; ; \qquad M_{DB} = +\frac{wL_1^2\,k_1}{8}\left[\frac{1}{\mu_2} - \frac{1}{\mu_1}\right]$$

$$M_{BC} = M_{BD} - M_{BA} \; ; \quad M_{DE} = M_{DB} - M_{DF} \; ; \quad V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} \; ; \quad V_F = \frac{M_{DF}}{L_1}$$

$$V_C = \frac{wL_1}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \; ; \quad V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DF}}{L_1} \; ; \quad H_C = \frac{M_{BC}}{h} \; ; \quad H_E = \frac{M_{DE}}{h}$$

(411)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \quad ; \quad \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \; ; \; \lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1} \quad ; \quad M_{BA} = M_{DF} = -\frac{wL_1^2}{8} \cdot \frac{3 + 2k_1}{\mu_1}$$

$$M_{BC} = M_{DE} = +\frac{3wL_1^2}{8\mu_1} \; ; \qquad M_{BD} = M_{DB} = -\frac{wL_1^2}{4} \cdot \frac{k_1}{\mu_1} \quad ; \quad H_C = H_E = H = \frac{M_{BC}}{h}$$

$$V_A = V_F = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} \quad ; \qquad V_C = V_E = \frac{wL_1}{2} - \frac{M_{BA}}{L_1}$$

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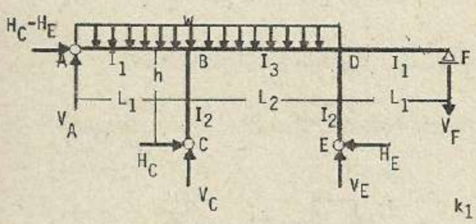
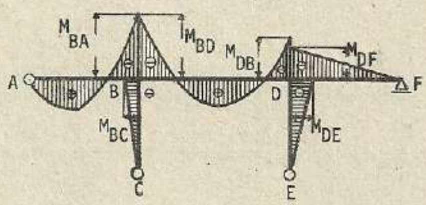
(412)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \quad ; \quad \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \quad ; \quad \lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = -\frac{wL_1^2}{16}\left[\frac{3+2k_1+4\lambda_2^2 k_1}{\mu_1} + \frac{1+2k_1}{\mu_2}\right] ; \quad M_{DF} = -\frac{wL_1^2}{16}\left[\frac{3+2k_1+4\lambda_2^2 k_2}{\mu_1} - \frac{1+2k_1}{\mu_2}\right]$$

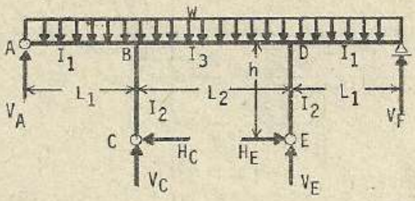
$$M_{BD} = -\frac{wL_1^2}{8}\left[\frac{k_1+2\lambda_2^2(1+k_2)}{\mu_1} + \frac{k_1}{\mu_2}\right] \quad ; \quad M_{DB} = \frac{wL_1^2}{8}\left[\frac{k+2\lambda_2^2(1+k_2)}{\mu_1} - \frac{k_1}{\mu_2}\right]$$

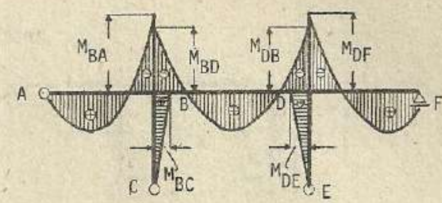
$$M_{BC} = M_{BD} - M_{BA} \quad ; \quad M_{DE} = M_{DB} - M_{DF} \; ; \quad H_C = -\frac{M_{BC}}{h} \quad ; \quad H_E = -\frac{M_{DE}}{h}$$

$$V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} \; ; \quad V_C = \frac{w(L_1 + L_2)}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \quad ; \quad V_F = -\frac{M_{DF}}{L_1}$$

$$V_E = \frac{wL_2}{2} - \frac{M_{DF}}{L_1} - \frac{M_{DB} - M_{BD}}{L_2}$$

(413)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k = \frac{I_1 h}{I_2 L_1} \quad ; \mu_1 = 2 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \quad ; \quad \lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = M_{DF} = -\frac{wL_1^2}{8} \cdot \frac{3+2k_1 + 2\lambda_2^2 k_2}{\mu_1} \quad ; \quad M_{BC} = M_{DE} = +\frac{wL_1^2}{8} \cdot \frac{3-2\lambda_2^2}{\mu_1}$$

$$M_{BD} = M_{DB} = -\frac{wL_1^2}{4} \cdot \frac{k_1 + \lambda_2^2(1+k_2)}{\mu_1} ; \quad H_C = H_E = H = \frac{M_{BC}}{h}$$

$$V_A = V_F = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} \quad ; \quad V_C = V_E + \frac{w(L_1 + L_2)}{2} - \frac{M_{BA}}{L_1}$$

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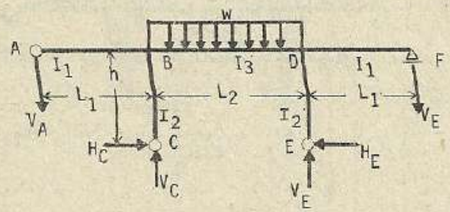
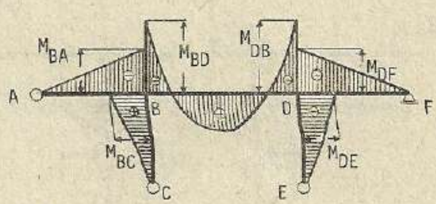
(414)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

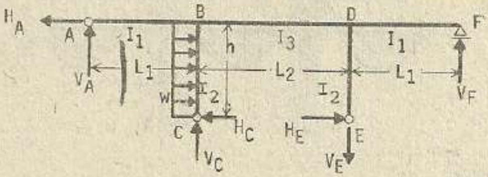
$$\mu_1 = 3 + 2k_1 + 3k_2 \quad ; \quad \mu_2 = 1 + 2k_1 + k_2 \quad ; \quad \lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1}$$

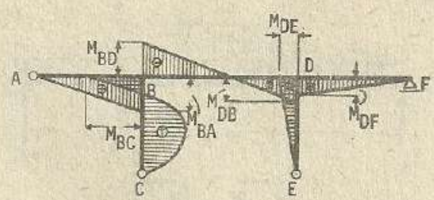
$$M_{BA} = M_{DF} = -\frac{wL_2^2 k_2}{4\mu_1} \quad ; \quad M_{BC} = M_{DE} = -\frac{wL_2^2}{4\mu_1}$$

$$M_{BD} = M_{DB} = -\frac{wL_2^2 (L+k_2)}{4\mu_1} ; \quad H_C = H_E = H = -\frac{M_{BC}}{h} \; ; \; V_A = V_F = -\frac{M_{BA}}{L_1}$$

$$V_C = V_E = \frac{wL_2}{2} - \frac{M_{BA}}{L_1}$$

(415)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} ; \quad \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \quad ; \quad \lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = + \frac{wh^2 k_2}{16} \left[\frac{3}{\mu_1} + \frac{1}{\mu_2} \right] ; \quad M_{DF} = + \frac{wh^2 k_2}{16} \left[\frac{3}{\mu_1} - \frac{1}{\mu_1} \right] ; \quad M_{BD} = - \frac{wh^2 k_1}{8} \left[\frac{1}{\mu_2} + \frac{1}{\mu_1} \right]$$

$$M_{DB} = + \frac{wh^2 k_1}{8} \left[\frac{1}{\mu_2} - \frac{1}{\mu_1} \right] ; \quad M_{BC} = M_{BD} - M_{BA} \quad : \quad M_{DE} = M_{DB} - M_{DF}$$

$$H_A = \frac{wh}{2} + \frac{M_{DE} - M_{BC}}{h} ; \quad H_C = \frac{wh}{2} + \frac{M_{BC}}{h} ; \quad H_E = \frac{M_{DE}}{h} : \quad V_A = \frac{M_{BA}}{L_1}$$

$$V_C = - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \quad ; \quad V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DB}}{L_1} ; \quad V_F = \frac{M_{DF}}{L_1}$$

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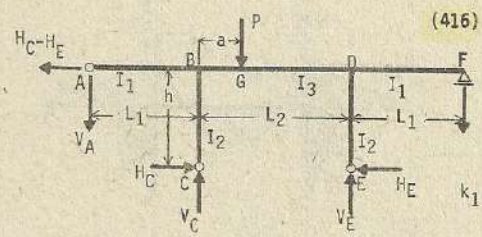
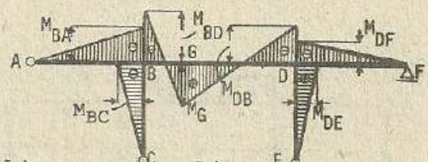
(416)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \quad ; \quad \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \quad ; \quad \lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1} \quad ; \quad \alpha = \frac{a}{L_2}$$

$$M_{BA} = - \frac{Pa\,(1 - \alpha) k_2}{2} \left[\frac{3}{\mu_1} + \frac{1 - 2\alpha}{\mu_2} \right] ; \quad M_{DF} = - \frac{Pa\,(1 - \alpha)\, k_2}{2} \left[\frac{3}{\mu_1} - \frac{1 - 2\alpha}{\mu_2} \right]$$

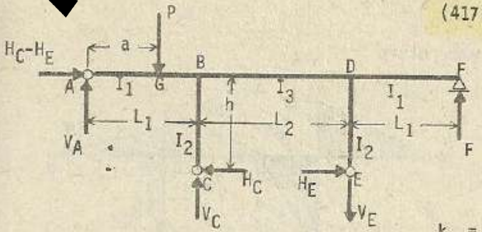
$$M_{BC} = - \frac{Pa\,(1 - \alpha)}{2} \left[\frac{3}{\mu_1} + \frac{1 - 2\alpha}{\mu_2} \right] ; \quad M_{DE} = - \frac{Pa\,(1 - \alpha)}{2} \left[\frac{3}{\mu_1} - \frac{1 - 2\alpha}{\mu_2} \right]$$

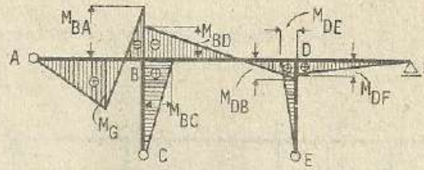
$$M_{BD} = M_{BA} + M_{BC} \quad ; \quad M_{DB} = M_{DF} + M_{DE} \quad ; \quad H_C = - \frac{M_{BC}}{h} \quad ; \quad H_E = - \frac{M_{DE}}{h}$$

$$V_A = - \frac{M_{BA}}{L_1} \quad ; \quad V_C = P\,(1 - \alpha) - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \quad ; \quad V_F = - \frac{M_{DF}}{L_1}$$

$$V_E = P\,\alpha + \frac{M_{BD} - M_{DB}}{L_2} - \frac{M_{DF}}{L_1}$$

(417)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \; ; \; \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \quad ; \quad \lambda_1 = \frac{L_1}{L_2} \; ; \quad \lambda_2 = \frac{L_2}{L_1} \quad ; \quad \alpha = \frac{a}{L_1}$$

$$M_{BA} = -\frac{Pa\,(1-\alpha^2)}{4}\left[\frac{1+2k_1}{\mu_2} + \frac{3+2k_1}{\mu_1}\right] \quad ; \quad M_{DF} = +\frac{Pa\,(1-\alpha^2)}{4}\left[\frac{1+2k_1}{\mu_2} - \frac{3+2k_1}{\mu_1}\right]$$

$$M_{BD} = -\frac{Pa\,(1-\alpha^2)k_1}{2}\left[\frac{1}{\mu_2} + \frac{1}{\mu_1}\right] \qquad M_{DB} = +\frac{Pa\,(1-\alpha^2)k_1}{2}\left[\frac{1}{\mu_2} - \frac{1}{\mu_1}\right]$$

$$M_{BC} = M_{BD} - M_{BA} \quad ; \quad M_{DE} = M_{DB} - M_{DF} \quad ; \quad H_C = \frac{M_{BC}}{h} \quad ; \quad H_E = \frac{M_{DE}}{h}$$

$$V_A = P\,(1-\alpha) + \frac{M_{BA}}{L_1} \; ; \quad V_C = P\alpha - \frac{M_{BA}}{L_1} + \frac{M_{DB}-M_{BD}}{L_2} ; \quad V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DF}}{L_1}$$

$$V_F = \frac{M_{DF}}{L_1}$$

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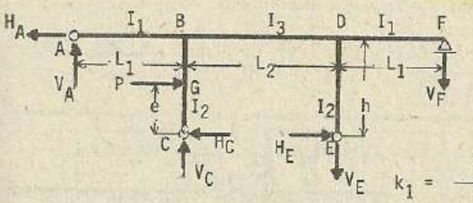
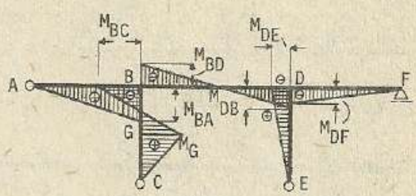
(418)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \; ; \; \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \quad ; \quad \lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1} \quad ; \quad \beta = \frac{e}{h}$$

$$M_{BA} = +\frac{Pe\,(1-\beta^2)k_2}{4}\left[\frac{3}{\mu_1} + \frac{1}{\mu_2}\right] \quad ; \quad M_{DF} = +\frac{Pe\,(1-\beta^2)k_2}{4}\left[\frac{3}{\mu_1} - \frac{1}{\mu_2}\right]$$

$$M_{BD} = -\frac{Pe\,(1-\beta^2)k_1}{2}\left[\frac{1}{\mu_2} + \frac{1}{\mu_1}\right] \qquad M_{DB} = +\frac{Pe\,(1-\beta^2)\,k_1}{2}\left[\frac{1}{\mu_2} - \frac{1}{\mu_1}\right]$$

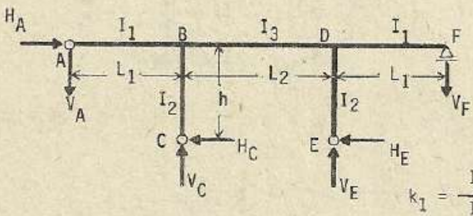
$$M_{BC} = M_{BD} - M_{BA} \quad ; \quad M_{DE} = M_{DB} - M_{DF} \quad ; \quad H_A = P\beta + \frac{M_{DE} - M_{BC}}{h} \quad ; \quad H_E = \frac{M_{DE}}{h}$$

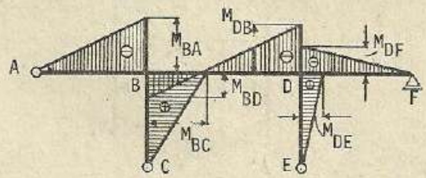
$$H_C = P\,(1-\beta) + \frac{M_{BC}}{h} \; ; \; V_A = \frac{M_{BA}}{L_1} \quad ; \quad V_C = -\frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \; ; \; V_F = \frac{M_{DF}}{L_1}$$

$$V_E = \frac{M_{DB}-M_{BD}}{L_2} + \frac{M_{DF}}{L_1}$$

(419)

Esfuerzos por efectos del cambio de temperatura





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \quad ; \quad \mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 \quad ; \quad \lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1} \quad ; \quad \alpha_1 = \frac{h^2}{L_1^2}$$

$$M_{BA} = - \frac{3EI_2 \varepsilon t L_1 k_2}{2h^2} \left[\frac{2\ \alpha_1 (3+2k_1) - 3\ \lambda_2}{\mu_1} + \frac{2 + \lambda_2}{\mu_2} \right]$$

$$M_{DF} = - \frac{3EI_2 \varepsilon t L_1 k_2}{2h^2} \left[\frac{2\ \alpha_1 (3+2k_1) - 3\lambda_2}{\mu_1} - \frac{2 + \lambda_2}{\mu_2} \right]$$

$$M_{BD} = + \frac{3EI_2 \varepsilon t L_1 k_1}{h^2} \left[\frac{2 + \lambda_2}{\mu_2} - \frac{2\alpha_1 k_2 + \lambda_2}{\mu_1} \right]$$

$$M_{DB} = - \frac{3EI_2 \varepsilon t L_1 k_1}{h^2} \left[\frac{2 + \lambda_2}{\mu_2} + \frac{2\alpha_1 k_2 + \lambda_2}{\mu_1} \right] ; \quad M_{BC} = M_{BD} - M_{BA} \quad ; \quad M_{DE} = M_{DB} - M_{DF}$$

$$H_A = H_C + H_E \quad ; \quad H_C = \frac{M_{BC}}{h} \quad ; \quad H_E = - \frac{M_{DE}}{h} \quad ; \quad V_A = - \frac{M_{BA}}{L_1}$$

$$V_C = - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \quad ; \quad V_F = - \frac{M_{DF}}{L_1} \quad ; \quad V_E = - \frac{M_{DF}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2}$$

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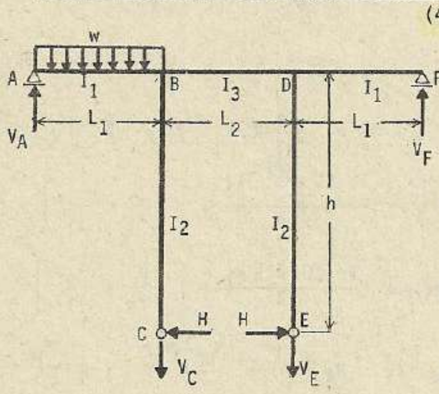
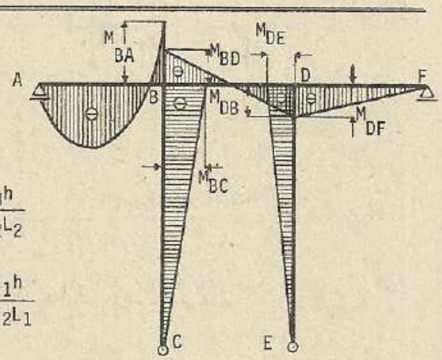
(420)





$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 1 + 2k_1 + k_2 ; \quad \lambda_1 = \frac{L_1}{L_2} \qquad \lambda_2 = \frac{L_2}{L_1}$$

$$M_{BA} = + \frac{wL_1^2}{4} \cdot \frac{1}{4} \left[- \frac{3+2k_1}{\mu_1} - \frac{2k_1 + 1}{\mu_2} \right]$$

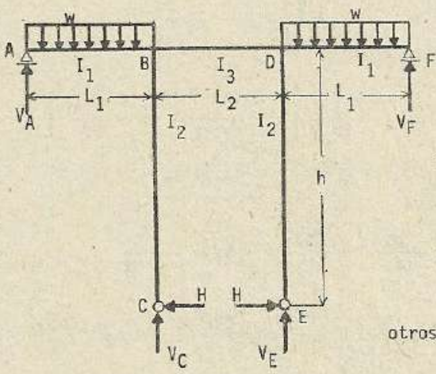
$$M_{DF} = + \frac{wL_1^2}{4} \cdot \frac{1}{4} \left[- \frac{3+2k_1}{\mu_1} + \frac{2k_1 + 1}{\mu_2} \right] ; \quad M_{BD} = + \frac{wL_1^2}{4} \cdot \frac{k_1}{2} \left[- \frac{1}{\mu_1} - \frac{1}{\mu_2} \right]$$

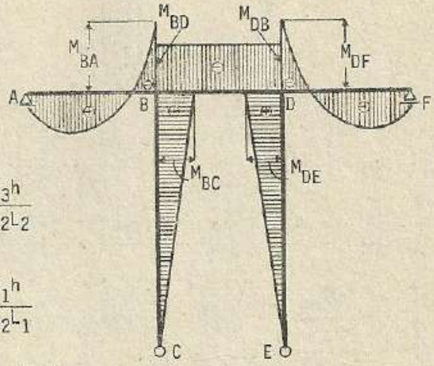
$$M_{DB} = +\frac{wL_1^2}{4}\cdot\frac{k_1}{2}\left[-\frac{1}{\mu_1}+\frac{1}{\mu_2}\right];\quad M_{BC} = M_{DE} = \frac{3}{4\mu_1}\cdot\frac{wL_1^2}{4}\ ;\quad H = \frac{M_{BC}}{h}$$

$$V_A = \frac{w}{2} - \frac{M_{BA}}{L_1}\ ;\quad V_C = \frac{w}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2}$$

$$V_E = \frac{M_{DB} - M_{BD}}{L_2} - \frac{M_{DF}}{L_1}\ ;\quad V_F = \frac{M_{DF}}{L_1}$$

(421)





$$k_1 = \frac{I_3 h}{I_2 L_2}$$

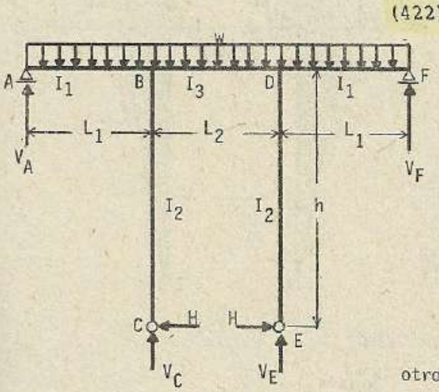
$$k_2 = \frac{I_1 h}{I_2 L_1}$$

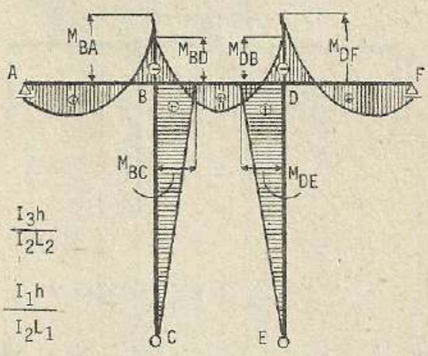
otros datos (420)

$$M_{BA} = M_{DF} = -\frac{wL_1^2}{8}\cdot\frac{3 + 2k_1}{\mu_1}\ ;\quad M_{BD} = -\frac{wL_1^2}{4}\cdot\frac{k_1}{\mu_1}$$

$$M_{BC} = \frac{3wL_1^2}{8\mu_1}\ ;\quad V_A = V_F = \frac{wL_1}{2} + \frac{M_{BA}}{L_1}\ ;\quad V_C = V_E = \frac{wL_1}{2} - \frac{M_{BA}}{L_1}$$

(422)





$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

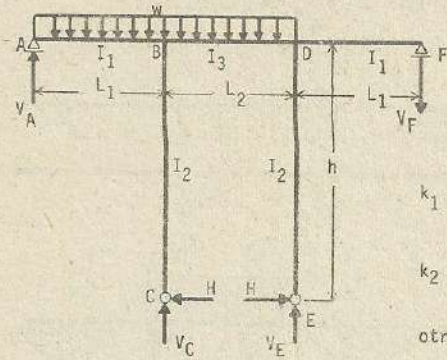
otros datos (420)

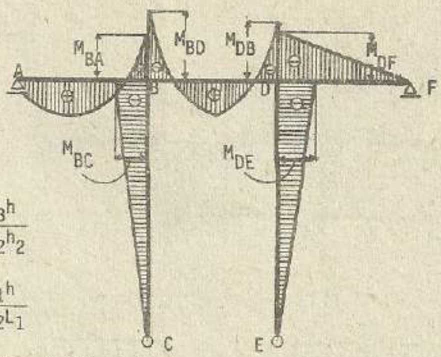
$$M_{BA} = M_{DF} = -\frac{wL_1^2}{8}\cdot\frac{3 + 2k_1 + 2\lambda_2^2 k_2}{\mu_1}$$

$$M_{BC} = M_{DE} = \frac{wL_1^2}{8}\cdot\frac{3 - 2\lambda_2^2}{\mu_1}\ ;\quad M_{BD} = M_{DB} = -\frac{wL_1^2}{4}\cdot\frac{k_1 + \lambda_2^2(1+k_2)}{\mu_1}$$

$$V_A = V_F = \frac{wL_1}{2} + \frac{M_{BA}}{L_1}\ ;\quad V_C = V_E = \frac{w\,(L_1+L_2)}{2} - \frac{M_{BA}}{L_1}\ ;\quad H = \frac{M_{BC}}{h}$$

(423)





$$k_1 = \frac{I_3 h}{I_2 h_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

otros datos (420)

$$M_{BA} = -\frac{wL_1^2}{16}\left[\frac{3+2k_1+4\lambda_2^2 k_2}{\mu_1} + \frac{2k_1}{\mu_2}\right] \qquad M_{DF} = -\frac{wL_1^2}{16}\left[\frac{3+2k_1+4\lambda_2^2 k_2}{\mu_1} - \frac{2k_1}{\mu_2}\right]$$

$$M_{BD} = -\frac{wL_1^2}{8}\left[\frac{k_1+2\lambda_2^2(1+k_2)}{\mu_1} + \frac{k_1}{\mu_2}\right] \quad ; \quad M_{DB} = -\frac{wL_1^2}{8}\cdot\left[\frac{k_1+2\lambda_2^2(1+k_2)}{\mu_1} - \frac{k_1}{\mu_2}\right]$$

$$M_{BC} = M_{DB} = -\frac{wL_1^2}{16}\cdot\frac{4\lambda_2^2-3}{\mu_1} \quad ; \quad V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1} \quad ; \quad V_F = -\frac{M_{DF}}{L_1} \quad ;$$

$$V_C = \frac{w(L_1+L_2)}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \quad ; \quad V_E = \frac{wL_2}{2} - \frac{M_{DF}}{L_1} - \frac{M_{DB}-M_{BD}}{L_2} \quad ; \quad H = \frac{M_{BC}}{h}$$

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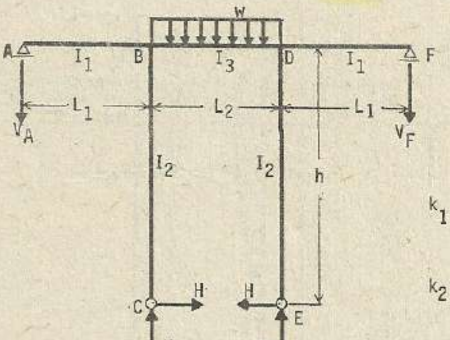
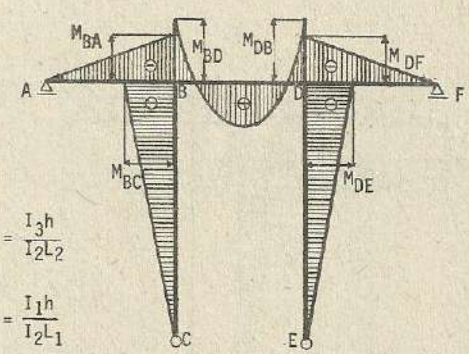
(424)





$$k_1 = \frac{I_3 h}{I_2 L_2}$$

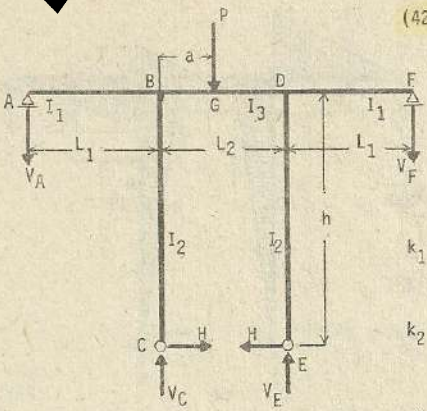
$$k_2 = \frac{I_1 h}{I_2 L_1}$$

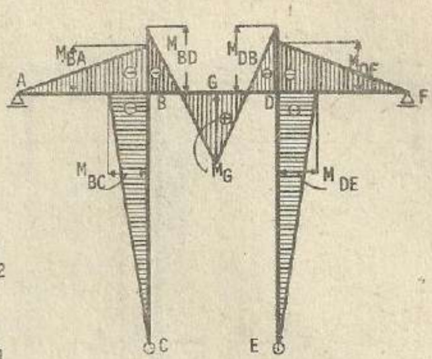
otros datos (420)

$$M_{BA} = M_{DF} = -\frac{wL_2^2\, k_2}{4\mu_1} \quad ; \quad M_{BD} = M_{DB} = -\frac{wL_2^2(1+k_2)}{4\mu_1} \quad ; \quad M_{BC} = M_{DE} = -\frac{wL_2^2}{4\mu_1}$$

$$V_A = V_F = -\frac{M_{BA}}{L_1} \quad ; \quad V_C = \frac{wL_2}{2} + \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \quad ; \quad V_E = \frac{wL_2}{2} + \frac{M_{BD} - M_{DB}}{L_2} - \frac{M_{DF}}{L_1}$$

(425)





$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\alpha = \frac{a}{L_2}$$

otros datos (420)

$$M_{BA} = -\frac{Pa(1-\alpha)k_2}{2}\left[\frac{3}{\mu_1} + \frac{1-2\alpha}{\mu_2}\right] \; ; \; M_{DF} = \frac{Pa(1-\alpha)k_2}{2}\left[\frac{3}{\mu_1} - \frac{1-2\alpha}{\mu_2}\right]$$

$$M_{BD} = -\frac{Pa(1-\alpha)}{2}\left[\frac{3(1+k_2)}{\mu_1} + \frac{(1-2\alpha)k_2}{\mu_2}\right] \; ; \; M_{DB} = -\frac{Pa(1-\alpha)}{2}\left[\frac{3(1+k_2)}{\mu_1} - \frac{(1-2\alpha)k_2}{\mu_2}\right]$$

$$M_{BC} = M_{DE} = -\frac{3Pa(1-\alpha)}{2\mu} \; ; \; V_C = P(1-\alpha) - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \; ; \; V_A = -\frac{M_{BA}}{L_1}$$

$$V_E = P\alpha + \frac{M_{BD} - M_{DB}}{L_2} + \frac{M_{DF}}{L_1} \; ; \; V_F = -\frac{M_{DF}}{L_1} \; ; \; H = -\frac{M_{BC}}{h}$$

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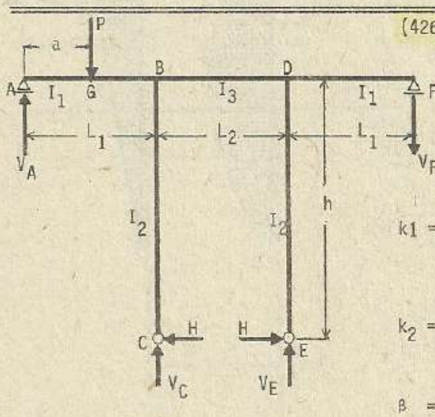
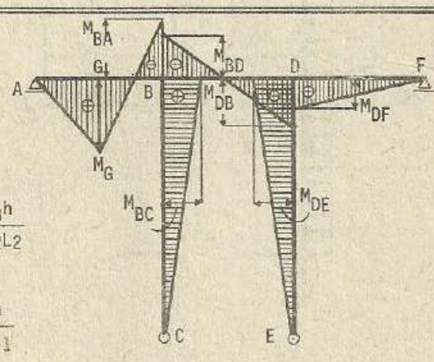
(426)





$$k1 = \frac{I_3 h}{I2L2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\beta = \frac{a}{L_1}$$

otros datos (420)

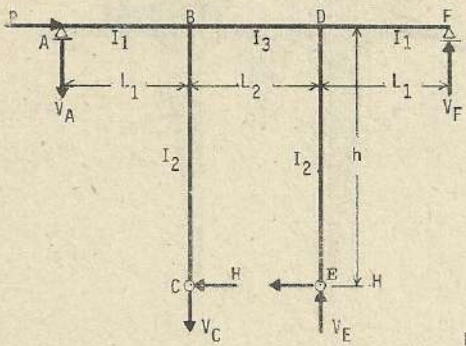
$$M_{BA} = -\frac{Pa(1-\beta^2)}{4}\left[\frac{2k_1+1}{\mu_2} + \frac{3+2k_1}{\mu_1}\right] ; \; M_{DF} = +\frac{pa(1-\beta^2)}{4}\left[\frac{2k_1+1}{\mu_2} + \frac{3+2k_1}{\mu_1}\right]$$

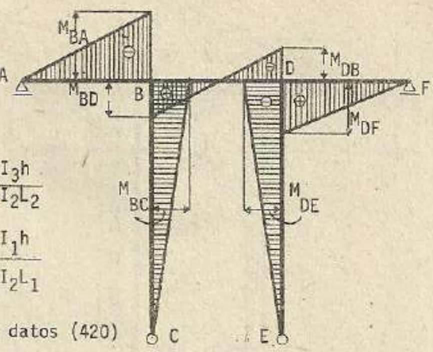
$$M_{BD} = -\frac{pa(1-\beta^2)k_1}{2}\left[\frac{1}{\mu_2} + \frac{1}{\mu_1}\right] \; ; \; M_{DB} = +\frac{Pa(1-\beta^2)k_1}{2}\left[\frac{1}{\mu_2} - \frac{1}{\mu_1}\right]$$

$$M_{BC} = M_{DE} = +\frac{3Pa(1-\beta^2)}{4\mu_1} \; ; \; V_E = \frac{M_{DB} - M_{BD}}{L_2} + \frac{M_{DF}}{L_1} \; ; \; V_A = P(1-\beta) + \frac{M_{BA}}{L_1}$$

$$V_C = P\beta - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L_2} \; ; \; V_F = \frac{M_{DF}}{L_1} \; ; \; H = \frac{M_{BC}}{h}$$

(427)





$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

otros datos (420)

$$M_{BA} = -\frac{Phk_2}{2\mu_2} \quad ; \quad M_{DF} = +\frac{Phk_2}{\mu_2} \quad ; \quad M_{BD} = +\frac{Phk_1}{\mu_2}$$

$$M_{DB} = -\frac{Phk_1}{\mu_2} \quad ; \quad M_{BC} = +\frac{Ph}{2} \quad ; \quad M_{DE} = -\frac{Ph}{2} \quad ; \quad H = \frac{P}{2} \quad ;$$

$$V_A = V_F = \frac{M_{DF}}{L_1} \quad ; \quad V_C = V_E = \frac{M_{BA}}{L_1} + \frac{2M_{BD}}{L_2}$$

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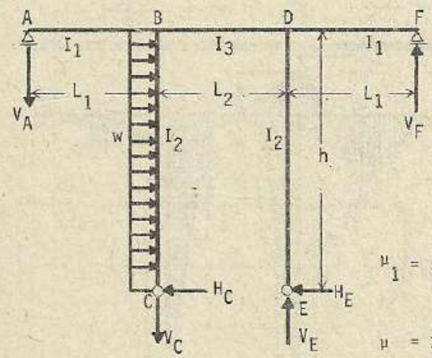
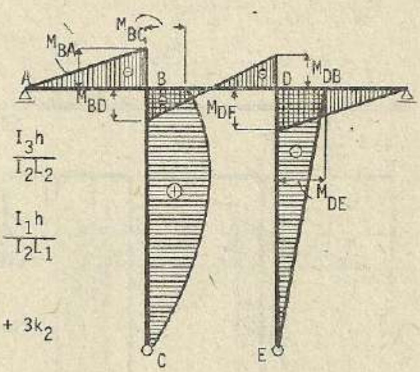
(428)





$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\mu_1 = 3 + 2\,k_1 + 3k_2$$

$$\mu = 2\,k_1 + k_2$$

$$\lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_1 = \frac{L_2}{L_1}$$

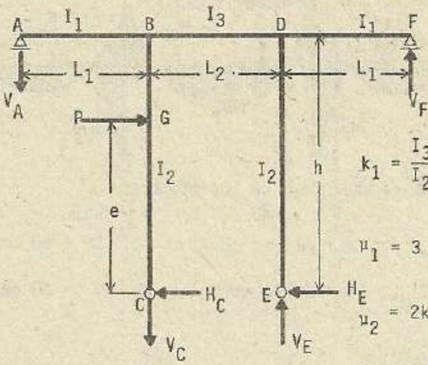
$$M_{BA} = -\frac{wh^2k_2}{16}\left[\frac{4}{\mu_2} - \frac{3}{\mu_1}\right]; \quad M_{DF} = +\frac{wh^2k_2}{16}\left[\frac{4}{\mu_2} + \frac{3}{\mu_1}\right]; \quad M_{BC} = M_{BD} - M_{BA}$$

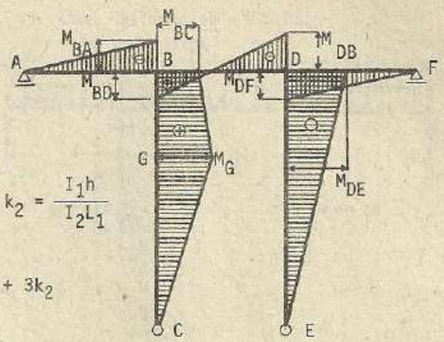
$$M_{BD} = +\frac{wh^2k_1}{8}\left[\frac{4}{\mu_2} - \frac{1}{\mu_1}\right]; \quad M_{DB} = -\frac{wh^2k_1}{8}\left[\frac{4}{\mu_2} + \frac{1}{\mu_1}\right]; \quad M_{DE} = M_{DB} - M_{DF}$$

$$H_E = -\frac{M_{DE}}{h} \quad ; \quad H_C = wh - H_E \quad ; \quad V_A = -\frac{M_{BA}}{L_1} \quad ; \quad V_C = +\frac{M_{BA}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2}$$

$$V_F = \frac{M_{DF}}{L_1} \quad ; \quad V_E = -\frac{M_{DF}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2}$$

(429)





$$k_1 = \frac{I_3 h}{I_2 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\mu_1 = 3 + 2k_1 + 3k_2$$

$$\mu_2 = 2k_1 + k_2$$

$$\lambda_1 = \frac{L_1}{L_2} \quad ; \quad \lambda_2 = \frac{L_2}{L_1} \quad ; \quad \beta = \frac{e}{h}$$

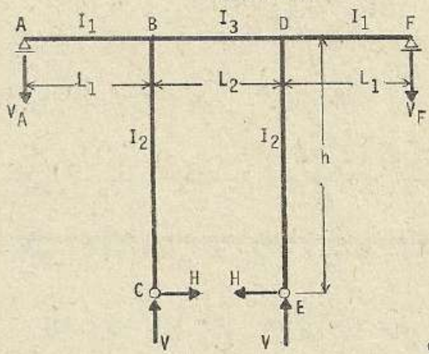
$$M_{BA} = -\frac{Pek_2}{4}\left[\frac{2}{\mu_2} - \frac{3\,(1-\beta^2)}{\mu_1}\right] \quad ; \quad M_{DF} = +\frac{Pek_2}{4}\left[\frac{2}{\mu_2} + \frac{3\,(1-\beta^2)}{\mu_1}\right]$$

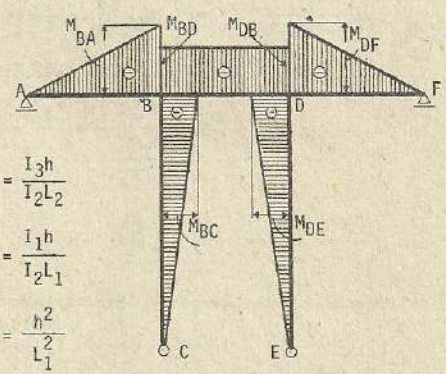
$$M_{BD} = +\frac{Pek_1}{2}\left[\frac{2}{\mu_2} - \frac{1-\beta^2}{\mu_1}\right] \quad ; \quad M_{DB} = -\frac{Pek_1}{2}\left[\frac{2}{\mu_2} + \frac{1-\beta^2}{\mu_1}\right]$$

$$M_{BC} = M_{BD} - M_{BA} \quad ; \quad M_{DE} = M_{DB} - M_{DF} \quad ; \quad H_E = -\frac{M_{DE}}{h} \quad ; \quad H_C = P - H_E \quad ; \quad V_A = -\frac{M_{BA}}{L_1}$$

$$V_C = \frac{M_{BD}}{L_1} + \frac{M_{BD}-M_{DB}}{L_2} ; \quad V_F = \frac{M_{DF}}{L_1} \quad ; \quad V_E = -\frac{M_{DF}}{L_1} + \frac{M_{BD} - M_{DB}}{L_2}$$

Esfuerzos debido a cambio de temperatura (430)





$$k_1 = \frac{I_3 h}{I_2 L_2}$$

$$k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\lambda_3 = \frac{h^2}{L_1^2}$$

otros datos (420)

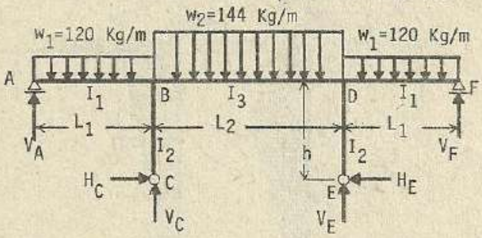
$$M_{BA} = M_{DF} = -\frac{3E\epsilon I_2 t L_1 k_2}{2h^2} \cdot \frac{2\lambda_3(3 + 2k_1) - 3\lambda_1}{\mu_1}$$

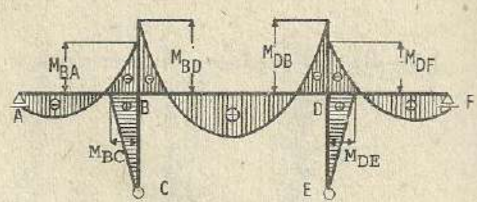
$$M_{BD} = M_{DB} = -\frac{3E\epsilon I_2 t L_1 k_1}{h^2} \cdot \frac{2\lambda_3 k_2 + \lambda_1}{\mu_1}$$

$$M_{BC} = M_{DE} = M_{BD} - M_{BA} \quad ; \quad V = -\frac{M_{BA}}{L_1} \quad ; \quad H = \frac{M_{BC}}{h}$$

Nota : En caso de descenso de temperatura, los esfuerzos tendrán signos contrarios a lo expuesto.

Solución de problema numérico





Altura útil (d_1) de los miembros AB , DF = 40 cm.

L_1 = 6 m. ; L_2 = 10 m. ; h = 5 m.

(d_2) de los miembros BC, DE = 50 cm.

(d_3) del miembro BD = 50 cm.

$$\frac{I_1}{I_3} = \left(\frac{d_1}{d_3}\right)^3 = 0.512 \quad ; \quad \frac{I_3}{I_2} = \left(\frac{d_3}{d_2}\right)^3 = 1$$

$$k_1 = \frac{I_3 h}{I_2 L_2} = 1 \times \frac{5}{10} = 0.5 \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_2} = 0.512 \times \frac{5}{6} = 0.43$$

$$M_{BA} = M_{DF} = \frac{-w_1L_1^2\,(3+2k_1)-2w_2L_2^2k_2}{8\,(3+2k_1+3k_2)} = \frac{-120\text{x}36(3+2\text{x}0.5)-2\text{x}144\text{x}100\text{x}0.43}{8\,(3+2\text{x}0.5+3\text{x}0.43)} = -\ 700.9 \text{ Kg - m}$$

$$M_{BC} = M_{DE} = \frac{3w_1L_1^2}{8\,\mu_1} - \frac{w_2L_2^2}{4\,\mu_1} = \frac{3w_1L_1^2 - 2w_2L_2^2}{8\,\mu_1} = \frac{3\text{x}120\text{x}36 - 2\text{x}144\text{x}100}{8\,(3+2\text{x}0.5+3\text{x}0.43)} = -374.3 \text{Kg m}$$

$$M_{BD} = M_{DB} = -\ 700.9 - 374.3 = -1075.2 \text{ Kg - m}$$

$$V_A = \frac{1}{L_1}\left[\frac{w_1L_1^2}{2} - (M_A + M_{BA})\right] = \frac{1}{6}\left[\frac{120\text{x}36}{2} - (0 + 700.9)\right] = 243.2 \text{ Kg}$$

$$V_C = w_1L_1 - V_A + \frac{w_2L_2}{2} = 720 - 243.2 + 720 = 1196.8 \text{ Kg}$$

$$H_C = H_E = -\frac{M_{BC}}{h} = +\frac{372.3}{5} = 74.9 \text{ Kg}$$

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$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L} \quad ; \quad k_3 = \frac{I_2 L_1}{I_4 L} \quad ; \quad m = k_2 + k_2k_3 + k_3$$

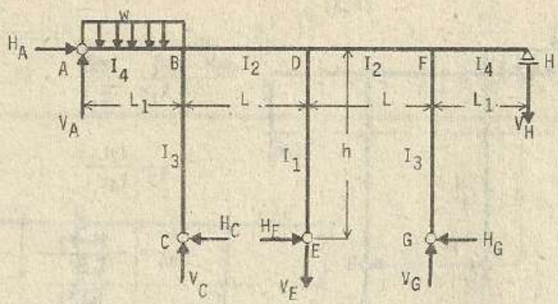
$$\mu_1 = 3m + k_2k_3 \qquad \mu_2 = \mu_1 + 8k_1m \quad ; \quad \lambda_1 = \frac{L_1}{L} \quad ; \quad \lambda_2 = \frac{L}{L_1}$$

$$M_{BA} = M_{BD} - M_{BC} \quad ; \quad M_{FH} = M_{FD} - M_{FG} \quad ; \quad V_H = -\frac{M_{FH}}{L_1} ; \quad H_C = \frac{M_{BC}}{h}$$

$$H_E = \frac{M_{DE}}{h} \quad ; \quad H_{DE} = -\frac{M_{FG}}{h} \quad ; \quad M_{BC} = +\frac{wL_1^2k_3}{16}\left[\frac{3+8k_1}{\mu_2} + \frac{3}{\mu_1}\right]$$

$$M_{FG} = -\frac{wL_1^2k_3}{16}\left[\frac{3+8k_1}{\mu_2} - \frac{3}{\mu_1}\right] ; \quad M_{BD} = +\frac{wL_1^2k_2k_3}{4}\left[\frac{1}{\mu_1} + \frac{2k_1}{\mu_2}\right]$$

$$M_{FD} = -\frac{wL_1^2k_2k_3}{4}\left[\frac{1}{\mu_1} - \frac{2k_1}{\mu_2}\right] \quad M_{DB} = +\frac{wL_1^2k_2k_3}{8}\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right]$$



$$M_{DF} = + \frac{wL_1^2 k_2 k_3}{8}\left[\frac{1}{\mu_1} - \frac{1}{\mu_2}\right]; \quad M_{DE} = \frac{wL_1^2 k_2 k_3}{4} \quad ; \quad V_A = \frac{wL_1}{2} + \frac{M_{BA}}{L_1}$$

$$V_C = \frac{wL_1}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L} ; \quad V_G = - \frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L}$$

$$V_E = \frac{1}{L}\left(M_{DB} - M_{BD} + M_{DF} - M_{FD}\right) \quad ; \quad H_A = H_C + H_G - H_E$$

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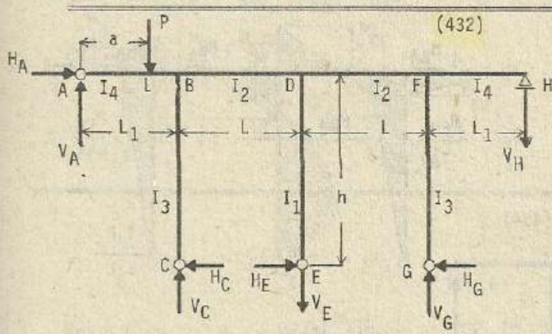

$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L}$$

$$k_3 = \frac{I_2 L_1}{I_4 L} \quad ; \text{ otros datos (431)}$$

$$\alpha = \frac{a}{L_1} ; \quad V_A = P\,(1-\alpha) + \frac{M_{BA}}{L_1}$$

$$V_C = P\alpha - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L}$$

$$V_E = \frac{1}{L}\left(M_{DB} - M_{BD} + M_{DF} - M_{FD}\right) \quad ; \quad V_G = - \frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L}$$

$$M_{BC} = + \frac{Pa\,(1-\alpha^2)k_3}{4}\left[\frac{3+8k_1}{\mu_2} + \frac{3}{\mu_1}\right]; \quad M_{FG} = - \frac{Pa\,(1-\alpha^2)k_3}{4}\left[\frac{3+8k_1}{\mu_2} - \frac{3}{\mu_1}\right]$$

$$M_{BD} = - Pa\,(1-\alpha^2)\,k_2k_3\left[\frac{1}{\mu_1} + \frac{1+2k_1}{\mu_2}\right] \quad M_{FD} = - Pa\,(1-\alpha^2)\,k_2k_3\left[\frac{1}{\mu_1} - \frac{1+2k_1}{\mu_2}\right]$$

$$M_{BA} = M_{BD} - M_{BC} \quad ; \quad M_{FH} = M_{FD} - M_{FG} ; \quad M_{DB} = + \frac{Pa\,(1-\alpha^2)k_2k_3}{2}\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right]$$

$$M_{DF} = + \frac{Pa\,(1-\alpha^2)k_2k_3}{2}\left[\frac{1}{\mu_1} - \frac{1}{\mu_2}\right]; \quad M_{DE} = + \frac{Pa\,(1-\alpha^2)k_2k_3}{\mu_2}$$

$$H_C = \frac{M_{BC}}{L} \quad ; \quad H_E = \frac{M_{DE}}{h} \quad ; \quad H_A = H_C + H_G - H_E \quad ; \quad H_G = - \frac{M_{FG}}{h} \quad ; \quad V_H = - \frac{M_{FH}}{L_1}$$

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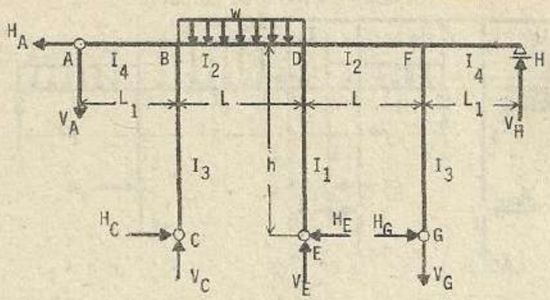

$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L}$$

$$k_3 = \frac{I_2 L_1}{I_4 L} \quad ; \quad \text{otros datos (431)}$$

$$M_{BA} = -\frac{wL^2 k_2}{8}\left[\frac{1+4k_1}{\mu_2} + \frac{1}{\mu_1}\right]$$

$$M_{FH} = +\frac{wL^2 k_2}{8}\left[\frac{1+4k_1}{\mu_2} - \frac{1}{\mu_1}\right]$$

$$M_{BC} = -\frac{wL_1^2 k_3}{8}\left[\frac{1+4k_1}{\mu_2} + \frac{1}{\mu_1}\right] , \quad M_{FG} = +\frac{wL_1^2 k_3}{8}\left[\frac{1+4k_1}{\mu_2} - \frac{1}{\mu_1}\right]$$

$$M_{DB} = -\frac{wL^2(m+k_2k_3)}{8}\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right] ; \quad M_{DF} = -\frac{wL^2(m+k_2k_3)}{8}\left[\frac{1}{\mu_1} - \frac{1}{\mu_2}\right]$$

$$M_{DE} = -\frac{wL^2}{4}\cdot\frac{m+k_2k_3}{\mu_2} ; \quad M_{BD} = M_{BA} + M_{BC} ; \quad M_{FD} = M_{FH} + M_{FG} ; \quad H = -\frac{M_{BC}}{h}$$

$$H_E = -\frac{M_{DE}}{h} ; \quad H_G = \frac{M_{FG}}{h} ; \quad V_A = -\frac{M_{BA}}{L_1} ; \quad V_C = \frac{wL}{2} - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L}$$

$$V_E = \frac{wL}{2} + \frac{M_{BD} - M_{DB} + M_{FD}}{L} ; \quad V_G = \frac{M_{FH}}{L_1} + \frac{M_{FD} - M_{DF}}{L} ; \quad V_F = \frac{M_{FH}}{L_1}$$

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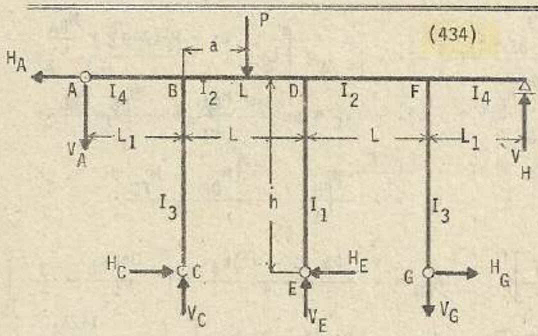

$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L}$$

$$k_3 = \frac{I_2 L_1}{I_4 L} \quad ; \quad \text{otros datos (431)}$$

$$\beta = \frac{a}{L}$$

$$M_{BD} = M_{BA} + M_{BC} \quad ; \quad M_{FD} = M_{FH} + M_{FG}$$

$$M_{DE} = M_{DB} - M_{DF}$$

$$M_{BA} = -\frac{Pa(1-\beta)k_2}{2}\left[\frac{(1-\beta)(3+4k_1)+4k_1}{\mu_2} + \frac{3(1-\beta)}{\mu_1}\right] ; \quad H_C = -\frac{M_{BC}}{h}$$

$$M_{FH} = +\frac{Pa(1-\beta)k_2}{2}\left[\frac{(1-\beta)(3+4k_1)+4k_1}{\mu_2} - \frac{3(1-\beta)}{\mu_1}\right] ; \quad H_E = -\frac{M_{DE}}{h}$$

$$M_{BC} = -\frac{Pa(1+\beta)k_3}{2}\left[\frac{(1-\beta)(3+4k_1)+4k_1}{\mu_2} + \frac{3(1-\beta)}{\mu_1}\right] ; \quad H_G = \frac{M_{FG}}{h}$$

$$M_{FG} = +\frac{Pa(1+\beta)k_3}{2}\left[\frac{(1-\beta)(3+4k_1)+4k_1}{\mu_2} - \frac{3(1-\beta)}{\mu_1}\right]$$

$$M_{DF} = -\frac{Pa(1-\beta)}{2}\left\{2(1+\beta)m - (2-\beta)(k_2+k_3)\right\}\cdot\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right] ; \quad V_A = -\frac{M_{BA}}{L_1}$$

$$M_{DB} = M_{DF}$$

$V_E = P\beta + \frac{1}{L}\left[(M_{BD} - M_{DB} - M_{DF} + M_{FD})\right]$; $V_F = \frac{M_{FH}}{L_1}$; $V_G = \frac{M_{FH}}{L_1} + \frac{M_{FD} - M_{DF}}{L}$

$V_C = P(1 - \beta) - \frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L}$; $H_A = H_C + H_G - H_E$

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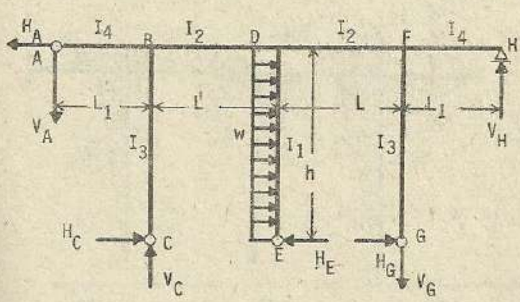


$k_1 = \frac{I_2 h}{I_1 L}$; $k_2 = \frac{I_2 h}{I_3 L}$; $k_3 = \frac{I_2 L_1}{I_4 L}$

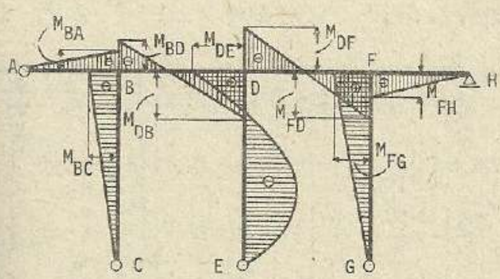
$m = k_2 + k_2 k_3 + k_3$; $\mu_1 = 3m + k_2 k_3$

$\mu_2 = \mu_1 + 8k_1 m$

$n = 3 + 8k_1 + 4k_2(1 + 2k_1)$

$\lambda_1 = \frac{L_1}{L}$; $\lambda_2 = \frac{L}{L_1}$

$M_{BA} = -\frac{wh^2}{4} \cdot \frac{k_1 k_2}{\mu_2}$; $M_{FH} = +\frac{wh^2}{4} \cdot \frac{k_1 k_2}{\mu_2}$



$M_{BC} = -\frac{wh^2}{4} \cdot \frac{k_1 k_3}{\mu_2}$; $M_{FG} = +\frac{wh^2}{4} \cdot \frac{k_1 k_3}{\mu_2}$

$M_{BD} = M_{BA} + M_{BC}$; $M_{FD} = M_{FH} + M_{FG}$

$M_{DB} = +\frac{wh^2}{2} \cdot \frac{k_1 m}{\mu_2}$; $M_{DF} = -\frac{wh^2}{2} \cdot \frac{k_1 m}{\mu_2}$

$M_{DE} = + wh^2 \frac{k_1 m}{2} = 2M_{DB}$; $H_C = H_G = \frac{M_{FG}}{h}$

$H_E = \frac{wh}{2} - \frac{M_{DE}}{h}$; $H_A = \frac{wh}{2} + \frac{M_{DE}}{h} + 2H_C$

$V_E = 0$; $V_A = V_H = \frac{M_{FH}}{L_1}$; $V_C = V_G = \frac{M_{FH}}{L_1} + \frac{M_{DB} - M_{FD}}{L}$

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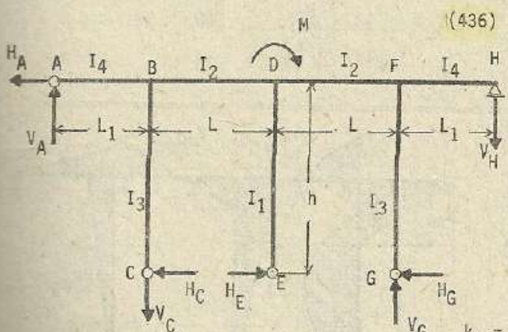

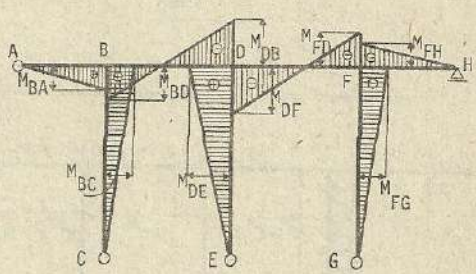

$k_1 = \frac{I_2 h}{I_1 L}$; $k_2 = \frac{I_2 h}{I_3 L}$; $k_3 = \frac{I_2 L_1}{I_4 L}$

$m = k_2 + k_2 k_3 + k_3$; $\mu_1 = 3m + k_2 k_3$; $\mu_2 = \mu_1 + 8k_1 m$; $\lambda_1 = \frac{L_1}{L}$ $\lambda_2 = \frac{L}{L_1}$

$n = 3 + 8k_1 + 4k_2(1 + 2k_1)$

$M_{BA} = +\frac{2Mk_1 k_2}{\mu_2}$; $M_{FH} = -\frac{2Mk_1 k_2}{\mu_2}$; $M_{BC} = +\frac{2Mk_1 k_3}{\mu_2}$; $M_{FG} = -\frac{2Mk_1 k_3}{\mu_2}$

$M_{BD} = M_{BA} + M_{BC}$; $M_{FD} = M_{FH} + M_{FG}$; $M_{DB} = -\frac{4Mk_1 m}{\mu_2}$; $M_{DF} = +\frac{4Mk_1 m}{\mu_2}$

$M_{DE} = + M \cdot \dfrac{\mu_1}{\mu_2}$; $M + M_{DB} - M_{DE} - M_{DF} = 0$; $H_C = H_G = \dfrac{M_{BC}}{h}$; $H_E = \dfrac{M_{DE}}{h}$

$H_A = H_E - 2H_C$; $V_A = V_H = \dfrac{M_{BA}}{L_1}$; $V_C = V_G = \dfrac{M_{BA}}{L_1} + \dfrac{M_{BD} - M_{DB}}{L}$; $V_E = 0$

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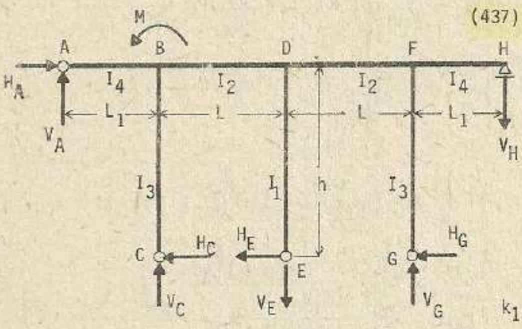

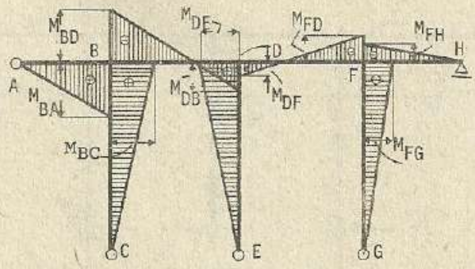

$k_1 = \dfrac{I_2 h}{I_1 L}$; $k_2 = \dfrac{I_2 h}{I_3 L}$; $k_3 = \dfrac{I_2 L_1}{I_4 L}$

$m = k_2 + k_2 k_3 + k_3$; $\mu_1 = 3m + k_2 k_3$; $\mu_2 = \mu_1 + 8k_1 m$; $\lambda_1 = \dfrac{L_1}{L}$; $\lambda_2 = \dfrac{L}{L_1}$

$n = 3 + 8k_1 + 4k_2(1+2k_1)$

$M_{BA} = + \dfrac{Mk_2}{2}\left[\dfrac{3+8k_1}{\mu_2} + \dfrac{3}{\mu_1}\right]$; $M_{FH} = - \dfrac{Mk_2}{2}\left[\dfrac{3+8k_1}{\mu_2} - \dfrac{3}{\mu_1}\right]$; $M_{BC} = + \dfrac{Mk_3}{2}\left[\dfrac{3+8k_1}{\mu_2} + \dfrac{3}{\mu_1}\right]$

$M_{BD} = - 2Mk_2k_3\left[\dfrac{1}{\mu_1} + \dfrac{1+2k_1}{\mu_2}\right]$; $M_{FD} = - 2Mk_2k_3\left[\dfrac{1}{\mu_1} - \dfrac{1+2k_1}{\mu_2}\right]$; $M_{DB} = + Mk_2k_3\left[\dfrac{1}{\mu_1} + \dfrac{1}{\mu_2}\right]$

$M_{BA} + M_{BC} - M_{BD} - M = 0$; $M_{FG} = - \dfrac{Mk_3}{2}\left[\dfrac{3+8k_1}{\mu_2} - \dfrac{3}{\mu_1}\right]$; $M_{DF} = + Mk_2k_3\left[\dfrac{1}{\mu_1} - \dfrac{1}{\mu_2}\right]$

$M_{DE} = + \dfrac{2Mk_2k_3}{\mu_2} = M_{DB} - M_{DF}$; $H_C = \dfrac{M_{BC}}{h}$; $H_E = \dfrac{M_{DE}}{h}$; $H_G = - \dfrac{M_{FG}}{h}$

$H_A = H_C + H_G - H_E$; $V_A = \dfrac{M_{BA}}{L_1}$; $V_H = - \dfrac{M_{FH}}{L_1}$; $V_C = - \dfrac{M_{BA}}{L_1} + \dfrac{M_{DB} - M_{BD}}{L}$

$V_G = - \dfrac{M_{FH}}{L_1} + \dfrac{M_{DF} - M_{FD}}{L}$; $V_E = \dfrac{M_{DB} - M_{BD} + M_{DF} - M_{FD}}{L}$

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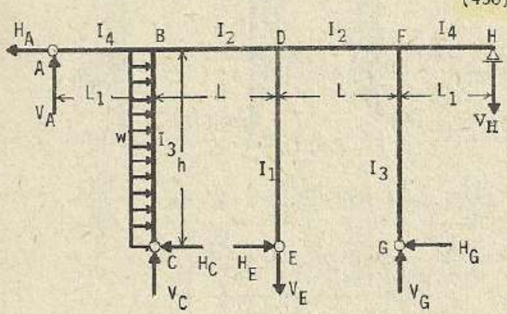

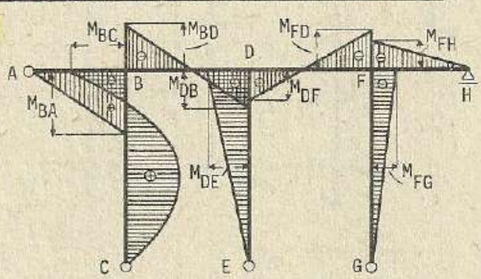

$k_1 = \dfrac{I_2 h}{I_1 L}$; $k_2 = \dfrac{I_2 h}{I_3 L}$; $k_3 = \dfrac{I_2 L_1}{I_4 L}$

$m = k_2 + k_2 k_3 + k_3$; $\mu_1 = 3m + k_2 k_3$; $\mu_2 = \mu_1 + 8k_1 m$ = $\lambda_1 = \dfrac{L_1}{L}$; $\lambda_1 = \dfrac{L}{L_1}$

$n = 3 + 8k_1 + 4k_2(1 + 2k_1)$

$$M_{BA} = +\frac{wh^2k_2}{16}\left[\frac{3+8k_1}{\mu_2}+\frac{3}{\mu_1}\right] ; \quad M_{FH} = -\frac{wh^2k_2}{16}\left[\frac{3+8k_1}{\mu_2}-\frac{3}{\mu_1}\right] ; \quad M_{BC} = M_{BD} - M_{BA}$$

$$M_{BD} = -\frac{wh^2k_2k_3}{4}\left[\frac{1}{\mu_1}+\frac{1+2k_1}{\mu_2}\right] ; \quad M_{FD} = -\frac{wh^2k_2k_3}{4}\left[\frac{1}{\mu_1}-\frac{1+2k_1}{\mu_2}\right] ; \quad M_{FG} = M_{FD} - M_{FH}$$

$$M_{DB} = +\frac{wh^2k_2k_3}{8}\left[\frac{1}{\mu_1}+\frac{1}{\mu_2}\right] ; \quad M_{DF} = +\frac{wh^2k_2k_3}{8}\left[\frac{1}{\mu_1}-\frac{1}{\mu_2}\right]$$

$$M_{DE} = +\frac{wh^2k_2k_3}{4\mu_2} = M_{DB} - M_{DF} ; \quad H_C = \frac{wh}{2}+\frac{M_{BC}}{h} ; \quad H_E = \frac{M_{DE}}{h} ; \quad H_G = -\frac{M_{FG}}{h}$$

$$H_A = \frac{wh}{2} - \frac{M_{BC}}{h} + H_E - H_G ; \quad V_A = \frac{M_{BA}}{L_1} ; \quad V_E = \frac{M_{DB} - M_{BD} + M_{DF} - M_{FD}}{L}$$

$$V_C = -\frac{M_{BA}}{L_1} + \frac{M_{DB} - M_{BD}}{L} ; \quad V_D = -\frac{M_{FH}}{L} + \frac{M_{DF} - M_{FD}}{L} ; \quad V_G = -\frac{M_{FH}}{L_1}$$

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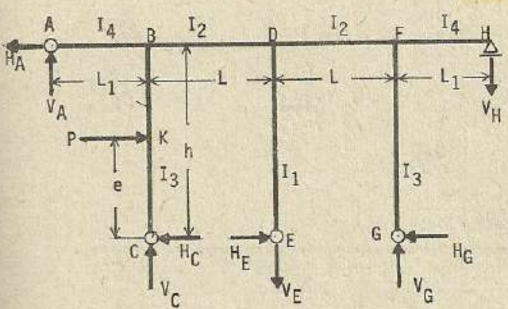
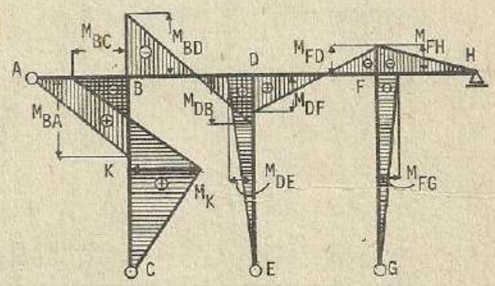




$$k_1 = \frac{I_2h}{I_1L} ; \quad k_2 = \frac{I_2h}{I_3L} ; \quad k_3 = \frac{I_2L_1}{I_4L}$$

$$m = k_2 + k_2k_3 + k_3$$

$$\mu_1 = 3m + k_2k_3 ; \quad \mu_2 = \mu_1 + 8k_1m ; \quad n = 3 + 8k_1 + 4k_2(1+2k_1) ; \quad \lambda_1 = \frac{L_1}{L}$$

$$M_{BA} = +\frac{Pe\,(1-\beta^2)\,k_2}{4}\left[\frac{3+8k_1}{\mu_2}+\frac{3}{\mu_1}\right] ; \quad \beta = \frac{e}{h} ; \quad \lambda_2 = \frac{L}{L_1}$$

$$M_{FH} = -\frac{Pe(1-\beta^2)k_2}{4}\left[\frac{3+8k_1}{\mu_2}-\frac{3}{\mu_1}\right] ; \quad M_{BD} = -Pe(1-\beta^2)k_2k_3\left[\frac{1}{\mu_1}+\frac{1+2k_1}{\mu_2}\right]$$

$$M_{FD} = -Pe(1-\beta^2)k_2k_3\left[\frac{1}{\mu_1}-\frac{1+2k_1}{\mu_2}\right] ; \quad M_{BC} = M_{BD} - M_{BA} ; \quad M_{FG} = M_{FD} - M_{FH}$$

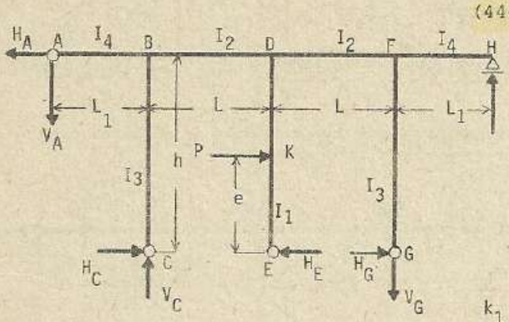
$$M_{DB} = +\frac{Pe(1-\beta^2)k_2k_3}{2}\left[\frac{1}{\mu_1}+\frac{1}{\mu_2}\right] ; \quad M_{DF} = +\frac{Pe(1-\beta^2)k_2k_3}{2}\left[\frac{1}{\mu_1}-\frac{1}{\mu_2}\right]$$

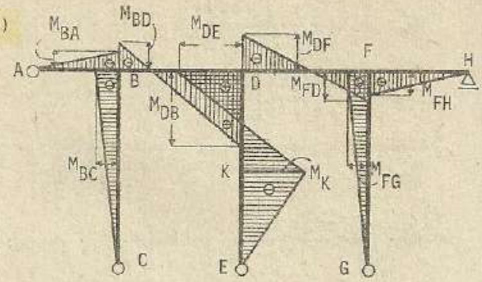
$$M_{DE} = +\frac{Pe(1-\beta^2)k_2k_3}{\mu_2} = M_{DB} - M_{DF} ; \quad H_E = \frac{M_{DE}}{h} ; \quad H_A = P\beta - \frac{M_{BC}}{h} + H_E - H_G$$

$$H_C = P(1-\beta) + \frac{M_{BC}}{h} ; \quad H_G = -\frac{M_{FG}}{h} ; \quad V_A = \frac{M_{BA}}{L_1} ; \quad V_C = -\frac{M_{BA}}{L_1} + \frac{M_{DB} + M_{BD}}{L}$$

$$V_H = -\frac{M_{FH}}{L_1} ; \quad V_E = \frac{M_{DB} - M_{BD} + M_{DF} - M_{FD}}{L} ; \quad V_G = -\frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L}$$

(440)





$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L} \quad ; \quad k_3 = \frac{I_2 L_1}{I_4 L}$$

$$m = k_2 + k_2 k_3 + k_3 \; ; \quad \mu_1 = 3m + k_2 k_3 \; ; \quad \mu_2 = \mu_1 + 8k_1 m \; ; \quad \lambda_1 = \frac{L_1}{L} \quad \lambda_2 = \frac{L}{L_1}$$

$$n = 3 + 8k_1 + 4k_2(1+2k_1) \quad ; \quad \beta = \frac{e}{h} \quad ; \quad M_{BA} = -\,Pe\,(1-\beta^2)\,\frac{k_1 k_2}{\mu_2}$$

$$M_{FH} = +\,Pe\,(1-\beta^2)\,\frac{k_1 k_2}{\mu_2} \quad ; \quad M_{BC} = -\,Pe(1-\beta^2)\,\frac{k_1 k_3}{\mu_2} \quad ; \quad M_{FG} = +Pe(1-\beta^2)\,\frac{k_1 k_3}{\mu_2}$$

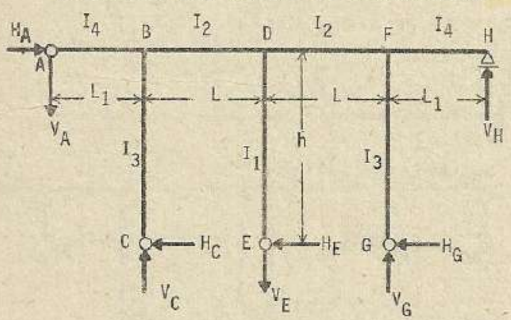
$$M_{BD} = M_{BA} + M_{BC} \quad ; \quad M_{FD} = M_{FH} + M_{FG} \; ; \quad M_{DB} = +\,2Pe\,(1-\beta^2)\,\frac{k_1 m}{\mu_2}$$

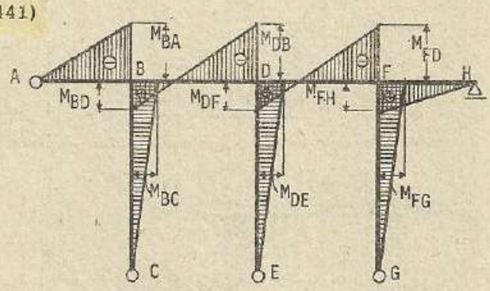
$$M_{DF} = -\,2\,Pe\,(1-\beta^2)\,\frac{k_1 m}{\mu_2} \quad ; \quad M_{DE} = +\,4Pe\,(1-\beta^2)\,\frac{k_1 m}{\mu_2} = 2M_{DB} \; ; \quad H_C = H_G = \frac{M_{FG}}{h}$$

$$H_E = P\,(1-\beta) - \frac{M_{DE}}{h} \quad ; \quad H_A = P\,\beta + \frac{M_{DE}}{h} + 2H_C \quad ; \quad V_E = 0 \quad ; \quad V_A = V_H = \frac{M_{FH}}{L_1}$$

$$V_C = V_G = \frac{M_{FH}}{L_1} + \frac{M_{DB} + M_{FD}}{L}$$

Esfuerzos por cambio de temperatura (441)





$$k_1 = \frac{I_2 h}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h}{I_3 L} \quad ; \quad k_3 = \frac{I_2 L_1}{I_4 L}$$

$$\lambda_3 = \frac{h^2}{L_1^2}$$ otros datos (431)

$$M_{BA} = -\,\frac{3E\varepsilon I_2 t L_1}{hL}\cdot\left[\frac{(\lambda_2+1)(3+8k_1-2k_2)}{\mu_2} - \frac{3\lambda_2 - \lambda_3\;(3+4k_2)}{\mu_1}\right]$$

$$M_{FH} = +\,\frac{3E\varepsilon I_2 t L_1}{hL}\cdot\left[\frac{(\lambda_2+1)(3+8k_1-2k_2)}{\mu_2} + \frac{3\lambda_2 - \lambda_3\;(3+4k_2)}{\mu_1}\right]$$

$$M_{BD} = +\,\frac{6E\varepsilon I_2 t L_1}{hL}\cdot\left[\frac{(1+\lambda_2)\left[k_3\,(3+4k_1)+k_2\right]}{\mu_2} - \frac{2\,(\lambda_3 k_2 + \lambda_2\,k_3)}{\mu_1}\right]$$

$$M_{FD} = -\frac{6E\varepsilon I_2 tL_1}{hL}\left[\frac{(1+\lambda_2)\left[k_3(3+4k_1)+k_2\right]}{\mu_2} + \frac{2(\lambda_3 k_2 + \lambda_2 k_3)}{\mu_1}\right] \quad ; \quad M_{BC} = M_{BD} - M_{BA}$$

$$M_{DB} = -\frac{6E\varepsilon I_2 tL_1}{hL}\left[\frac{(1+\lambda_2)(2m+k_3)}{\mu_2} - \frac{\lambda_3 k_2 + \lambda_2 k_2}{\mu_1}\right] \quad ; \quad M_{FG} = M_{FD} - M_{FB}$$

$$M_{DF} = +\frac{6E\varepsilon I_2 tL_1}{hL}\left[\frac{(1+\lambda_2)(2m+k_3)}{\mu_2} - \frac{\lambda_3 k_2 + \lambda_2 k_3}{\mu_1}\right] \quad ; \quad M_{DE} = M_{DB} - M_{DF}$$

$$H_C = \frac{M_{BC}}{h} \quad ; \quad H_E = -\frac{M_{DE}}{h} \quad ; \quad H_G = -\frac{M_{FG}}{h} \quad ; \quad H_A = H_C + H_E + H_G \quad ; \quad V_A = -\frac{M_{BA}}{L_1}$$

$$V_H = \frac{H_{FH}}{L} \quad ; \quad V_C = -\frac{M_{BA}}{L} + \frac{M_{DB} - M_{BD}}{L} \quad ; \quad V_G = -\frac{M_{FH}}{L_1} + \frac{M_{DF} - M_{FD}}{L}$$

$$V_E = \frac{-M_{BD} + M_{DB} + M_{DF} - M_{FD}}{L}$$

; Nota : En caso de descenso de temperatura los signos de los esfuerzos serán contrarios a los presentados.

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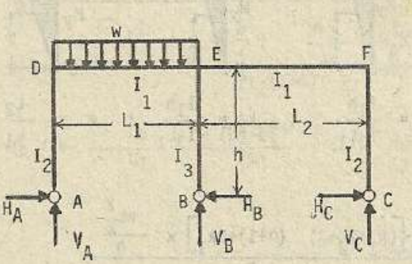
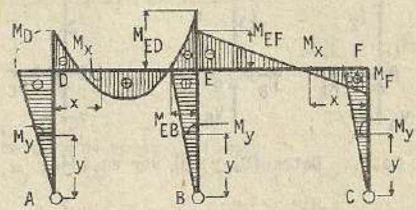
(442)





$$k_1 = \frac{I_1 h}{I_2 L_1} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{L_2}{L_1}$$

$$D = k_2\left(3\alpha + \frac{2}{3}k_2 + \frac{2}{3}\alpha k_2\right) + (k_1+k_2+3)\left[\frac{3}{2}\alpha^2 - \frac{2}{3}(1+\alpha)(k_1+k_2+3\alpha)\right] + \frac{3}{2}(k_1+k_2+3\alpha)$$

$$H_A = \frac{-\frac{1}{24}\left[(k_1+k_2+3\alpha)(1+\alpha) + 3\alpha k_1\right] \times \frac{wL_1^2}{h}}{D} \quad ; \quad H_C = \frac{\frac{1}{24}(k_2+\alpha k_2-3\alpha k_1) \times \frac{wL_1^2}{h}}{D}$$

$$H_B = \frac{-\frac{1}{24}\left[(k_1+3\alpha)(1+\alpha) + 6\alpha k_1\right] \times \frac{wL_1^2}{h}}{D} \quad ; \quad V_A = \frac{3\left[\left(\frac{1}{4}+\frac{\alpha}{3}\right)wL_1^2 - H_A h + H_C h\alpha\right]}{2(L_1+L_2)}$$

$$V_B = w\frac{L_1}{L_2}\left(\frac{L_1}{2} + L_2\right) - V_A\frac{(L_1+L_2)}{L_2} \quad ; \quad V_C = V_A\frac{L_1}{L_2} - \frac{wL_1^2}{2L_2}$$

Momentos de flexión en las columnas : AD $\longrightarrow$ $M_y = -H_A y$; BE $\longrightarrow$ $M_y = H_B y$

CF $\longrightarrow$ $M_y = H_C y$

luego, $M_{DA} = -H_A h$; M_{EB} (columna) $= H_B h$; $H_{FC} = H_C h$

Momentos de flexión en las vigas,

DE $\longrightarrow$ $M_x = -H_A h + V_A x - \frac{wx^2}{2}$

EF $\longrightarrow$ $M_x = H_C h + V_A\frac{L_1}{L_2}x - \frac{wL_1^2}{2L_2}x$

luego,

$$M_{ED} = -H_A h + V_A L_1 - \frac{wL_1^2}{2} \quad ; \quad M_{EF} = H_C h + V_A L_1 - \frac{wL_1^2}{2}$$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$

$$M = 12h\left[(\alpha+1)(4k^2+3\alpha) + 2k(\alpha^2+5\alpha+1)\right]$$

$$H_A = \frac{\left[k(5\alpha+2) + 3\alpha(\alpha+1)\right] wL_1^2}{M} \quad ; \quad H_C = \frac{-k(1-2\alpha)\, wL_1^2}{M}$$

$$H_B = \frac{\left[k(7\alpha+1) + 3\alpha(\alpha+1)\right] wL_1^2}{M} \quad ; \quad V_A = \frac{3\left[\left(\frac{1}{4}+\frac{\alpha}{3}\right) wL_1^2 - H_A h + H_C h\alpha\right]}{2(L_1+L_2)}$$

$$V_B = w\frac{L_1}{L_2}\left(\frac{L_1}{2}+L_2\right) - V_A\frac{(L_1+L_2)}{L_2} \quad ; \quad V_C = V_A\frac{L_1}{L_2} - \frac{wL_1^2}{2L_1}$$

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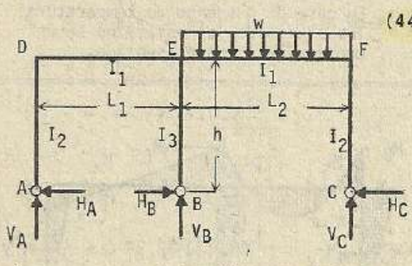
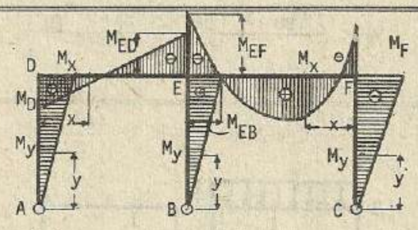
(443)





$$k_1 = \frac{I_1 h}{I_2 L_1} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{L_2}{L_1}$$

Nota : Datos (D) y (M) ver en (442)

$$H_A = \frac{\frac{\alpha}{24}(k_2-3k_1+\alpha k_2) \times \frac{wL_2^2}{h}}{D} \quad ; \quad H_C = \frac{-\frac{\alpha}{24}\left[(k_1+k_2+3)(\alpha+1)+3k_1\right] \times \frac{wL_2^2}{h}}{D}$$

$$H_B = \frac{-\frac{\alpha}{24}\left[(k_1+3)(\alpha+1)+6k_1\right] \times \frac{wL_2^2}{h}}{D} \qquad V_A = V_C\frac{L_2}{L_1} - \frac{wL_2^2}{2L_1}$$

$$V_C = \frac{3\left[wL_2^2\left(\frac{1}{3}+\frac{\alpha}{4}\right) - H_A h + H_C h\alpha\right]}{2L_2(1+\alpha)} \qquad V_B = \frac{wL_2^2\left(\frac{L_2}{2}+L_1\right)}{L_1} - V_C\frac{L_1+L_2}{L_1}$$

Momentos de flexión en las columnas, $AD \longrightarrow M_y = H_A y$; $BE \longrightarrow M_y = -H_B y$

$CF \longrightarrow M_y = -H_C y$

luego, $M_{DA} = H_A h$; $M_{EB} = -H_B h$; $M_{FC} = -H_C h$

Momentos de flexión en las vigas, $DE \longrightarrow M_x = H_A h + V_A x$

$EF \longrightarrow M_x = -H_C h + V_C x - \frac{wx^2}{2}$

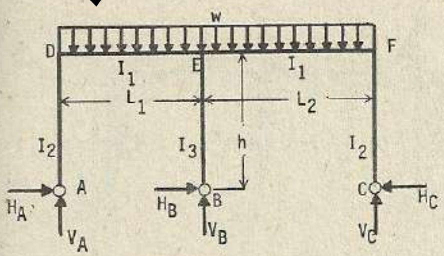
luego, $M_{ED} = H_A h + V_C L_2 - \frac{wL_2}{2}$

$M_{EF} = -H_C h + V_C L_2 - \frac{wL_2}{2}$

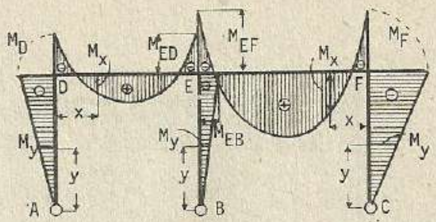
Si $I_2 = I_3$, entonces $k_1 = k_2 = k$ $\quad H_A = \frac{\alpha k(2-\alpha) wL_2^2}{M}$; $H_B = \frac{\alpha\left[k(\alpha+7) + 3(\alpha+1)\right]}{M}$

$$H_C = \frac{\alpha\left[k(2\alpha+5) + 3(\alpha+1)\right] wL_2^2}{M} \quad ; \quad V_C = \frac{3\left[wL_2^2\left(\frac{1}{3}+\frac{\alpha}{4}\right) - H_A h + H_C h\alpha\right]}{2L_2(1+\alpha)}$$

$$V_B = \frac{wL_2^2\left(\frac{L_2}{2}+L_1\right)}{L_1} - V_C\frac{(L_1+L_2)}{L_1} \quad ; \quad V_A = V_C\frac{L_2}{L_1} - \frac{wL_2^2}{2L_1}$$



(444)



$$k_1 = \frac{I_1 h}{I_2 L_1} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{L_2}{L_1}$$

otros datos (442)

$$H_A = \frac{-\frac{1}{24h}\left[(k_1+k_2+3\alpha)(1+\alpha) + 3\alpha k_1\right] wL_1^2 - \frac{\alpha}{24h}(k_2-3k_1+\alpha k_2)\, wL_2^2}{D}$$

$$H_C = \frac{-\frac{\alpha}{24h}\left[(k_1+k_2+3)(1+\alpha) + 3k_1\right] wL_2^2 - \frac{1}{24h}(k_2+\alpha k_2 - 3\alpha k_1)\, wL_1^2}{D}$$

$$H_B = \frac{-\frac{\alpha}{24}\left[(k_1+3)(\alpha+1)+6k_1\right]\frac{wL_2^2}{h} \quad \frac{1}{24}\left[(k_1+3\alpha)(1+\alpha)+6\alpha k_1\right]\frac{wL_1^2}{h}}{D}$$

$$V_C = \frac{3\left[wL_2^2\left(\frac{1}{3}+\frac{\alpha}{4}\right) - \frac{wL_1^2}{12} - H_A h + H_C h\alpha\right]}{2L_2(1+\alpha)} \; ; \; V_B = -\frac{V_C(L_1+L_2)}{L_1} + \frac{w(L_1+L_2)^2}{2L_1}$$

$$V_A = V_C\frac{L_2}{L_1} - \frac{w(L_2^2-L_1^2)}{2L_1} \; ;$$

Momentos de flexión en las columnas

AD ⟶ $M_y = -H_A y$; BE ⟶ $M_y = -H_B y$

CF ⟶ $M_y = -H_C y$

luego, $M_{DA} = -H_A h$; $M_{EB} = -H_B h$; $M_{FC} = -H_C h$

Momentos de flexión en las vigas, DE ⟶ $M_x = -H_A h + V_A x - \frac{wx^2}{2}$

EF ⟶ $M_x = -H_C h + V_C x - \frac{wx^2}{2}$

luego, $M_{ED} = -H_A h + V_A L_1 - \frac{wL_1^2}{2}$

$M_{EF} = -H_C h + V_C L_2 - \frac{wL_2^2}{2}$

Si $I_2 = I_3$, entonces $K_1 = k_2 = k$

$$H_A = \frac{\left[k(5\alpha+2) + 3\alpha(\alpha+1)\right] wL_1^2 - \alpha k(2-\alpha)\, wL_2^2}{M}$$

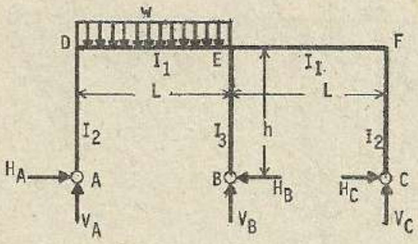
$$H_C = \frac{\alpha\left[k(2\alpha+5) + 3(\alpha+1)\right] wL_2^2 - k(2\alpha-1)\, wL_1^2}{M}$$

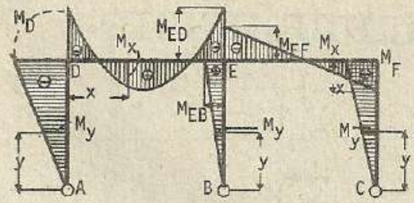
$$H_B = \frac{\alpha\left[k(\alpha+7)+3(\alpha+1)\right] wL_2^2 - \left[k(7\alpha+1)+3\alpha(\alpha+1)\right] wL_1^2}{M}$$

$$V_C = \frac{3\left[wL_2^2\left(\frac{1}{3}+\frac{\alpha}{4}\right) - \frac{wL_1^2}{12} - H_A h + H_C h\,\alpha\right]}{2L_2(1+\alpha)}$$

$$V_B = \frac{V_C(L_1+L_2)}{L_1} + \frac{w(L_1+L_2)^3}{2L_1} ; \quad V_A = V_C\frac{L_2}{L_1} - \frac{w(L_2 - L_1)}{2L_1}$$

(445)





$$k_1 = \frac{L_1 h}{I_2 L} \quad ; \quad k_2 = \frac{L_1 h}{I_3 L}$$

$$E = (4k_1^2 + 8k_1k_2 + 15k_1 + 6k_2 + 9) \quad ; \quad N = 24\cdot(4k+3)\cdot(k+1)$$

$$H_A = \frac{(5k_1+2K_2+6)}{8E}\cdot\frac{wL^2}{h} \quad ; \quad H_B = \frac{(4k_1+3)}{4E}\cdot\frac{wL^2}{h} \quad ; \quad H_C = \frac{(3k_1 - 2k_2)}{8E}\cdot\frac{wL^2}{h}$$

$$V_A = \frac{(7k_1^2 + 14k_1k_2 + 27k_1 + 12k_2 + 18)}{4E}\cdot wL \quad ; \quad V_B = \frac{(5k_1^2 + 10k_1k_2 + 18k_1 + 6k_2 + 9)}{2E}\cdot wL$$

$$V_C = -\frac{(k_1+2k_2+3)k_1}{4E}\cdot wL \quad ; \quad M_D = -\frac{(5k_1+2k_2+6)\ wL^2}{8E} \quad ; \quad M_{EB} = \frac{(4k_1+3)\ wL^2}{4E}$$

$$M_F = \frac{(3k_1 - 2k_2)\ wL^2}{8E} \quad ; \quad M_{EF} = -\frac{(2k_1 + 4k_1k_2 + 3k_1 + 2k_2)\ wL^2}{8E}$$

$$M_{ED} = -\frac{(2k_1^2+4k_1k_2 + 11k_1 + 2k_2 + 6)\ wL^2}{8E}$$

Esfuerzo de flexión en las columnas, AD $\longrightarrow M_y = -H_A y$

BE $\longrightarrow M_y = -H_B y$

CF $\longrightarrow M_y = H_C y$

Momentos de flexión en las vigas, DE $\longrightarrow M_x = -H_A h + V_A x - \frac{wx^2}{2}$

EF $\longrightarrow M_x = H_C h + V_C x$

Si $I_2 = I_3$, entonces $K_1 = k_2 = k$

$$H_A = \frac{(7k+6)}{N}\cdot\frac{wL^2}{h} \; ; \quad H_B = \frac{1}{12\ (k+1)}\cdot\frac{wL^2}{h} \; ; \quad H_C = \frac{k}{N}\cdot\frac{wL^2}{h} \; ; \quad V_A = \frac{7k+6}{4\ (4k+3)}\cdot wL$$

$$V_B = \frac{5k+3}{2\ (4k+3)}\cdot wL \; ; \quad V_C = -\frac{k}{4\ (4k+3)}\cdot wL \; ; \quad M_D\ \frac{7k+6}{N}\cdot wL^2 \; ; \quad M_{EB} = \frac{1}{12\ (k+1)}\cdot wL^2$$

$$M_{ED} = -\frac{6k^2+13k+6}{N}\cdot wL^2 \quad ; \quad M_{EF} = -\frac{(6k+5)}{N}\cdot wL^2 \quad ; \quad M_F = \frac{k}{N}\cdot wL^2$$

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(446)

$$k_1 = \frac{I_1 h}{I_2 L} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L}$$; otros datos (445)

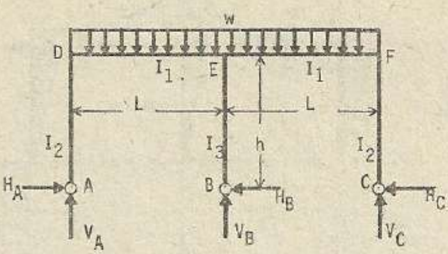
$$H_A = H_C = \frac{(k_1+2k_2+3)}{4E}\cdot\frac{wL^2}{h} \quad ; \quad H_B = 0 \quad ; \quad V_B = \frac{(5k_1^2+10k_1k_2+ 18k_1 + 6k_2+9)}{E}\cdot wL$$

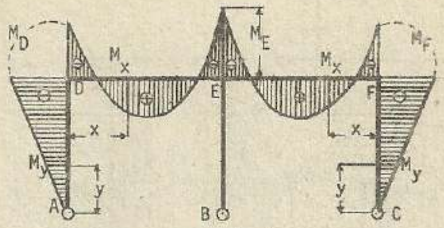
$$V_A = V_C = \frac{(3k_1^2+6k_1k_2+12k_1+6k_2+9)}{2E}\cdot wL \quad ; \quad M_D = M_F = -\frac{(k_1 + 2k_2+3)}{4E}\cdot wL^2$$

$$M_{EB} = 0 \quad ; \quad M_{ED} = M_{EF} = -\frac{(2k_1+4k_1k_2+7k_1 + 2k_2 + 3)}{4E}\cdot wL^2$$

Momentos por flexión en las columnas, AD y CF $\longrightarrow M_y = -H_a y$

BE $\longrightarrow M_y = 0$





Esfuerzos de momentos en las vigas, DE y EF $\qquad M_x = -H_A h + V_A x - \frac{wx^2}{2}$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$, donde

$$H_A = H_C = \frac{wL^2}{4\,(4k+3)\,h} \;;\quad H_B = 0 \;;\quad V_A = V_C = -\frac{3(k+1)}{2\,(4k+3)}wL \;;\quad V_B = \frac{(5k+3)}{(4k+3)}\cdot wL$$

$$M_D = M_F = -\frac{wL^2}{4\,(4k+3)} \;;\quad M_{EB} = 0 \;;\quad M_{ED} = M_{EF} = -\frac{2k+1}{4\,(4k+3)}\cdot wL^2$$

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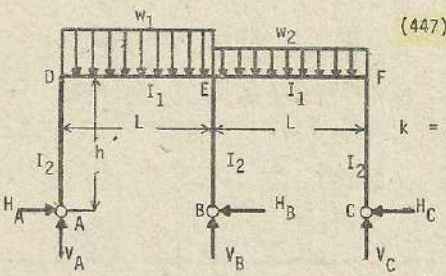
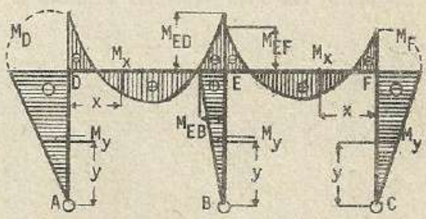
(447)



$$k = \frac{I_1 h}{I_2 L}$$



$N = 24\,(4k+3)\,(k+1)$

$$H_A = \frac{(7k+6)w_1 - kw_2}{N}\cdot\frac{L^2}{h} \;;\quad H_B = \frac{(w_1 - w_2)}{12\,(k+1)}\cdot\frac{L^2}{h} \;;\quad H_C = \frac{(7k+6)w_2 - kw_1}{N}\cdot\frac{L^2}{h}$$

$$V_A = \frac{(7k+6)\,w_1 - kw_2}{4\,(4k+3)}\cdot L \;;\quad V_B = \frac{(5k+3)(w_1+w_2)}{2(4k+3)}\cdot L \;;\quad V_C = \frac{(7k+6)w_2 - kw_1}{4\,(4k+3)}\cdot L$$

$$M_D = -\frac{(7k+6)w_1 - kw_2}{N}\cdot L^2 ;\quad M_{EB} = \frac{(w_1 + w_2)\,L^2}{12\,(k+1)} \;;\quad M_F = -\frac{(7k+6)w_2 - kw_1}{N}\cdot L^2$$

$$M_{ED} = -\frac{w_1\,(6k_1+13k+6) + w_2 k(6k+5)}{N}\cdot L^2 \;;\quad M_{EF} = -\frac{w_2\,(6k^2+13k+6)+w_1 k(6k+5)}{N}\cdot L^2$$

Momentos por flexión en las columnas, AD $\longrightarrow$ $M_y = -H_A y$

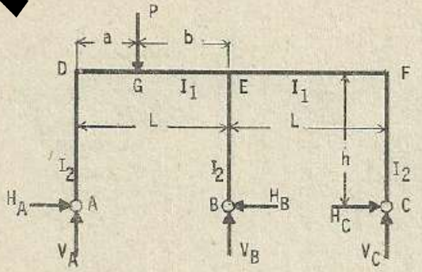
BE $\longrightarrow$ $M_y = +H_B y$

CF $\longrightarrow$ $M_y = -H_C y$

Momentos por flexión en las vigas :

DE $\longrightarrow$ $M_x = -H_A x + V_A x - \frac{w_1 x^2}{2}$

EF $\longrightarrow$ $M_x = -H_C x + V_C x - \frac{w_2 x^2}{2}$



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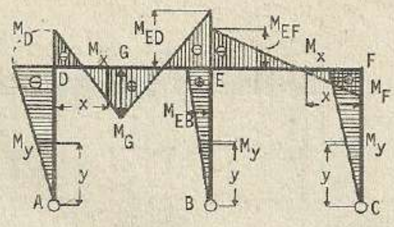
$$k = \frac{I_1 h}{I_2 L}$$



$$H_A = \frac{a\,(4k+3) + b\,(10k+9)}{4(k+1)\,(4k+3)} \cdot \frac{Pab}{hL^2} \quad ; \quad H_B = \frac{1}{2\,(k+1)} \cdot \frac{Pab}{hL^2}$$

$$H_C = \frac{a\,(4k+3) - b\,(2k+3)}{4(k+1)\,(4k+3)} \cdot \frac{Pab}{hL^2} \quad ; \quad V_A = \frac{\left[2k(3ab+2b^2+2L^2) + 3(3ab+b^2+L^2)\right]}{2\,(4k+3)L^3} \cdot Pb$$

$$V_B = \frac{\left[2k(2L^3-ab^2-2b^2L)+3(L^3-2ab^2-b^2L)\right]}{(4k+3)L^3} \cdot P \quad ; \quad V_C = \frac{a\left[3(b-a) - 2k(b+2a)\right]}{2\,(4k+3)L^3}\, Pb$$

$$M_D = -\frac{a\,(4k+3) + b\,(10k+9)}{4\,(k+1)(4k+3)} \cdot \frac{Pab}{L^2} \quad ; \quad M_E = \frac{Pab}{2(k+1)L^2} \quad ; \quad M_F = \frac{a(4k+3) - b(2k+3)}{4(k+1)\,(4k+3)} \cdot \frac{Pab}{L^2}$$

$$M_G = \frac{\left[4k^2(3ab+2b^2+2L^2) + 2k\,(12ab+4b^2+L^2) + 3\,(4ab+L^2)\right]}{4\,(k+1)\,(4k+3)\,L^3} \cdot Pab$$

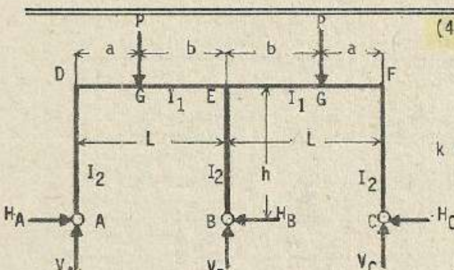
Momentos por flexión en las columnas , AD $\longrightarrow M_y = -H_A y$; BE $\longrightarrow M_y = H_B y$

CF $\longrightarrow M_y = H_C y$

Momentos de flexión en las vigas : DE $\longrightarrow M_x = -H_A h + V_A x \qquad x \leq a$

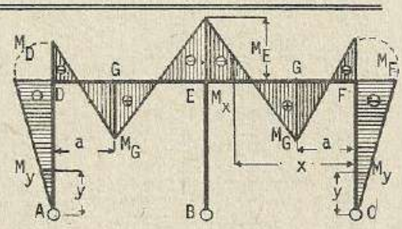
$M_x = -H_A h + V_A x - P\,(x-a) \qquad x \geq a$

EF $\longrightarrow M_x = H_C h + V_C x$



(449)

$$k = \frac{I_1 h}{I_2 L}$$



$$H_A = H_C = \frac{3ab^2 P}{(4k+3)hL^2} \quad ; \quad H_B = 0$$

$$V_A = V_C = \frac{2k(a+2L) + 3\,(2a+L)}{(4k+3)} \cdot \frac{Pb^2}{L^3} \quad ; \quad V_B = \frac{4k(2L^3-ab^2-2b^2L) + 6\,(L^3-3ab^2-b^2L)}{(4k+3)} \cdot \frac{P}{L^3}$$

$$M_D = M_F = -\frac{3ab^2}{(4k+3)} \cdot \frac{P}{L^2} \quad ; \quad M_E\ (\text{VIGA}) = -\frac{2k\,(a+L) + 3a}{(4k+3)} \cdot \frac{Pab}{L^2}$$

$$M_G = \frac{2k\,(a+2L) + 6a}{(4k+3)} \cdot \frac{Pab^2}{L^2} \; ;$$

Momentos de flexión en las columnas AD y CF

$$M_y = -H_A y = -H_C y$$

Esfuerzos de flexión en las vigas DE y EF :

$$M_x = -H_A h + V_A x = -H_C h + V_C x \;\text{-------}\; x \geq a$$

$M_x = H_A h + V_A x - P(x-a)$

$= -H_C h + V_C x - P(x-a) \qquad x \geq a$

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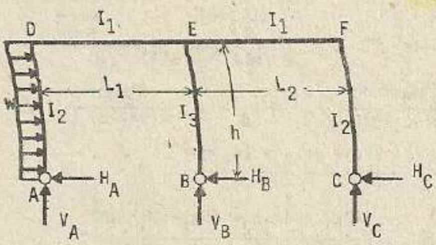

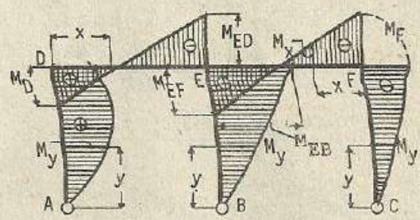

$k_1 = \dfrac{I_1 h}{I_2 L_1} \quad ; \quad k_2 = \dfrac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \dfrac{L_2}{L_1}$

$F = (1+\alpha)(9\alpha+3\alpha k_2+3k_2+8k_1k_2+4k_1^2) + 3k_1\,(1+8\alpha+\alpha^2)$

$P = 4k^2\,(1+\alpha) + 2k\,(1+5\alpha+\alpha^2) + 3\alpha(1+\alpha) \quad ; \quad R = (k_1+2k_2+3)\,(4k_1+3)$

$$H_A = \frac{wh\left[4(1+\alpha)(3\alpha k_2+3k_2+8k_1k_2+3k_1^2) + 3k_1(4+24\alpha+3\alpha^2) + 12(\alpha+k_1)(3\alpha+\alpha k_2+k_2) + 18\alpha(\alpha+2)\right]}{8F}$$

$$H_B = \frac{wh\left[18(\alpha+1)\alpha+20k_1^2+12k_1+51\alpha k_1+\alpha k_1(15\alpha+20k_1)\right]}{8F} ; \quad H_C = \frac{wh\left[18\alpha+33\alpha k_1+4\,(3+5k_1)\,(1+\alpha)\,k_2\right]}{8F}$$

$$V_A = -\frac{wh^2\left[6\alpha^2(3+k_1+k_2) + \alpha\,(6k_2+15k_1+k_1k_2+8k_1^2) + 3k_1\,(3k_2-k_1)\right]}{4L_1F}$$

$$V_B = -\frac{wh^2\left[k_1(9\alpha+7k_1+3-3\alpha^2-4\alpha k_1) + k_1k_2\,(7\alpha-1) + (1-\alpha)\,(9\alpha+3k_2+3\alpha k_2)\right]}{2L_1F}$$

$$V_C = \frac{wh^2\left[k_1(33\alpha+6+11k_1) + k_1\,(6+6\alpha) + k_1k_2\,(7+15\alpha) + 18\alpha\right]}{4L_1F}$$

$$M_D = \frac{wh^2\left[12\,(\alpha+k_1)\,(3\alpha+\alpha k_2+k_2) - 3\alpha k_1\,(\alpha+8) - 4k_1^2\,(1+\alpha) - 18\alpha^2\right]}{8F}$$

$$M_{ED} = -\frac{wh^2\left[18\alpha^2+\alpha k_1\,(18+15\alpha+20k_1) + 6k_1k_2 - 2k_1^2 - 10\alpha k_1k_2\right]}{8F}$$

$$M_{EF} = \frac{wh^2\left[33\alpha k_1+10\alpha k_1k_2 - 6k_1k_2 + 18\alpha+12k_1 + 22k_1^2\,)\right]}{8F}$$

$$M_{EB} = -\frac{wh^2\left[18(1+\alpha)\alpha + 20k_1^2 + 12k_1+51\alpha k_1+\alpha k_1(15\alpha+20k_1)\right]}{8F}$$

$$M_F = -\frac{wh^2\left[18\alpha+33\alpha k_1+4\,(3+5_1)\,(1+\alpha)\,k_2\right]}{8F}$$; Si $I_2 = I_3$, entonces $k_2 = k_1 = k$, donde :

$$H_A = \frac{wh\left[56(1+\alpha)k^2 + (24+144\alpha+33\alpha^2)\,k + 18\,(2+3\alpha)\alpha\right]}{24P}$$

$$H_B = \frac{wh\left[20(1+\alpha)k^2 + (12+51\alpha+15\alpha^2)\,k + 18\,(1+\alpha)\alpha\right]}{24P}$$

$$H_C = \frac{wh\left[20\,(1+\alpha)k^2+(12+45\alpha)k+18\alpha\right]}{24P} ; \quad V_A = -\frac{wh^2\left[(2+3\alpha)\,k^2+(7+4\alpha)\,\alpha k+6\alpha^2\right]}{4L_1P}$$

$$V_B = -\frac{wh^2\left[(6+3\alpha)\,k^2+(4+9\alpha-4\alpha^2)k+9(1-\alpha)\alpha\right]}{6L_1P} ; \quad V_C = \frac{wh^2\left[(6+5\alpha)k^2+(4+13\alpha)k+6\alpha\right]}{4L_1P}$$

$$M_D = \frac{wh^2\left[8\,(1+\alpha)k^2 + 3\,(8+3\alpha)\,\alpha k + 18\alpha^2\right]}{24P} ; \quad M_{ED} = -\frac{wh^2\left[(4+10\alpha)k^2 + (18+15\alpha)\alpha k+18\alpha^2\right]}{24P}$$

$$M_{EF} = \frac{wh^2\left[(16+10\alpha)k^2 + (12+33\alpha)\,k + 18\alpha\right]}{24P}; \quad M_F = -\frac{wh^2\left[20\,(1+\alpha)\,k^2 + (12+45\alpha)\,k + 18\alpha\right]}{24P}$$

$$M_{EB} = -\frac{wh^2\left[20\,(1+\alpha)\,k^2 + (12+51\alpha+15\alpha^2)\,k + 18\,(1+\alpha)\alpha\right]}{24P}$$

Si $L_1 = L_2 = L$, entonces $k_1 = \frac{I_1 h}{I_2 L}$; $k_2 = \frac{I_1 h}{I_3 L}$, luego

$$H_A = \frac{(24k_1^2 + 88k_1k_2 + 129k_1 + 72k_2 + 90)}{16R}\,wh \;; \quad H_B = \frac{(5k_1+6)}{8\,(k_1 + 2k_2 + 3)}\,wh$$

$$H_C = \frac{(40k_1k_2 + 24k_2 + 33k_1 + 18)}{16R}\,wh \;; \quad V_A = -\frac{(5k_1+6)wh^2}{3(4k_1+3)L} \;; \quad V_B = -\frac{3k_1wh^2}{4(4k_1+3)\,L}$$

$$V_C = \frac{(11k_1+6)\,wh^2}{8\,(4k_1+3)\,L} \;; \quad M_D = \frac{(-8k_1^2 + 24k_1k_2 + 9k_1 + 24k_2 + 18)\,wh^2}{16R}$$

$$M_{ED} = -\frac{(18k_1^2 - 4k_1k_2 + 33k_1 + 18)\,wh^2}{16R} \;; \quad M_{EF} = \frac{(22k_1^2 + 4k_1k_2 + 45k_1 + 18)\,wh^2}{16R}$$

$$M_{EB} = -\frac{(5k_1+6)}{8(k_1+2k_2+3)}\,wh^2 \;; \quad M_F = -\frac{(40k_1k_2 + 24k_2 + 33k_1 + 18)}{16R}\,wh^2$$

Si $L_1 = L_2 = L$, $I_1 = I_2 = I_3$, o sea cuando $K_1 = k_2 = k$

$$H_A = \frac{(112k^2+201k+90)}{48\,(k+1)\,(4k+3)}\cdot wh \;; \quad H_B = \frac{(5k+6)}{24\,(k+1)}wh \;; \quad H_C = \frac{(40k^2+57k+18)}{48\,(k+1)(4k+3)}\,wh$$

$$V_A = -\frac{(5k+6)wh^2}{8\,(4k+3)L} \;; \quad V_B = -\frac{3kwh^2}{4\,(4k+3)L} \;; \quad V_C = \frac{(11k+6)wh^2}{8\,(4k+3)L}$$

$$M_D = \frac{(16k^2+33k+18)wh^2}{48\,(1+k)(4k+3)} \;; \quad M_{ED} = -\frac{(14k^2+33k+18)wh^2}{48\,(k+1)(4k+3)} \;; \quad M_{EF} = \frac{(26k^2+45k+18)wh^2}{48\,(k+1)(4k+3)}$$

$$M_{EB} = -\frac{(5k+6)\;\;wh^2}{24\,(k+1)} \;; \quad M_F = -\frac{(40k^2+57k+18)wh^2}{48\,(k+1)(4k+3)}$$

Esfuerzos de momentos por flexión de las columnas.

AD $\longrightarrow M_y = H_A y - \frac{wx^2}{2}$; BE $\longrightarrow M_y = -H_B y$; CF $\longrightarrow M_y = -H_C y$

Momentos en las vigas : DE $\longrightarrow M_x = H_A h - \frac{wh^2}{2} + V_A x$

EF $\longrightarrow M_x = -H_C h + V_C x$

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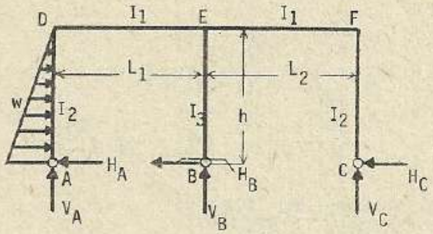

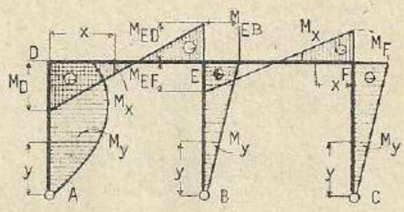

$$k_1 = \frac{I_1 h}{I_2 L_1} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{L_2}{L_1}$$

otros datos (450)

$$H_A = W\,\frac{\left[44(1+\alpha)k_1^2+(40+332\alpha+33\alpha^2)\,k_1+124(1+\alpha)\,k_1k_2+20(2+5\alpha+3\,\alpha^2)k_2+30(4+5\alpha)\alpha\right]}{20F}$$

$$H_B = W\,\frac{\left[36(\alpha+1)k_1^2+(27\alpha^2+87\alpha+20)K_1+30\alpha(\alpha+1)\right]}{20F}; \quad H_C = W\,\frac{\left[61\alpha k_1+36(1+\alpha)\,k_1k_2+20(1+\alpha)k_2+30\alpha\right]}{20F}$$

$$V_A = -\,\frac{Wh\left[(40\alpha-21)k_1^2+\alpha(57+30\alpha)k_1+(39-\alpha)k_1k_2+30(1+\alpha)\alpha k_2+90\alpha^2\right]}{30L_1F}$$

$$V_B = -\,\frac{Wh\left[(82-40\alpha)k_1^2+(30+126\alpha-30\alpha^2)k_1+(82\alpha+2)k_1k_2+30(\alpha+1)(1-\alpha)k_2+90\,(1-\alpha)\right]}{30L_1F}$$

$$V_C = \frac{Wh\left[61k_1^2+(183\alpha+30)k_1+(81\alpha+41)k_1k_2+30(1+\alpha)k_2+90\alpha^2\right]}{30L_1F}$$

$$M_D = Wh\,\frac{\left[-28(1+\alpha)k_1^2+(36-21\alpha)\alpha k_1+52(1+\alpha)k_1k_2+60(1+\alpha)\alpha k_2+90\alpha^2\right]}{60F}$$

$$M_{ED} = -\,\frac{Wh\left[(108\alpha-14)k_1^2+\alpha(81\alpha+78)k_1+(26-54\alpha)k_1k_2+90\alpha^2\right]}{60F}$$

$$M_{EF} = \frac{Wh\left[122k_1^2+(83\alpha+60)k_1+(54\alpha-26)k_1k_2+90\,\alpha)\right]}{60F}$$

$$M_{EB} = \frac{Wh\left[36(1+\alpha)k_1^2+(27\alpha^2+87\alpha+20)k_1+30\alpha(1+\alpha)\right]}{20F}$$

$$M_F = -\,\frac{wh\left[61\alpha k_1+36(1+\alpha)\,k_1k_2+20(1+\alpha)k_2+30\,\alpha\right]}{20F}$$

Si $L_2 = L_3$, entonces $K_1 = k_2 = k$, luego

$$H_A = \frac{W\left[168(1+\alpha)k^2+(80+432\alpha+93\alpha^2)k+30(4+5\alpha)\alpha\right]}{60P}$$

$$H_B = \frac{W\left[36\,(1+\alpha)k^2+(20+87\alpha+27\alpha^2)k+30(1+\alpha)\,\alpha\right]}{60P}; \quad H_C = \frac{W\left[36(1+\alpha)k^2+(20+81\alpha)k+30\alpha\right]}{60P}$$

$$V_A = -\,\frac{Wh\left[(13+6)k^2+(20\alpha+29)\alpha k+30\alpha\right]}{30L_1P} \quad ; \quad V_B = -\,\frac{Wh\left[7(2+\alpha)k^2+(10+21\alpha-10\alpha^2)k+15(1-\alpha)\right]}{15L_1P}$$

$$V_C = \frac{Wh\left[(27\alpha+34)k^2+(71\alpha+20)k+30\alpha\right]}{30L_1P} \quad ; \quad M_D = \frac{Wh\left[8\,(1+\alpha)k^2+(32+13\alpha)k+30\alpha^2\right]}{60P}$$

$$M_{ED} = -\,\frac{Wh\left[(4+18\alpha)k^2+\alpha(27\alpha+26)k+30\,\alpha^2\right]}{60P} \quad ; \quad M_{EF} = -\,\frac{Wh\left[(32+18\alpha)k^2+(20+61\alpha)k+30\alpha\right]}{60P}$$

$$M_{EB} = \frac{Wh\left[(36+36\alpha)k^2+(27\alpha^2+87\alpha+20)\,k+30\alpha(\alpha+1)\right]}{60P}; \quad M_F = -\,\frac{Wh\left[36(1+\alpha)k^2+(20+81\alpha)k+30\alpha\right]}{60P}$$

Si $L_1 = L_2 = L$, entonces $k_1 = \dfrac{I_1 h}{I_2 L}$; $k_2 = \dfrac{I_1 h}{I_3 L}$

$H_A = W\dfrac{(88k_1^2 + 248k_1k_2 + 405k_1 + 200k_2 + 270)}{40R}$; $H_B = W\dfrac{(9k_1+10)}{20\,(k_1+2k_2+3)}$

$H_C = W\dfrac{(72k_1k_2 + 40k_2 + 61k_1 + 30)}{40\,R}$; $V_A = -\dfrac{Wh\,(19k_1+30)}{60L\,(4k_1+3)}$; $V_B = -\dfrac{7Whk_1}{10L(4k_1+3)}$

$V_C = \dfrac{Wh(61k_1+30)}{60L(4k_1+3)}$; $M_D = Wh\dfrac{(-56k_1^2 + 104k_1k_2 + 15k_1 + 120k_2 + 90)}{120R}$

$M_{ED} = -Wh\dfrac{(94k_1^2 - 28k_1k_2 + 159k_1 + 90)}{120R}$; $M_{EF} = Wh\dfrac{(122k_1^2 + 28k_1k_2 + 243k_1 + 90)}{120R}$

$M_{EB} = -Wh\dfrac{(9_1+10)}{20(k_1+2k_2+3)}$; $M_F = -Wh\dfrac{72k_2k_1 + 40k_2 + 61k_1 + 30)}{40R}$

Si $L_1 = L_2 = L$, $I_3 = I_2$, entonces $k_1 = k_2 = k$ donde : $H_A = W\dfrac{(336k^2+605k+270)}{120(k+1)(4k+3)}$

$H_B = W\dfrac{(9k+10)}{60(k+1)}$; $H_C = W\dfrac{(72k^2+101k+30)}{120(k+1)(4k+3)}$; $V_A = -\dfrac{Wh\,(19k+30)}{60L(4k+3)}$

$V_B = -\dfrac{7kWh}{10L(4k+3)}$; $V_C = \dfrac{Wh(61k+30)}{60L(4k+3)}$; $M_D = Wh\dfrac{(16k^2+45k+30)}{120\,(k+1)(4k+3)}$

$M_{ED} = -Wh\dfrac{(22k^2+53k+30)}{120(k+1)(4k+3)}$; $M_{EF} = Wh\dfrac{(50k^2+81k+30)}{120(k+1)(4k+3)}$; $M_{EB} = -Wh\dfrac{(9k+10)}{60(k+1)}$

$M_F = Wh\dfrac{(50k^2+81k+30)}{120(k+1)(4k+3)}$

Esfuerzos de momentos por flexión en las columnas :

AD ⟶ $M_y = H_A y - \dfrac{W}{h^2}\left(hy^2 - \dfrac{y^3}{3}\right)$; BE ⟶ $M_y = -H_B y$; CF ⟶ $M_y = -H_C y$

Esfuerzos de momentos por flexión en las vigas :

DE ⟶ $M_x = H_A h - \dfrac{2}{3}\,Wh + V_A x$

EF ⟶ $M_x = -H_C h + V_C x$

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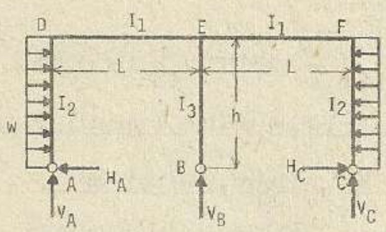
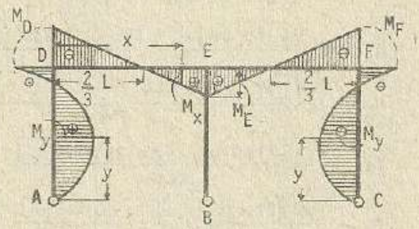


$k_1 = \dfrac{I_1 h}{I_2 L}$

$k_2 = \dfrac{I_1 h}{I_3 L}$



$H_A = H_C = \dfrac{3(k_1+1)}{2\,(4k_1+3)}\,wh$; $H_B = 0$

$V_A = V_C = \dfrac{3k_1}{4(4k_1+3)}\cdot\dfrac{wh^2}{2}$; $V_B = \dfrac{3k_1}{2(4k_1+3)}\cdot\dfrac{wh^2}{2}$; $M_D = M_F = -\dfrac{k_1}{2(4k_1+3)}\,wh^2$

$M_{ED} = M_{EF} = \dfrac{k_1}{4\,(4k_1+1)}\,wh^2$; $M_{EB} = 0$

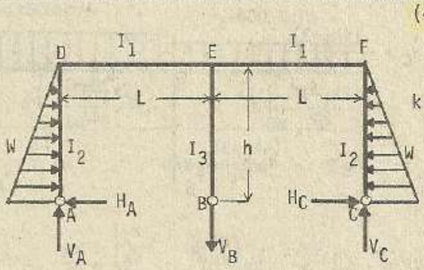
Momentos por flexión en las columnas AD y CF :

$$M_y = H_A y - \frac{wy^2}{2} = H_C y - \frac{wy^2}{2}$$

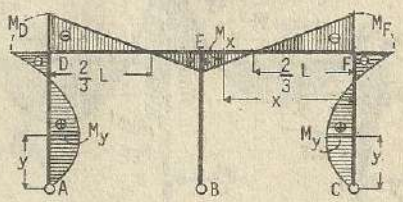
Momentos por flexión en las vigas : DE y FE :

$$M_x = H_A h - \frac{wh^2}{2} + V_A x = H_C h - \frac{wh^2}{2} + V_C x$$

(453)



$$k = \frac{I_1 h}{I_2 L}$$



$$H_A = H_C = \frac{(11k+10)}{5\ (4k+3)} \cdot W \quad ; \quad H_B = 0$$

$$V_A = V_C = \frac{7k}{10(4k+3)} W \quad ; \quad V_B = -\frac{7k}{5\ (4k+3)} W \quad ; \quad M_D = M_F = -\frac{7k}{15(4k+3)} Wh$$

$$M_{ED} = M_{EF} = \frac{7k}{30\ (4k+3)} \cdot Wh \quad ; \quad M_{EB} = 0$$

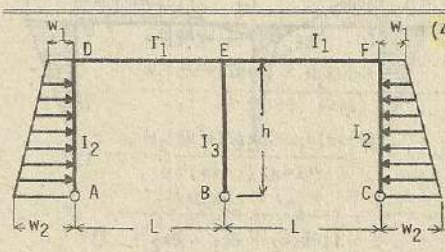
Momentos por flexión o doblado en las columnas AD y CF,

$$M_y = H_A y - \frac{W}{h^2} \left(hy^2 - \frac{y^3}{3}\right) = H_C y - \frac{W}{h^2} \left(hy^2 - \frac{y^3}{3}\right)$$

Momentos por flexión en las vigas DE y FE,

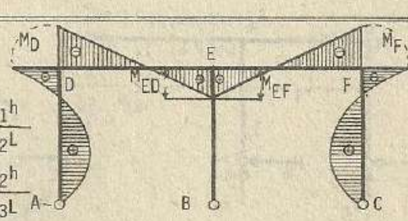
$$M_x = H_A h - \frac{2}{3} Wh + V_A x = H_C h - \frac{2}{3} Wh + V_C x$$

(454)



$$k_1 = \frac{I_1 h}{I_2 L}$$

$$k_2 = \frac{I_2 h}{I_3 L}$$



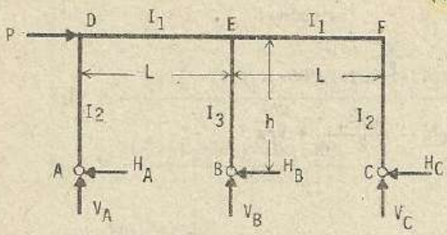
$$M_D = M_F = -\frac{4k_1 h^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k_1)}$$

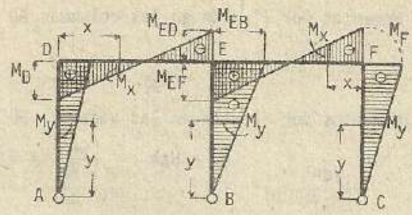
$$M_{ED} = M_{EF} = \frac{2k_1 h^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k_1)}$$

Si $I_2 = I_3$, $k = \frac{I_1 h}{I_2 L}$ donde $M_D = M_F = -\frac{4kh^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k)}$

$$M_{ED} = M_{EF} = \frac{2kh^2}{120} \cdot \frac{(8w_1 + 7w_2)}{(3+4k)}$$

(455)





$G = (k_1 + 2k_2 + 3)$ $\qquad k_1 = \dfrac{I_1 h}{I_2 L} \quad ; \quad k_2 = \dfrac{I_1 h}{I_3 L}$

$H_A = \dfrac{(4k_2+3)}{4G} \cdot P \quad ; \quad H_B = \dfrac{(2k_1+3)}{2G} \cdot P \quad H_C = \dfrac{(4k_2+3)}{4G} \cdot P; \quad V_A = -\dfrac{Ph}{2L} \quad ; \quad V_B = 0$

$V_C = \dfrac{Ph}{2L} \quad ; \quad M_D = \dfrac{(4k_2+3)}{4G} Ph \quad ; \quad M_{EB} = -\dfrac{(2k_1+3)}{2G} \cdot Ph \; ; \; M_{ED} = -\dfrac{(2k_1+3)}{4G} \cdot Ph$

$M_{EF} = \dfrac{(2k_1+3)}{4G} \cdot Ph \quad ; \quad M_F = -\dfrac{(4k_2+3)}{4G} \cdot Ph$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$

$H_A = H_C = \dfrac{4k+3}{12\,(k+1)} \cdot P \; ; \quad H_B = \dfrac{2k+3}{6\,(k+1)} \cdot P \quad ; \quad V_A = -\dfrac{Ph}{2L} \quad ; \quad V_B = 0 \quad ; \quad V_C = \dfrac{Ph}{2L}$

$M_D = \dfrac{(4k+3)}{12(k+1)} \cdot Ph \quad ; \quad M_{EB} = -\dfrac{(2k+3)}{6\,(k+1)} \cdot Ph \quad ; \quad M_{ED} = -\dfrac{(2k+3)}{12(k+1)} \cdot Ph$

$M_{EF} = \dfrac{(2k+3)}{12(k+1)} \cdot Ph \; ; \quad M_F = -\dfrac{(4k+3)}{12(k+1)} \cdot Ph$

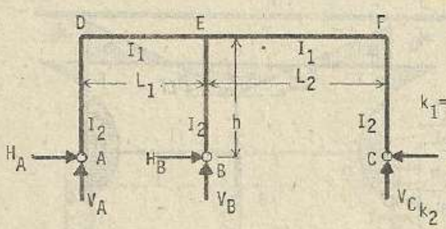
Momentos por flexión en las columnas,

AD $\longrightarrow M_y = H_A y \quad ; \quad$ BE $\longrightarrow M_y = -H_B y \quad ; \quad$ CF $\longrightarrow M_y = -H_C y$

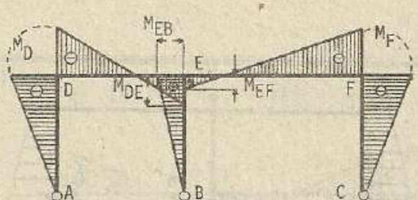
Momentos de flexión en las vigas : DE $\longrightarrow M_x = H_A h + V_A x$

EF $\longrightarrow M_x = -H_C h + V_C x$

Esfuerzos por cambio de temperatura (456)



$k_1 = \dfrac{I_1 h}{I_2 L_1} \qquad k_2 = \dfrac{I_1 h}{I_2 L_2}$



$S = (16k_1k_2 + 8k_1 + 8k_2 + 3)$

$H_A = \dfrac{4E\varepsilon tI_1(4k_1+8k_2+3)}{Sh^2} \quad ; \quad H_B = \dfrac{16E\varepsilon tI_1(k_1 - k_2)}{Sh^2} \; ; \quad H_C = \dfrac{4E\varepsilon tI_1(4k_2+8k_1+3)}{Sh^2}$

$V_A = \dfrac{6E\varepsilon tI_1(4k_1+8k_2+3)}{ShL_1} \quad ; \quad V_B = \dfrac{6E\varepsilon tI_1\left[(4k_1+8k_2+3)L_2 + (4k_2+8k_1+3)L_1\right]}{ShL_1L_2}$

$V_C = \dfrac{6E\varepsilon tI_1(4k_2+8k_1+3)}{ShL_2} \quad ; \quad M_D = -\dfrac{4E\varepsilon tI_1(4k_1+8k_2+3)}{Sh} \; ; \; M_{EB} = \dfrac{16\,E\varepsilon tI_1\,(k_1 - k_2)}{Sh}$

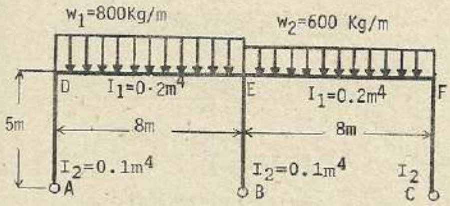
$M_{ED} = \dfrac{2E\varepsilon tI_1(4k_1+8k_2+3)}{Sh} \quad ; \quad M_{EF} = \dfrac{3E\varepsilon tI_1(4k_2+8k_1+3)}{Sh} \quad ; \quad M_F = -\dfrac{4\,E\varepsilon tI_1(4k_2+8k_1+3)}{Sh}$

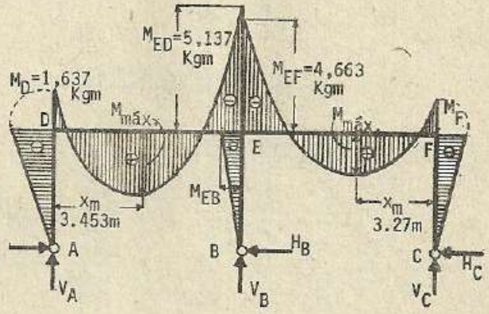
Si $L_1 = L_2 = L$, entonces $k_1 = k_2 = k$, donde $H_A = H_C = \frac{12E\varepsilon t I_1}{(4k+3)h^2}$; $H_B = 0$

$V_A = V_C = \frac{18E\varepsilon t I_1}{(4k+3)hL}$; $V_B = \frac{36E\varepsilon t I_1}{(4k+3)hL}$; Nota : En caso de descenso de temperatura, se modifican los signos de los esfuerzos

$M_D = M_F = -\frac{12E\varepsilon t I_1}{(4k+3)h}$; $M_E = \frac{6E\varepsilon t I_1}{(4k+3)h}$

Solución de problema numérico :





$$k = \frac{I_1 h}{I_2 L} = \frac{0.2x5}{0.1x8} = 1.25$$

$$H_A = \frac{\left[(7x+6)w_1 - kw_2\right]L^2}{24(4k+3)(k+1)\ h} = \frac{\left[(1.25x7+6)x800 - 1.25x600\right]x8x8}{24x(4x1.25+3)\ (1.25+1)x5} = 327\ Kg$$

$$V_A = \frac{(7k+6)w_1 - kw_2}{4(4k+3)}\ L = \frac{\left[(1.25x7+6)x800 - 1.25x600\right]x8}{4(4x1.25+3)} = 2{,}762.5\ Kg$$

$$H_B = \frac{(w_1-w_2)L^2}{12(k+1)\ h} = \frac{(800-600)x8x8}{12(1.25+1)x5} = 95\ Kg \quad ; \quad V_B = \frac{(5k+3)\ (w_1+w_2)}{2(4k+3)}L$$

$$= \frac{(5x1.25+3)(800+600)}{2x(4x1.25+3)} \ x\ 8 = 6{,}475\ Kg$$

$$H_C = \frac{\left[(7k+6)w_2 - kw_1\right]L^2}{24(4k+3)(k+1)\ h} = \frac{\left[(7x1.25+6) - 1.25x800\right]\ x8\ x\ 8}{24\ (4x1.25+3)(1.25+1)x5} = 233\ Kg$$

$$V_C = \frac{\left[(7k+6)w_2 - kw_1\right]L}{4\ (4k+3)} = \frac{\left[(7x1.25+6)x600 - 1.25x800\right]x8}{4x\ (4x1.25+3)} = 1{,}962.5\ Kg$$

$$M_D = -\frac{(7k+6) - kw_2}{24(4k+3)(k+1)}\ L^2 = -1{,}637\ Kgm \quad ; \quad M_{EB} = \frac{(w_1-w_2)}{12\ (k+1)}\ L^2 = 474\ Kgm$$

$$M_{ED} = -\frac{w_1(6k^2+13k+6) + w_2k\ (6k+5)}{24\ (4k+3)\ (k+1)}\ L^2 = -5{,}137\ Kgm$$

$$M_{EF} = -\frac{w_2(6k^2+13k+6)+w_1k(6k+5)}{24\ (4k+3)\ (k+1)}L^2 = -4{,}663\ Kgm$$

$$M_F = -\frac{(7k+6)w_2 - kw_1}{24\ (4k+3)\ (k+1)}\ L^2 = -1{,}163\ Kgm$$

Ubicación del punto de máximo momento, desde el extremo (D)

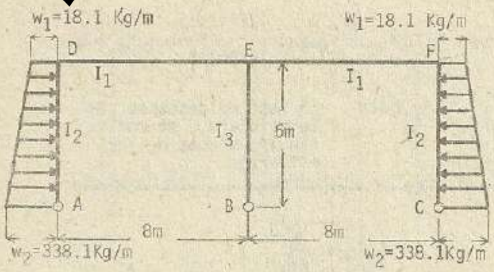
$$V_A = w_1x_m \longrightarrow x_m = \frac{V_A}{w_1} = \frac{2{,}762.5}{800} = 3.453\ m. \quad \therefore\ M_{max} = M_D + V_Ax_m - \frac{w_1x_m^2}{2}$$

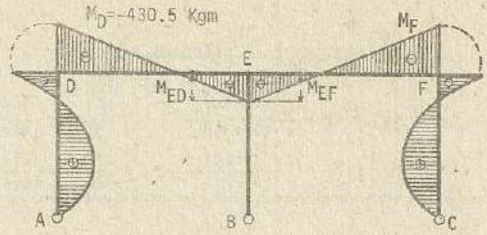
$$= 3{,}132.6\ Kgm.$$

Desde el extremo (F) ; $V_C = w_2x_m \longrightarrow x_m = \frac{V_C}{w_2} = \frac{1{,}962.5}{600} = 3.27\ m.$

$$\therefore\ M_{máx.} = M_F + V_Cx_m - \frac{w_2x_m^2}{2} = -1{,}163 + 6417.4 - 3207.9$$

$$= 2{,}046.5\ Kgm$$





Altura útil (d_1) del miembro DE : 48 cm.

(d_2) del miembro soporte : 44 cm.

$$\frac{d_1}{d_2} = \frac{48}{44} = 1.1 \rightarrow \frac{I_1}{I_2} = \left(\frac{d_1}{d_2}\right)^3 = (1.1)^3 = 1.33$$

$$k_1 = \frac{I_1 h}{I_2 L} = 1.33 \times \frac{6}{8} = 1$$

$$M_D = M_F = -\frac{4k_1h^2}{120} \cdot \frac{(8w_1+7w_2)}{(3+4k_1)} = -\frac{4x1x6x6}{120} \cdot \frac{(8x18.1+338.1)}{(3+4x1)} = -430.5 \text{ Kgm}$$

$$M_{ED} = M_{EF} = \frac{2k_1h^2}{120} \cdot \frac{(8w_1+7w_2)}{(3+4k_1)} = \frac{2x1x6x6}{120} \cdot \frac{(8x18.1+7x338.1)}{(3+4x1)} = 215.3 \text{ Kgm}$$

$$M_{EB} = 0$$

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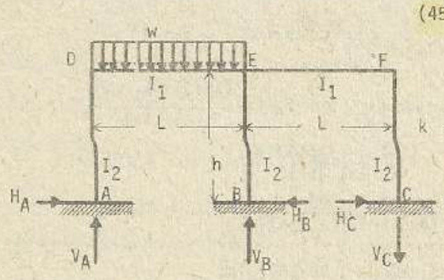
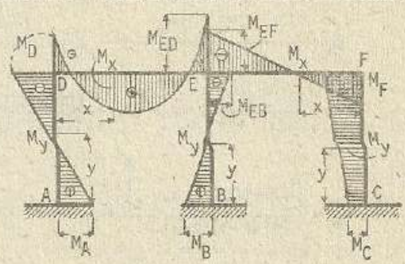
(457)



$$k = \frac{I_1 h}{I_2 L}$$



$$H = (k+1)(6k^2 + 9k+1) \quad ; \quad T = (6k^2 + 9k+1)$$

$$H_A = \frac{(7k^2 + 9k+1)}{8H} \cdot \frac{wL^2}{h} \; ; \quad H_B = \frac{(8k+1)}{8T} \cdot \frac{wL^2}{h} \; ; \quad H_C = \frac{k^2}{8H} \cdot \frac{wL^2}{h}$$

$$V_A = \frac{(21k^3+56k^2+40k+4)wL}{8H} \; ; \quad V_B = \frac{(5k+4)\;kwL}{8(k+1)} \; ; \quad V_C = \frac{(3k^2+5k+1)wL}{8H}$$

$$M_A = \frac{(6k^2+8k+1)wL^2}{24H} \; ; \quad M_B = \frac{(9k+1)\;wL^2}{24T} \; ; \quad M_C = \frac{kwL^2}{24H}$$

$$M_D = -\frac{(15k^2+19k+2)\;wL^2}{24H} \; ; \quad M_{ED} = -\frac{(9k^3+27k^2+19k+2)\;wL^2}{24H}$$

$$M_{EF} = -\frac{(9k^2+12k+2)\;kwL^2}{24H} \; ; \quad M_{EB} = -\frac{(15k+2)wL^2}{24T} \; ; \quad M_F = \frac{(3k+1)wL^2}{24H}$$

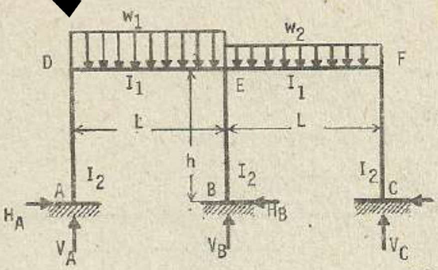
Momentos por flexión en las columnas :

AD $\longrightarrow$ $M_y = M_A - H_A y$; BE $\rightarrow$ $M_y = M_B - H_B y$; CF $\longrightarrow$ $M_y = M_C + H_C y$

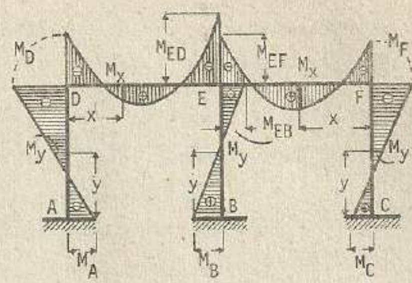
Momentos por flexión en las vigas: DE $\rightarrow$ $M_x = M_A - H_A h + V_A x - \frac{wx^2}{2}$

EF $\rightarrow$ $M_x = M_C + H_C h - V_C x$

(458)



$$k = \frac{I_1 h}{I_2 L}$$



otros datos (457)

$$H_A = \frac{(7k^2+9k+1)w_1L^2 - k^2w_2L^2}{8Hh} \quad ; \quad H_B = \frac{(8k+1)\ (w_1-w_2)\ L^2}{8Th}$$

$$H_C = \frac{(7k^2+9k+1)w_2L^2 - K^2w_1L^2}{8Hh} \quad ; \quad V_A = \frac{(21k^3+56k^2+40k+4)w_1L - (3k^2+5k+1)kw_1L}{8H}$$

$$V_C = \frac{(21k^3+56k^2+40k+4)w_2L - (3k^2+5k+1)kw_1L}{8H}$$

$$V_B = \frac{(5k+4)(w_1+w_2)\ L}{8\ (k+1)}$$

$$M_A = \frac{(6k^2+8k+1)w_1L^2 + kw_2L^2}{24H} \quad ; \quad M_B = \frac{(9k+1)(w_1-w_2)L^2}{24T} \quad ; \quad M_C = \frac{(6k^2+8k+1)w_2L^2+kw_1L^2}{24H}$$

$$M_D = -\frac{(15k^2+19k+2)w_1L^2 - (3k+1)kw_2L^2}{24H} \quad ; \quad M_{EB} = -\frac{(15k+2)\ (w_1 - w_2)L^2}{24T}$$

$$M_{EF} = -\frac{(9k^2+12k+2)w_1L^2 + (9k^3+27k^2+19k+2)\ w_2L^2}{24H}$$

$$M_F = -\frac{(15k^2+19k+2)w_2L^2 - (3k+1)\ kw_1L^2}{24H}$$

$$M_{ED} = -\frac{(9k^3+27k^2+19k+2)w_1L^2 + k\ (9k^2+12k+2)\ w_2L^2}{24H}$$

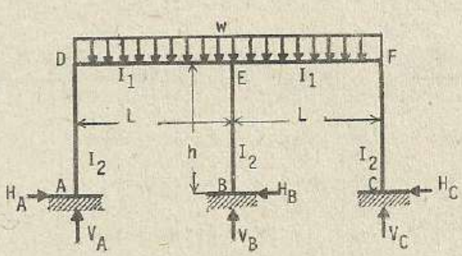
Momentos por flexión en las columnas, AD $\longrightarrow$ $M_y = M_A - H_Ay$

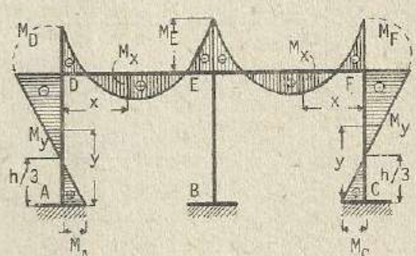
BE $\longrightarrow$ $M_y = M_B - H_By$; CF $\longrightarrow$ $M_y = M_C - H_Cy$

Momentos por flexión en las vigas, DE $\longrightarrow$ $M_x = M_A - H_Ah + V_Ax - \frac{w_1x^2}{2}$

FE $\longrightarrow$ $M_x = M_C - H_Ch + V_Cx - \frac{w_2x^2}{2}$

En caso donde $w_1 = w_2 = w$





$$H_A = H_C = \frac{wL^2}{8\ (k+1)h} \quad ; \quad H_B = 0 \quad ; \quad V_A = V_C = \frac{(3k+4)wL}{8(k+1)} \quad ; \quad V_B = \frac{(5k+4)wL}{4(k+1)}$$

$$M_A = M_C = \frac{wL^2}{24\ (k+1)} \quad ; \quad M_B = 0 \quad ; \quad M_D = M_F = \frac{-wL^2}{12(k+1)} \ ; \ M_{ED} = M_{EF} = -\frac{(3k+2)\ wL^2}{24\ (k+1)}$$

$$M_{EB} = 0$$

Esfuerzos de momentos por flexión en las columnas

AD y CF $\longrightarrow M_y = M_A - H_A y = M_C - H_C y$

momentos en las vigas : DE y FE $\longrightarrow M_x = M_A - H_A h - V_A x - \frac{wx^2}{2}$

Ubicación del punto donde se genera el máximo momento de doblado o flexión

$$x_m = \frac{(5k+4)}{8(k+1)} \cdot L \longrightarrow M_{máx} = \frac{(27k^2+40k+6)}{384(k+1)^2} \cdot wL^2$$

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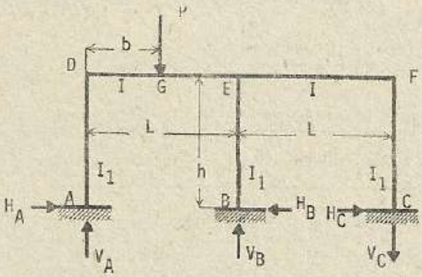
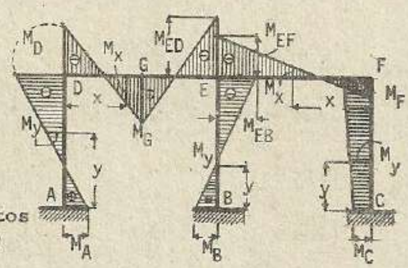
(459)



$$k = \frac{I_h}{I_1 L}$$

otros datos ver (457)



$$H_A = Pb\left(\frac{b}{L} - 1\right)\frac{\left[\frac{b}{L}(9k^2+13k+1) - (15k^2+20k+2)\right]}{2Hh} \; ; \quad H_B = -Pb\left(\frac{b}{L} - 1\right)\frac{\left(\frac{b}{L} + 12k+1\right)}{2Th}$$

$$H_C = -Pb\left(\frac{b}{L} - 1\right)\frac{\left[\frac{b}{L}(9k^2+14k+2) - (3k^2+7k+1)\right]}{2Hh}$$

$$V_A = P\left(1 + \frac{b\left[\frac{b^2}{L^2}(3k^3+18k^2+22k+4) - 3\frac{b}{L}(7k^2+11k+2) - 3k(5k^2+9k+3)\right]}{2HL}\right)$$

$$V_B = -Pb\frac{\left[\frac{b^2}{L^2}(k+4) - 6\frac{b}{L} - 3k\right]}{2(k+1)L} \; ; \quad V_C = -Pb\left(\frac{b}{L} - 1\right)\frac{3\left[\frac{b}{L}(k^3+5k^2+5k) + (k^3 - 2k)\right]}{2HL}$$

$$M_A = Pb\frac{\left[\frac{b^2}{L^2}(2k^2+2k-1) - \frac{b}{L}(6k^2+7k-1) + (4k^2+5)\right]}{2H}$$

$$M_B = -Pb\left(\frac{b}{L} - 1\right)\frac{\left[-\frac{b}{L}(k+1) + (5k+1)\right]}{2T}$$

$$M_C = Pb\left(\frac{b}{L} - 1\right)\frac{\left[\frac{b}{L}(4k^2+7k+2) - (2k^2+4k+1)\right]}{2H}$$

$$M_D = \frac{Pb\left[-\frac{b^2}{L^2}(7k^2+11k+2) + \frac{b}{L}(18k^2+26k+4) - (11k^2+15k+2)\right]}{2H}$$

$$M_{ED} = Pb\left(\frac{b}{L} - 1\right)(k+2)\frac{\left[\frac{b}{L}(3k^2+5k+1) + k(3k+2)\right]}{2H} \; ; \quad M_{EB} = Pb\left(\frac{b}{L} - 1\right)\frac{\left[\frac{b}{L}(k+2) + 7k\right]}{2T}$$

$$M_{EF} = Pb\left(\frac{b}{L} - 1\right)\frac{\left[\frac{b}{L}(k+2)(3k+4)k + k(3k^2+k-3)\right]}{2H}$$

$$M_F = -Pb\left(\frac{b}{L} - 1\right)\frac{\left[\frac{b}{L}(5k^2+7k) - (k^2+3k)\right]}{2H}$$

$$M_G = \frac{Pb\left[\frac{b^3}{L^3}(3k^3+18k^2+22k+4) - 4\frac{b^2}{L^2}(7k^2+11k+2) - \frac{b}{L}(15k^3+9k^2-17k-4) + k(12k^2+19k+5)\right]}{2H}$$

Momentos en las columnas, AD $\longrightarrow M_y = M_A - H_A y$; BE $\longrightarrow M_y = M_B - H_B y$

CF $\longrightarrow M_y = M_C + H_C y$

Momentos en las vigas : DE $\longrightarrow$ $M_x = M_D + V_A x$ $\quad x \leqslant b$

$$M_x = M_D + V_A x - P\,(x-b) \qquad x \geqslant b$$

EF $\longrightarrow$ $M_x = M_F - V_C x$

Si $b = \frac{L}{2}$; $H_A = PL\,\frac{(21k^2+27k+3)}{16Hh}$; $H_B = PL\,\frac{(24k+3)}{16Th}$; $H_C = PL\cdot\frac{3k^2}{16Hh}$

$V_A = P\,\frac{(39k^3+108k^2+80k+8)}{16H}$; $V_B = P\,\frac{(11k+8)}{16\,(k+1)}$; $V_C = P\cdot\frac{3\,(3k^3+5k^2+k)}{16H}$

$M_A = PL\,\frac{(6k^2+8k+1)}{16H}$; $M_B = PL\cdot\frac{(9k+1)}{16T}$; $M_C = PL\cdot\frac{k}{16H}$; $M_D = -PL\cdot\frac{(15k^2+19k+2)}{16H}$

$M_{ED} = -PL\cdot\frac{(k+2)(9k^2+9k+1)}{16H}$; $M_{EF} = -PL\cdot\frac{k(9k^2+12k+2)}{16H}$; $M_{EB} = -PL\cdot\frac{(15k+2)}{16T}$

$M_F = PL\cdot\frac{k\,(3k+1)}{16H}$; $M_G = PL\cdot\frac{39k^3+78k^2+42k+4}{32H}$

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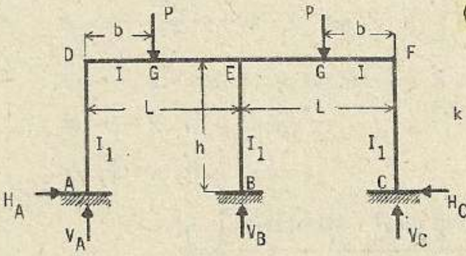
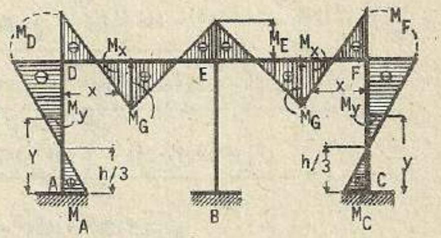
(460)



$$k = \frac{I_h}{I_1 L}$$



$H_A = H_C = 3Pb\cdot\frac{(\frac{b}{L}-1)^2}{2\,(k+1)\,h}$; $H_B = 0$; $V_A = V_C = P\left[-\frac{b\left[\frac{b^2}{L^2}(k+4)-6\frac{b}{L}-3k\right]}{2\,(k+1)\,L}\right]$

$V_B = -Pb\,\frac{\left[\frac{b^2}{L^2}(k+4)-6\frac{b}{L}-3k\right]}{(k+1)L}$; $M_A = M_C = Pb\cdot\frac{(\frac{b}{L}-1)^2}{2\,(k+1)}$; $M_B = 0$

$M_D = M_F = -Pb\cdot\frac{(\frac{b}{L}-1)^2}{(k+1)}$; $M_E = Pb\,(\frac{b}{L}-1)\cdot\frac{\left[\frac{b}{L}(k+2)+k\right]}{2(k+1)}$

Si $b = \frac{L}{2}$

$H_A = H_C = \frac{3}{16\,(k+1)}\cdot P\cdot\frac{L}{h}$; $H_B = 0$; $V_A = V_C = \frac{(5k+8)}{16(k+1)}\cdot P$

$V_B = \frac{(11k+8)}{8(k+1)}\cdot P$; $M_A = M_C = \frac{1}{16\,(k+1)}\cdot PL$; $M_B = 0$; $M_D = M_F = -\frac{1}{8(k+1)}PL$

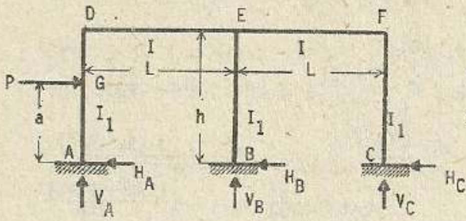
$M_E = -\frac{(3k+2)}{16(k+1)}\cdot PL$; $M_G = \frac{(5k+4)}{32(k+1)}\cdot PL$

Esfuerzos de momentos por flexión o doblado en las columnas :

AD y CF $\longrightarrow$ $M_y = M_A - H_A y$

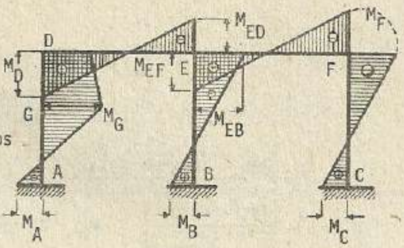
Momentos en las vigas : DE y EF $\longrightarrow$ $M_x = M_A - H_A h + V_A x$ $\qquad x \leqslant b$

$$M_x = M_A - H_A h + V_A x - P\,(x-b) \qquad x \geqslant b$$



$K = \frac{I\,h}{I_1 L}$

Otros datos (457)



$$H_A = P\left\{1 - \frac{a^2\left[(72k^3+144k^2+69k+6) - \frac{a}{h}(48k^3+81k^2+29k+2)\right]}{6h^2H}\right\}$$

$$H_B = P\,\frac{a^2\left[3(12k^2+12k+1) - \frac{a}{h}(24k^2+12k+1)\right]}{6h^2T}$$

$$H_C = P\,\frac{a^2\left[(36k^3+72k^2+30k+3) - \frac{a}{h}(24k^3+45k^2+13k+1)\right]}{6h^2H}$$

$$V_A = P\,\frac{a^2\left[-6k^3 - 6k^2 + 3k + \frac{a}{h}(9k^3+13k^2+k)\right]}{2hLH}$$

$$V_B = P\,\frac{a^2\left[-(18k^2+27k-9)k + \frac{a}{h}(18k^2+27k+3)k\right]}{2hLH}$$

$$V_C = P\,\frac{a^2\left[3(4k^2+7k+2)k - \frac{a}{h}(9k^2+14k+2)k\right]}{2hLH}$$

$$M_A = -P\left\{1 - \frac{a^2\left[3(18k^3+41k^2+24k+2) - \frac{a}{h}(24k^3+48k^2+23k+2)\right]}{6hH}\right\}$$

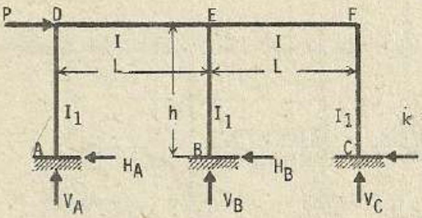
$$M_B = P\,\frac{a^2\left[3(6k^2+7k+1) - \frac{a}{h}(12k^2+9k+1)\right]}{6hT} \quad ; \quad M_D = P\,\frac{a^2\left[-6k^3-7k^2+k+\frac{a}{h}(8k^3+11k^2+k)\right]}{2hH}$$

$$M_C = P\,\frac{a^2\left[(18k^3+39k^2+21k+3) - \frac{a}{h}(12k^3+24k^2+10k+1)\right]}{6hH}$$

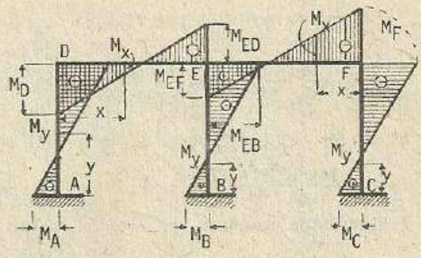
$$M_{ED} = -P\,\frac{a^2(k+2)\left(1 + k\frac{a}{h}\right)k}{2hH} \quad ; \quad M_{EF} = P\,\frac{a^2k\left[(6k^2+10k+3) - \frac{a}{h}(5k^2+7k+1)\right]}{2hH}$$

$$M_{EB} = -P\,\frac{a^2k\left[(6k+5) - \frac{a}{h}(4k+1)\right]}{2hT} \quad ; \quad M_F = -P\,\frac{a^2k\left[(6k^2+11k+3) - \frac{a}{h}(4k^2+7k+1)\right]}{2hH}$$

(462)



$k = \frac{I\,h}{I_1 L}$



$(6k^2+9k+1) = T$

$H_A = \frac{(12k^2+15k+2)}{6T} \cdot P$; $H_B = \frac{(6k^2+12k+1)}{3T} \cdot P$; $H_C = \frac{(12k^2+15k+2)}{6T} \cdot P$

$V_A = -\frac{(3k+4)\,k}{2T} \cdot \frac{Ph}{L}$; $V_B = 0$; $V_C = \frac{(3k+4)}{2T} \cdot \frac{Ph}{L}$; $M_A = -\frac{(6k^2+9k+2)}{6T} \cdot Ph$

$M_B = \frac{(3k^2+6k+1)}{3T} \cdot Ph$; $M_C = \frac{(6k^2+9k+2)}{6T} \cdot Ph$; $M_D = \frac{(k+1)k}{T} \cdot Ph$

$M_{ED} = -\frac{k\,(k+2)}{2T} \cdot Ph$; $M_{EF} = \frac{(k+2)k}{2T} \cdot Ph$; $M_{EB} = -\frac{(k+2)k}{T} \cdot Ph$; $M_F = -\frac{(k+1)k}{T} \cdot Ph$

Momentos por doblado en las columnas, AD ⟶ $M_y = M_A + H_A y$

BE ⟶ $M_y = M_B - H_B y$; CF ⟶ $M_y = M_C - H_C y$

Momentos en las vigas : DE ⟶ $M_x = M_A + H_A h + V_A x$

FE ⟶ $M_x = M_C - H_C h + V_C x$

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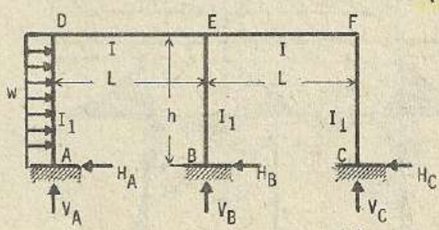
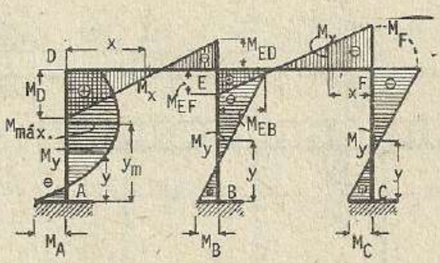
(463)





$k = \frac{I\,h}{I_1 L}$; otros datos (457)

$H_A = \frac{32k^3+83k^2+58k+6}{8H} \cdot wh$; $H_B = \frac{8k^3+20k^2+13k+1}{8H} \cdot wh$; $H_C = \frac{8k^3+17k^2+9k+1}{8H} \cdot wh$

$V_A = -\frac{(k^2+5k+5)}{8H} \cdot \frac{wh^2}{L}$; $V_B = \frac{(6k^2+9k+1)k}{8H} \cdot \frac{wh^2}{L} = \frac{k}{8\,(k+1)} \cdot \frac{wh}{L}$

$V_C = \frac{(7k^2+14k+6)k}{8H} \cdot \frac{wh^2}{L}$; $M_A = -\frac{24k^3+64k^2+47k+6}{24H} \cdot wh^2$; $M_B = \frac{12k^2+19k+3}{24T} \cdot wh^2$

$M_C = \frac{12k^3+28k^2+18k+3}{24H} \cdot wh^2$; $M_D = \frac{(5k+7)k}{24H} \cdot wh^2$ $M_{ED} = -\frac{(k+2)(3k+4)k}{24H} \cdot wh^2$

$M_{EF} = \frac{(9k^2+19k+9)k}{24H} \cdot wh^2$; $M_{EB} = \frac{(12k+17)k}{24T} \cdot wh^2$; $M_F = -\frac{(12k^2+23k+9)k}{24H} \cdot wh^2$

Esfuerzos de momentos por flexión en las columnas,

$$AD \longrightarrow M_y = M_A + H_A y - \frac{wy^2}{2} \quad ; \quad BE \longrightarrow M_y = M_B - H_B y \quad ; \quad CF \longrightarrow M_y = M_C - H_C y$$

Ubicación del punto donde se genera el momento máximo,

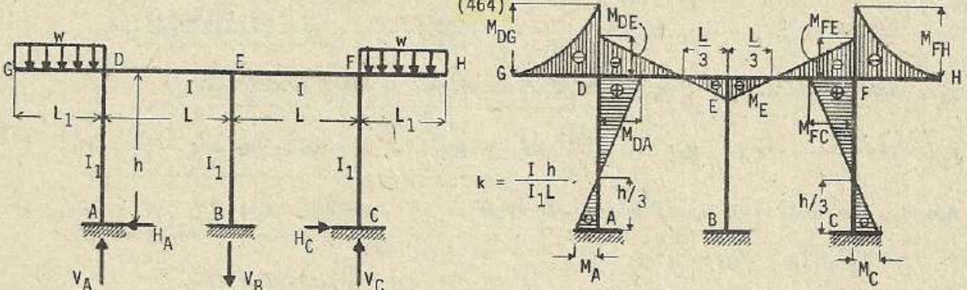
$$y_m = \frac{16k^3+27k^2+22k+2}{8H} \cdot h \quad \therefore \quad M_{máx.} = \left[\frac{(5k+7)k}{24H} + \frac{(16k^3+37k^2+22k+2)^2}{128(k+1)^2\,(6k^2+9k+1)^2} \right] wh^2$$

Esfuerzos de momentos por flexión en las vigas:

$$DE \longrightarrow M_x = M_A + H_A h - \frac{wh^2}{2} + V_A x$$

$$FE \longrightarrow M_x = M_C - H_C h + V_C x$$

(464)

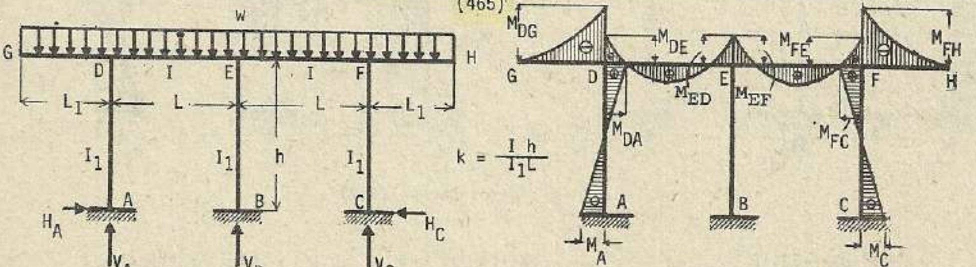


$$H_A = H_C = \frac{3wL_1^2}{4(1+k)h} \quad ; \quad H_B = 0 \quad ; \quad V_A = V_C = \frac{\left[\frac{L}{L_1}(4k+4) + 3k\right]}{4\,(1+k)\,L} wL_1^2$$

$$V_B = \frac{3kwL_1^2}{2\,(1+k)L} \quad ; \quad M_A = M_C = -\frac{wL_1^2}{4\,(1+k)} \quad ; \quad M_B = 0 \quad ; \quad M_{DE} = M_{FE} = -\frac{k}{2\,(1+k)} wL_1^2$$

$$M_{DG} = M_{FH} = -\frac{wL_1^2}{2} \quad ; \quad M_{DA} = M_{FC} = \frac{wL_1^2}{2(1+k)} \quad ; \quad M_{EF} = M_{ED} = \frac{k}{4\,(1+k)} wL_1^2 \quad M_{EB} = 0$$

(465)

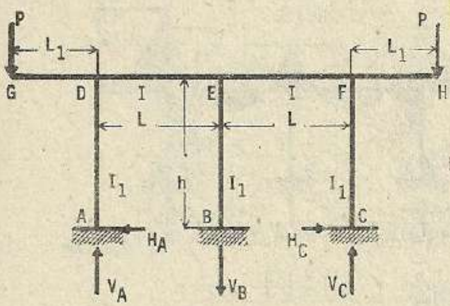


$$H_A = H_C = w\left[\frac{L^2 - 6L_1^2}{8(1+k)h}\right] \quad ; \quad V_A = V_C = wL\,\frac{(3k+4)}{8\,(k+1)} + \frac{\left[\frac{L}{L_1}(4k+4)+3k\right]}{4\,(1+k)L} wL_1^2$$

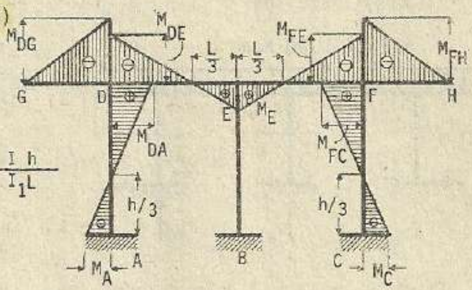
$$V_B = wL\,\frac{4+5k}{4(1+k)} - wL_1^2\,\frac{3k}{2(1+k)L} \quad ; \quad M_A = M_C = \frac{wL^2}{24\,(1+k)} - \frac{wL_1^2}{4(1+k)} ; \quad M_{DG} = M_{FH} = -\frac{wL_1^2}{2}$$

$$M_{DE} = M_{FE} = -\frac{wL^2}{12(1+k)} - \frac{kwL_1^2}{2(1+k)} \quad ; \quad M_{DA} = M_{FC} = (M_{DG} - M_{DE}) = (M_{FH} - M_{FE})$$

$$M_E = -\frac{(2+3k)}{24(1+k)} wL_1^2 + \frac{k}{4\,(1+k)} wL_1^2$$



(466)



$$k = \frac{I\ h}{I_1 L}$$

$$H_A = H_C = \frac{3PL_1}{2(1+k)\ h} \quad ; \quad H_B = 0 \quad ; \quad V_A = V_C = \frac{\left[\frac{L}{L_1}(2+k) + 3k\right]}{2\ (1+k)\ L} PL_1$$

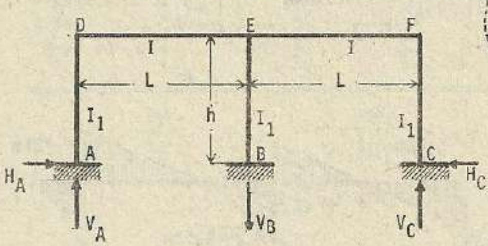
$$V_B = \frac{3kPL_1}{(1+k)L} \quad ; \quad M_A = M_C = -\frac{PL_1}{2\ (1+k)} \quad ; \quad M_B = 0 \quad ; \quad M_{DE} = M_{FE} = -\frac{KPL_1}{(1+k)}$$

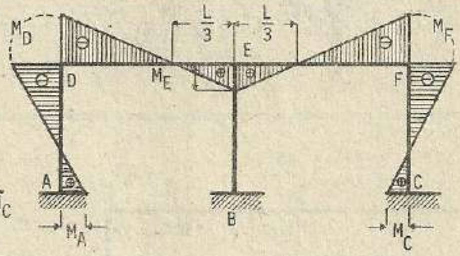
$$M_{DG} = M_{FH} = -PL_1 \quad ; \quad M_{DA} = M_{FC} = \frac{PL_1}{(1+k)} \quad ; \quad M_{ED} = M_{EF} = \frac{kPL_1}{2(1+k)}$$

$$M_{EB} = 0$$

(467)

Esfuerzos por cambio de temperatura





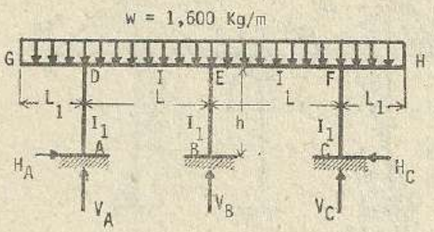
$$k = \frac{I\ h}{I_1 L} \quad ; \quad H_A = H_C = 3E\varepsilon tI \frac{(4k+1)}{H^2\ (k+1)k} \quad ; \quad H_B = 0 \quad ; \quad V_A = V_C = 9E\varepsilon tI \frac{1}{hL(k+1)}$$

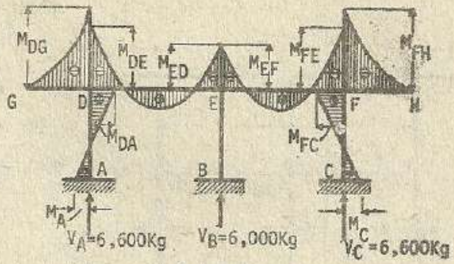
$$V_B = 18E\varepsilon tI \frac{1}{hL(k+1)}$$

$$M_A = M_C = 3E\varepsilon tI \frac{(2k+1)}{h\ (k+1)\ k} \quad ; \quad M_B = 0 \quad ; \quad M_D = M_F = -6E\varepsilon tI \frac{1}{h(k+1)}$$

$$M_E = 3E\varepsilon tI \frac{1}{h(k+1)}$$

Nota : En caso de descenso de temperatura, los signos son opuestos a las presentadas.





Datos : $L_1 = 2m.$; $L = 4m$; $h = 3m.$
$I = 0.4\ m^4$; $I_1 = 0.3\ m^4$

$$k = \frac{I\ h}{I_1 L} = \frac{0.4 \times 3}{0.3 \times 4} = 1$$

$$H_A = H_C = w\left[\frac{L^2 - 6L_1^2}{8(1+k)h}\right] = 1{,}600 \times \frac{4x4 - 6x2x2}{8(1+1)x3} = -\ 266\ Kg$$

$$V_A = V_C = w\frac{L(3k+4)}{8(k+1)} + \frac{\left[\frac{L}{L_1}(4k+4) + 3k\right]}{4(1+k)L} wL_1^2 = 1{,}600 \times \frac{4x(3x1+4)}{8\ (1+1)} + \frac{\left[\frac{4}{2}(4x1+4) + 3x1\right]}{4(1+1)x4}$$

$$\times\ 1{,}600x2x2 = 6{,}600\ Kg.$$

$$V_B = wL\frac{(4+5k)}{4(1+k)} - wL_1^2 \frac{3k}{2(1+k)L} = 1{,}600x4\ \frac{(4+5x1)}{4(1+1)} - 1{,}600x2x2 \cdot \frac{3x1}{2(1+1)x4} = 6{,}000\ Kg.$$

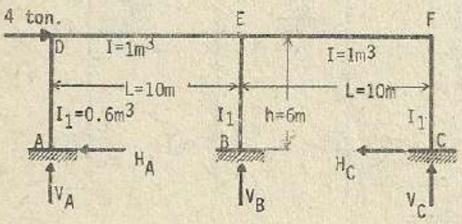
$$M_A = M_C = \frac{wL^2}{24(1+k)} - \frac{wL_1^2}{4\ (1+k)} = \frac{1{,}600x4x4}{24(1+1)} - \frac{1{,}600x2x2}{4\ (1+1)} = -\ 266\ Kg\text{-}m$$

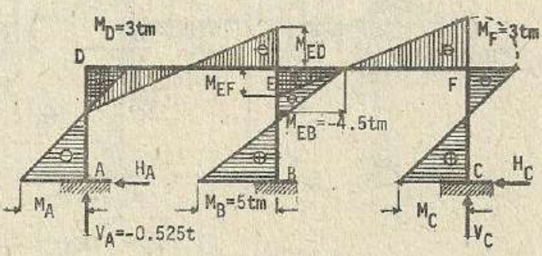
$$M_{DG} = M_{FH} = -\frac{wL_1^2}{2} = -\frac{1{,}600x2x2}{2} = -\ 3{,}200\ Kg\text{-}m\ ;\quad M_{DE} = M_{FE} = -\frac{wL^2}{12(1+k)} - \frac{kwL_1^2}{2\ (1+k)}$$

$$M_{DA} = M_{FC} = 3{,}200 - 2{,}666 = 534\ Kg\text{-}m \qquad = -\frac{1{,}600x4x4}{12(1+1)} - \frac{1x1{,}600x2x2}{2(1+1)} = -2{,}666\ Kg-m$$

$$M_E = -\frac{(2+3k)}{24(1+k)} wL^2 + \frac{k}{4\ (1+k)} wL_1^2 = -\frac{(2+3x1)}{24\ (1+1)} \times 1{,}600x4x4 + \frac{1}{4\ (1+1)} \times 1{,}600x2x2 =$$

$$-\ 1{,}866\ Kg\text{-}m$$





$$k = \frac{I\ h}{I_1 L} = \frac{1 \times 6}{0.6x10} = 1\ ;\quad H_A = \frac{(12k^2+15k+2)}{6(6k^2+9k+1)} P = \frac{(12+15+2)}{6(6+9+1)} x4 = 1.208\ t.$$

$$H_B = \frac{(6k^2+12k+1)}{3(6k^2+9k+1)} \cdot P = \frac{(6+12+1)}{3(6+9+1)} \times 4 = 1.583\ t.$$

$$H_C = \frac{(12k^2+15k+2)}{6\ (6k^2+9k+1)} \cdot P = \frac{(12+15+2)}{6(6+9+1)} \times 4 = 1.208\ t.$$

$$V_A = \frac{(3k+4)k}{2(6k^2+9k+1)} \cdot \frac{Ph}{L} = -\frac{(3+4)}{2\ (6+9+1)} \cdot \frac{4x6}{10} = -\ 0.525\ t.$$

$$V_B = 0 \quad ; \quad V_C = \frac{(3k+4)k}{2\,(6k^2+9k+1)} \; \frac{Ph}{L} = \frac{(3+4)}{2\,(6+9+1)} \times \frac{4x6}{10} = 0.525 \text{ t.}$$

$$M_A = -\frac{(6k^2+9k+2)}{6\,(6k^2+9k+1)}\,Ph = -\frac{(6+9+2)}{6(6+9+1)} x4x6 = -4.25 \text{ tm}$$

$$M_B = \frac{(3k^2+6k+1)}{3\,(6k^2+9k+1)}\,Ph = \frac{(3+6+1)}{3\,(6+9+1)} x4x6 = 5 \text{ tm.}$$

$$M_C = \frac{(6k^2+9k+2)}{6(6k^2+9k+1)}\,Ph = \frac{(6+9+2)}{6(6+9+1)} \times 4 \times 6 = 4.25 \text{ tm}$$

$$M_D = \frac{(k+1)k}{(6k^2+9k+1)}\,Ph = \frac{(1+1)x1}{(6+9+1)} x4x6 = 3 \text{ tm}$$

$$M_{ED} = -\frac{(k+2)k}{2\,(6k^2+9k+1)}\,Ph = -\frac{(1+2)}{2\,(6+9+1)} x4x6 = -2.25 \text{ tm.}$$

$$M_{EF} = \frac{(k+2)k}{2\,(6k^2+9k+1)}\,Ph = \frac{(1+2)}{2\,(6+9+1)} x4x6 = 2.25 \text{ tm.}$$

$$M_{EB} = -\frac{(k+2)k}{(6k^2+9k+1)}\,Ph = -\frac{(1+2)}{(6+9+1)} x4x6 = -4.5 \text{ tm.}$$

$$M_F = -\frac{(k+1)k}{(6k^2+9k+1)}\,Ph = -\frac{(1+1)}{(6+9+1)} x4x6 = -3 \text{ tm}$$

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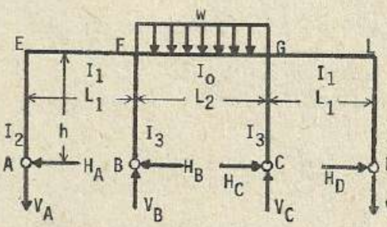
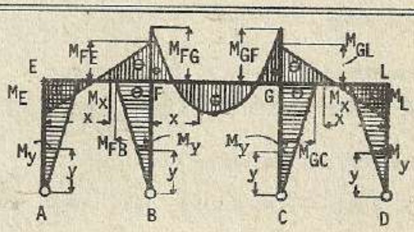
(468)





$$k_1 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} \quad ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

$$\left[\, 9 + 12k_1 + 12k_2 + 6\alpha k_1 + 2k_1k_2\,(6+4\alpha) \,\right] = J$$

$$H_A = H_D = \frac{k_1\, wL_2^2}{2Jh} \quad ; \quad H_B = H_C = \frac{(3+4k_2)\, wL_2^2}{4Jh} \quad ; \quad V_A = V_D = \frac{(3+2k_1)k_1 wL_2^2}{2JL_1}$$

$$V_B = V_C = \frac{\left[\, J + k_1 \frac{L_2}{L_1}\,(3+k_2) \,\right] wL_2}{2J} \qquad M_E = M_L = \frac{k_1 wL_2^2}{2J} \qquad M_{FE} = M_{GL} = -\frac{k_1(1+k_2) wL_2^2}{J}$$

$$M_{FG} = M_{GF} = -\frac{\left[\, 3 + 4(k_1+k_2+k_1k_2) \,\right] wL_2^2}{4J} \quad ; \quad M_{FB} = M_{GC} = -\frac{(3+4k_2)\; wL_2^2}{4J}$$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$; $\left[\, 9 + 12k^2 + 24k + 2\alpha k\,(3+4k) \,\right] = U$

$$H_A = H_D = \frac{kwL_2^2}{2Uh} \quad ; \quad H_B = H_C = \frac{(4k+3)wL_2^2}{2Uh} \quad ; \quad V_A = V_D = \frac{k\,(3+2k)wL_2^2}{2UL_1}$$

$$V_B = V_C = \frac{\left[U + k\,\frac{L_2}{L_1}\,(3+k) \right] wL_2}{2U} \quad ; \quad M_E = M_L = \frac{KwL_2^2}{2U} \quad ; \quad M_{FE} = M_{GL} = -\,\frac{k(k+1)\ wL_2^2}{U}$$

$$M_{FG} = M_{GF} = -\,\frac{(4k^2+8k+3)\ \ wL_2^2}{4U} \quad ; \quad M_{FB} = M_{GC} = -\,\frac{(4k+3)\ wL_2^2}{4U}$$

Si $L_1 = L_2 = L$, entonces $I_o = I_1$ $\qquad (20k_1k_2+12k_2+18k_1+9) = V$

$$H_A = H_D = \frac{k_1wL^2}{2Vh} \quad ; \quad H_B = H_C = \frac{(3+4k_2)\ wL^2}{4Vh} \quad ; \quad V_A = V_D = \frac{k_1(3+2k_2)wL}{2V}$$

$$V_B = V_C = \frac{(22k_1k_2+12k_2+21k_1+9)wL}{2V} \quad ; \quad M_E = M_L = \frac{k_1wL^2}{2V}$$

$$M_{FE} = M_{GL} = -\,\frac{k_1(1+k_2)\ wL^2}{V} \quad ; \quad M_{FG} = M_{GF} = -\,\frac{\left[3 + 4(k_1+k_2+k_1k_2)\right]wL^2}{4V}$$

$$M_{FB} = M_{GC} = -\,\frac{(3+4k_2)wL^2}{4V}$$

Si $L_1 = L_2 = L$, $\quad I_2 = I_3$ $\qquad (20k^2+30k+9) = W$

$$H_A = H_D = \frac{kwL^2}{2Wh} \quad ; \quad H_B = H_C = \frac{(4k+3)wL^2}{4Wh} \quad ; \quad V_A = V_D = \frac{k(2k+3)\ wL}{2W}$$

$$V_B = V_C = \frac{(22k^2+33k+9)\ wL}{2W} \quad ; \quad M_E = M_L = \frac{kwL^2}{2W} \quad ; \quad M_{FE} = M_{GL} = -\,\frac{k(k+1)wL^2}{W}$$

$$M_{FG} = M_{GF} = -\,\frac{(4k^2+8k+3)wL^2}{4W} \quad ; \quad M_{FB} = M_{GC} = -\,\frac{(4k+3)wL^2}{4W}$$

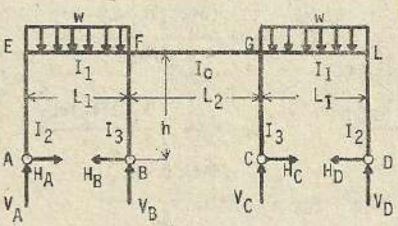
Momentos por flexión en las columnas :

AE y DL ⟶ $M_y = H_Ay = H_Dy$; BF y CG ⟶ $M_y = -H_By = -H_Cy$

Momentos por flexión en las vigas :

EF y GL ⟶ $M_x = H_Ah - V_Ax = H_Dh - V_Dx$

FG ⟶ $M_x = H_Ah - H_Bh - V_A(L_1+x) + V_Bx - \frac{wx^2}{2}$



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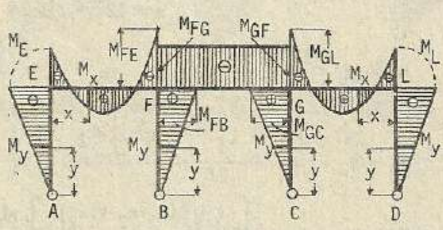

$$k_1 = \frac{I_1h}{I_3L_1} \quad ; \quad \alpha = \frac{I_oL_1}{I_1L_2} \quad ; \quad \alpha k_1 = \frac{I_oh}{I_3L_2} \quad ; \quad k_2 = \frac{I_1h}{I_2L_1}$$

Otros datos (468)

$$H_A = H_D = \frac{(3+6k_1+2\alpha k_1)wL_1^2}{4Jh} \quad ; \quad H_B = H_C = \frac{3(1+k_2)wL_1^2}{4Jh}$$

$$V_A = V_D = \frac{\left[9 + 15k_1 + 9k_2 + 6\alpha k_1 + 6k_1k_2(2+\alpha)\ wL_1\right]}{2J}$$

$$V_B = V_C = \frac{\left[9 + 9k_1 + 15k_2 + 6\alpha k_1 + 2k_1 k_2 (6+5\alpha) \right] wL_1}{2J} \quad ; \quad M_E = M_L = - \frac{(3+6k_1+2\alpha k_1) wL_1^2}{4J}$$

$$M_{FE} = M_{GL} = - \frac{(3+6k_2 + 2\alpha k_1 + 4\alpha k_1 k_2) wL_1^2}{4J} \quad ; \quad M_{FG} = M_{GF} = - \frac{(3k_2 + 2\alpha k_1 + 4\alpha k_1 k_2) wL_1^2}{4J}$$

$$M_{FB} = M_{GC} = \frac{3(1+k_2) wL_1^2}{4J}$$

Si $I_2 = I_3$, entonces $k_1 = k_2 = k$; $H_A = H_D = \frac{(3+6k+2\alpha k) wL_1^2}{4Uh}$

$$H_B = H_C = \frac{3(1+k) wL_1^2}{4Uh} \quad ; \quad V_A = V_D = \frac{\left[6(2+\alpha) k^2 + 6\alpha k + 24k + 9 \right] wL_1}{2U}$$

$$V_B = V_C = \frac{\left[2(6+5\alpha) k^2 + 6\alpha k + 24k + 9 \right] wL_1}{2U} \quad ; \quad M_E = M_L = \frac{(3+6k+2\alpha k) \cdot wL_1^2}{4U}$$

$$M_{FE} = M_{GL} = - \frac{(3+6k+2\alpha k + 4\alpha k^2) wL_1^2}{4U} \quad ; \quad M_{FG} = M_{GF} = - \frac{(3+2\alpha+4\alpha k) k wL_1^2}{4U}$$

$$M_{FB} = M_{GC} = \frac{3(1+k) wL_1^2}{4U}$$

Si $L_1 = L_2 = L$, $I_0 = I_1$; $H_A = H_D = \frac{(3+8k_1) wL^2}{4Vh}$; $H_B = H_C = \frac{3(1+k_2) wL^2}{4Vh}$

$$V_A = V_D = \frac{(18k_1 k_2 + 9k_2 + 21k_1 + 9)\ wL}{2V} \quad ; \quad V_B = V_C = \frac{(22k_1 k_2 + 15k_1 + 15k_2 + 9)\ wL}{2V}$$

$$M_E = M_L = - \frac{(3+8k_1) wL^2}{4V} \quad ; \quad M_{FE} = M_{GL} = - \frac{(3+6k_2+2k_1+4k_1 k_2)\ wL^2}{4V}$$

$$M_{FG} = M_{GF} = - \frac{(3k_2+2k_1+4k_1 k_2) wL^2}{4V} \quad ; \quad M_{FB} = M_{GC} = \frac{3(1+k_2)\ wL^2}{4V}$$

Si $L_1 = L_2 = L$, $I_2 = I_3$; $H_A = H_D = \frac{(3+8k) wL^2}{4Wh}$; $H_B = H_C = \frac{3\ (1+k) wL^2}{4Wh}$

$$V_A = V_D - \frac{3(6k^2+10k+3) wL}{2W} \quad ; \quad V_B = V_C = \frac{(22k^2+30k+9) wL}{2W} \quad ; \quad M_E = M_L = - \frac{(8k+3) wL^2}{4W}$$

$$M_{FE} = M_{GL} = - \frac{(4k^2+8k+3) wL^2}{4W} \quad ; \quad M_{FG} = M_{GF} = \frac{(4k^2+5k) wL^2}{4W} \quad ; \quad M_{FB} = M_{GC} = \frac{3(1+k) wL^2}{4W}$$

Momentos por flexión en las columnas :

AE y DL $\longrightarrow$ $M_y = - H_A y = - H_D y$ BF y CG $\longrightarrow$ $M_y = H_B y = H_C y$

Momentos por flexión en las vigas :

$$EF \text{ y } GL \longrightarrow M_x = - H_A h + V_A x - \frac{wx^2}{2}$$

$$= - H_D h + V_D x - \frac{wx^2}{2}$$

FG $\longrightarrow$ $M_x = M_{FG} = M_{GF}$ (constante)

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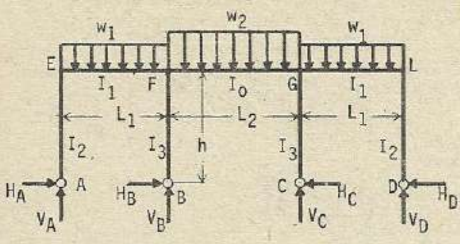

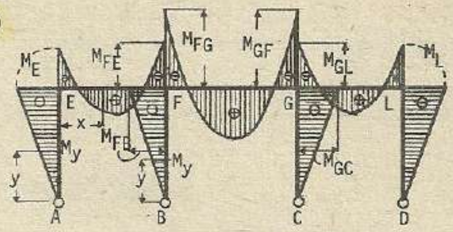

$$k_1 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{I_o h_1}{I_1 L_2} \quad ; \quad \alpha k_1 = \frac{I_o h}{I_3 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \quad ; \quad \text{otros datos (468)}$$

$$H_A = H_D = \frac{w_1 L_1^2\,(3+6k_1+2\alpha k_1) - 2w_2 L_2^2\,k_1}{4hJ} \quad ; \quad H_B = H_C = \frac{w_2 L_2^2(3+4k_2) - 3w_1 L_1^2(1+2k_2)}{4hJ}$$

$$V_A = V_D = \frac{\left[9+9k_2+15k_1+6\alpha k_1+6k_1k_2(2+\alpha)\right] w_1 L_1^2 - k_1(3+2k_2)\,w_2 L_2^2}{2JL_1}$$

$$V_B = V_C = \frac{w_1 L_1\left[9+15k_2+9k_1+6k_1\alpha+2k_1k_2(6+5\alpha)\right] + w_2 L_2\left[J + k_1\frac{L_2}{L_1}(3+k_2)\right]}{2J}$$

$$M_E = M_L = -\frac{w_1 L_1^2(3+6k_1+2\alpha k_1) - 2w_2 L_2^2 k_1}{4J} \quad ; \quad M_{FB} = M_{GC} = -\frac{w_2 L_2^2(3+4k_2) - 3w_1 L_1^2(1+2k_2)}{4hJ}$$

$$M_{FE} = M_{GL} = M_E + V_A L_1 - w_1\frac{L_1^2}{2} \quad ; \quad M_{FG} = M_{GF} = M_E + V_A L_1 - w_1\frac{L_1^2}{2} + M_{FB}$$

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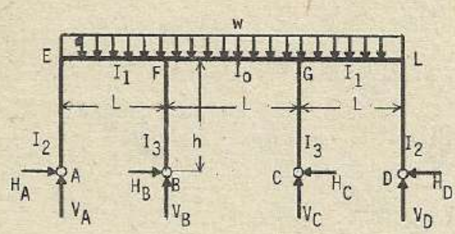
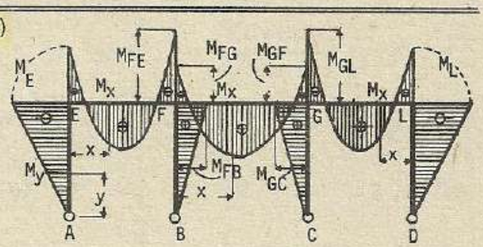




$$k_1 = \frac{I_2 h}{I_3 L} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L} \quad ; \quad \text{otros datos (468)}$$

$$H_A = H_D = \frac{(6k_1+3)}{4V}\cdot\frac{wL^2}{h} \quad ; \quad H_B = H_C = \frac{k_2}{2V}\cdot\frac{wL^2}{h} \quad ; \quad V_A = V_D = \frac{(9k_2+18k_1+16k_1k_2+9)\;wL}{2V}$$

$$V_B = V_C = \frac{(27k_2+36k_1+44k_1k_2+18)wL}{2V} \quad ; \quad M_E = M_L \; \frac{(6k_1+3)wL^2}{4V} \quad ; \quad M_{FB} = M_{GC} = \frac{K_2}{2V}\,wL^2$$

$$M_{FE} = M_{GL} = -\frac{6(k_2+k_1) + 8k_1k_2 + 3}{4V}\cdot wL^2 \qquad M_{FG} = M_{GF} = -\frac{4k_2+6k_1+8k_1k_2+3}{4V}\,wL^2$$

Si $I_2 = I_3$, $k_1 = k_2 = k$

$$H_A = H_D = \frac{(6k+3)\;\;wL^2}{4Wh} \quad ; \quad H_B = H_C = \frac{k}{2Wh}\cdot\frac{wL^2}{h} \quad ; \quad V_A = V_C = \frac{(16k^2+27k+9)}{2W}\cdot wL$$

$$V_B = V_C = \frac{44k^2+63k+18}{2W}\cdot wL \; ; \; M_E = M_L = -\frac{(6k+3)wL^2}{4W} \quad ; \quad M_{FB} = M_{GC} = \frac{k}{2W}\cdot wL^2$$

$$M_{FE} = M_{GL} = -\frac{(8k^2+12k+3)}{4W}\cdot wL^2 \quad ; \quad M_{FG} = M_{GF} = -\frac{(8k^2+10k+3)}{4W}\cdot wL^2$$

Esfuerzos de momentos por flexión en las columnas

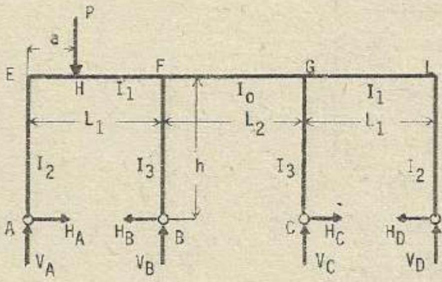
AE y DL ——— $M_y = -H_A y = -H_D y$; BF y CG ——— $M_y = H_B y = H_C y$

Momentos en las vigas :

EF y LG ——— $M_x = -H_A h + V_A x - \frac{wx^2}{2}$

$= -H_A h + V_D x - \frac{wx^2}{2}$

FG ——— $M_x = -H_A h + H_B h + V_A (L+x) + V_B x - w \frac{(L+x)^2}{2}$



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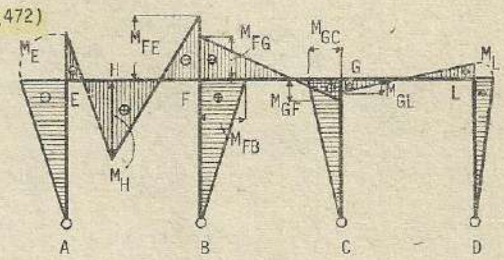

$k_1 = \frac{I_1 h}{I_3 L_1}$; $\alpha = \frac{I_0 L_1}{I_1 L_2}$; $\alpha k_1 = \frac{I_0 h}{I_3 L_2}$; $k_2 = \frac{I_1 h}{I_2 L_1}$; $m = \left(1 - \frac{a}{L_1}\right)$

$n = 2(1+2\alpha)(k_1 + k_2) + 3(2+\alpha)$; $\lambda_1 = \left(1 - \frac{a}{L_1}\right)(3 + 2\alpha k_1 + 2k_1) + 2k_1$

$\lambda_2 = 9 + 12k_1 + 12k_2 + 6\alpha k_1 + 2k_1 k_2 (6+4\alpha)$

$H_A = \frac{3Pam}{2h}\left[\frac{\lambda_1}{\lambda_2} + \frac{1+\alpha m}{n}\right]$; $H_B = \frac{Pam}{2h}\left[\frac{3(3+2k_2)\frac{a}{L_1} + 6k_2}{\lambda_2} + \frac{3(1+\alpha m)}{n}\right]$

$H_C = \frac{Pam}{2h}\left[\frac{3(3+2k_2)\frac{a}{L_1} + 6k_2}{\lambda_2} - \frac{3(1+\alpha m)}{n}\right]$

$H_D = \frac{3Pam}{2h}\left[\frac{\lambda_1}{\lambda_2} - \frac{1+\alpha m}{n}\right]$; $M_E = -\frac{3Pam}{2}\left[\frac{\lambda_1}{\lambda_2} + \frac{1+\alpha m}{n}\right]$

$M_{FE} = -\frac{Pam}{2}\left\{\frac{1}{\lambda_2}(3+\alpha k_1)\left[3\frac{a}{L_1} + 2k_2\left(1+\frac{a}{L_1}\right)\right] + \frac{1}{n}\left[3\left(1+\frac{\alpha a}{L_1}\right) + 2\alpha\left(1+\frac{a}{L_1}\right)(k_1+k_2)\right]\right\}$

$M_{FG} = -\frac{Pa\alpha m}{2}\left\{\frac{1}{\lambda_2} 2k_1 \left[(3+2k_1)\frac{a}{L_1} + 2k_1\right] + \frac{1}{n}\left[2(3+k_1+k_2)\frac{a}{L_1} + 2(k_1+k_2) - 3\right]\right\}$

$M_{FB} = \frac{Pam}{2}\left\{\frac{1}{\lambda_2}\left[3(3+2k_2)\frac{a}{L_1} + 6k_1\right] + \frac{1}{n}\left[3(1+\alpha m)\right]\right\}$

$M_{GF} = \frac{Pa\alpha m}{2}\left\{\frac{1}{n}\left[2(3+k_1+k_2)\frac{a}{L_1} + 2(k_1+k_2) - 3\right] - \frac{1}{\lambda_2}\left[2k_1(3+2k_2)\frac{a}{L_1} + 2k_2\right]\right\}$

$M_{GL} = \frac{Pam}{2}\left\{\frac{1}{n}\left[3\left(1+\alpha\frac{a}{L_1}\right) + 2\alpha\left(1+\frac{a}{L_1}\right)(k_1+k_2)\right] - \frac{1}{\lambda_2}\left[(3+2\alpha k_1)\left(3\frac{a}{L_1} + 2k_2\left(1+\frac{a}{L_1}\right)\right)\right]\right\}$

$$M_{GC} = \frac{Pam}{2}\left[\frac{1}{\lambda_2}\left[3\,(3+2k_2)\,\frac{a}{L_1} + 6k_2\right] - \frac{1}{n}\left[3\,(1+\alpha m)\right]\right]$$

$$M_L = -\frac{3Pam}{2}\left\{\frac{1}{\lambda_2}\left[m\,(3+2\alpha k_1+2k_1) + 2k_1) + 2k_2\right] - \frac{1}{n}\,(1+\alpha m)\right\}; \quad V_A = Pm + \frac{M_{FE} - M_E}{L_1}$$

$$V_B = P\,\frac{a}{L_1} + \frac{M_E - M_{FE}}{L_1} + \frac{M_{GF} - M_{FG}}{L_2}; \quad V_C = \frac{M_{GF} - M_{FG}}{L_2} + \frac{M_{GL} - M_L}{L_1} \quad ; \quad V_D = \frac{M_{GL} - M_L}{L}$$

Si $L_1 = L_2 = L$, $I_2 = I_3$, $I_0 = I_1$, o sea $k_1 = k_2 = k$, $\alpha = 1$

$$H_A = \frac{3Pa\,(1 - \frac{a}{L})}{2h}\left[\frac{3(1+2k) - (3+4k)\,\frac{a}{L}}{(20k^2+30k+9)} + \frac{2 - a\,\frac{\alpha}{L}}{3(4k+3)}\right]$$

$$H_B = \frac{Pa\,(1 - \frac{a}{L})}{2h}\left[\frac{3(3+2)\,\frac{a}{L} + 6k}{(20k^2+30k+9)} + \frac{\left(2 - \frac{a}{L}\right)}{(4k+3)}\right]$$

$$H_C = \frac{Pa\,(1 - \frac{a}{L})}{2h}\left[\frac{3\,(3+2k)\,\frac{a}{L} + 6k}{(20k^2+30k+9)} - \frac{(2 - \frac{a}{L})}{(4k+3)}\right]$$

$$H_D = \frac{3Pa\,(1 - \frac{a}{L})}{2h}\left[\frac{3(1+2k) - (3+4k)\,\frac{a}{L}}{(20k^2+30k+9)} - \frac{(2 - \frac{a}{L})}{3(4k+3)}\right]$$

$$M_E = -\frac{3Pa\,(1 - \frac{a}{L})}{2}\left[\frac{3(1+2k) - (3+4k)\,\frac{a}{L}}{(20k^2+30k+9)} + \frac{(2 - \frac{a}{L})}{3(4k+3)}\right]$$

$$M_{FE} = -\frac{Pa\,(1 - \frac{a}{L})}{2}\left[\frac{\frac{a}{L}\,(3+2k)^2 + 2k\,(3+2k)}{(20k^2+30k+9)} + \frac{1 + \frac{a}{L}}{3}\right]$$

$$M_{FG} = -\frac{Pa\,(1 - \frac{a}{L})}{2}\left[\frac{2k\left[(3+2k)\,\frac{a}{L} + 2k\right]}{(20k^2+30k+9)} + \frac{(6+4k)\,\frac{a}{L} + 4k - 3}{3(4k+3)}\right]$$

$$M_{FB} = \frac{Pa\,(1 - \frac{a}{L})}{2}\left[\frac{3(3+2k)\cdot\frac{a}{L} + 6k}{(20k^2+30k+9)} + \frac{2 - \frac{a}{L}}{4k+3}\right]$$

$$M_{GF} = \frac{Pa\,(1 - \frac{a}{L})}{2}\left[\frac{(6+4k)\,\frac{a}{L} + 4k - 3}{3\,(4k+3)} - \frac{2k\left[(3+2k)\,\frac{a}{L} + 2k\right]}{(20k^2+30k+9)}\right]$$

$$M_{GL} = \frac{Pa\,(1 - \frac{a}{L})}{2}\left[\frac{1 + \frac{a}{L}}{3} - \frac{\frac{a}{L}\,(3+2k)^2 + 2k(3+2k)}{(20k^2+30k+9)}\right]$$

$$M_{GC} = \frac{Pa\,(1 - \frac{a}{L})}{2}\left[\frac{3(3+2k)\,\frac{a}{L} + 6k}{(20k^2+30k+9)} - \frac{2 - \frac{a}{L}}{4k+3}\right]$$

$$M_L = -\frac{3Pa(1 - \frac{a}{L})}{2}\left[\frac{3(1+2k) - \frac{a}{L}\,(3+4k)}{(20k^2+30k+9)} - \frac{2 - \frac{a}{L}}{3\,(4k+3)}\right]$$

$$V_A = P\,(1 - \frac{a}{L}) + \frac{Pa\,(1 - \frac{a}{L})}{2L}\left[\frac{-\frac{a}{L}(4k^2+24k+18) - (4k^2-12k-9)}{(20k^2+30k+9)} + \frac{9 - (1 + \frac{a}{L})\,(4k+6)}{3(4k+3)}\right]$$

$$V_B = P\frac{a}{L} + \frac{Pa\left(1-\frac{a}{L}\right)}{2L}\left[\frac{\frac{a}{L}(4k^2+24k+18)+(4k^2-12k-9)}{20k^2+30k+9} + \frac{(4k+6)\left(1+3\frac{a}{L}\right)+8k-15}{3(4k+3)}\right]$$

$$V_C = \frac{Pa\left(1-\frac{a}{L}\right)}{2L}\left[\frac{-\frac{a}{L}(4k^2+24k+18)-(4k^2-12k-9)}{20k^2+30k+9} + \frac{(4k+6)\left(1+3\frac{a}{L}\right)+8k-15}{3(4k+3)}\right]$$

$$V_D = \frac{Pa\left(1-\frac{a}{L}\right)}{2L}\left[\frac{-\frac{a}{L}(4k^2+24k+18)-(4k^2-12k-9)}{(20k^2+30k+9)} + \frac{\left(1+\frac{a}{L}\right)(4k+6)-9}{3(4k+3)}\right]$$

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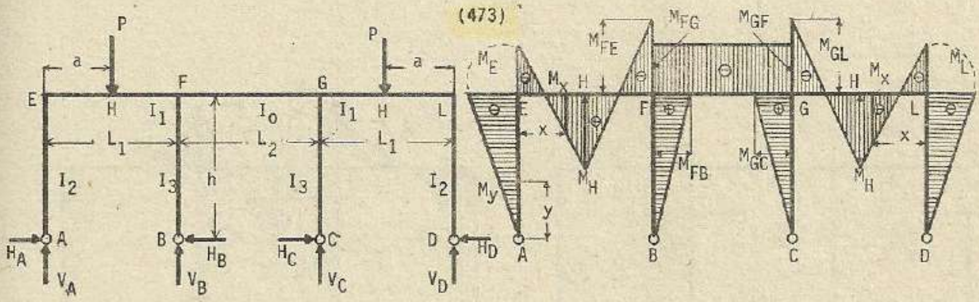

$$k_1 = \frac{I_1 h}{I_3 L_1}\ ;\quad \alpha = \frac{I_0 L_1}{I_1 L_2}\ ;\quad \alpha k_1 = \frac{I_0 h}{I_3 L_2}\ ;\quad k_2 = \frac{I_1 h}{I_2 L_1}\ ;$$

otros datos (468)

$$H_A = H_D = \frac{3Pa\left(1-\frac{a}{L_1}\right)}{h}\left[\frac{\left(1-\frac{a}{L_1}\right)(3+2\alpha k_1+2k_1)+2k_1}{J}\right]$$

$$H_B = H_C = \frac{3Pa\left(1-\frac{a}{L_1}\right)}{h}\left[\frac{(3+2k_2)\frac{a}{L_1}+2k_2}{J}\right]$$

$$M_E = M_L = -3Pa\left(1-\frac{a}{L_1}\right)\left[\frac{\left(1-\frac{a}{L_1}\right)(3+2\alpha k_1+2k_1)+2k_1}{J}\right]$$

$$M_{FE} = M_{GL} = -Pa\left(1-\frac{a}{L_1}\right)\left[\frac{(3+2\alpha k_1)\left[3\frac{a}{L_1}+2k_2\left(1+\frac{a}{L_1}\right)\right]}{J}\right]$$

$$M_{FG} = M_F = -Pa\left(1-\frac{a}{L_1}\right)\left[\frac{2k_1\left[(3+2k_2)\frac{a}{L_1}+2k_2\right]}{J}\right];\quad M_{FB} = M_{GC} = 3Pa\left(1-\frac{a}{L_1}\right)\left[\frac{(3+2k_2)\frac{a}{L_1}+2k_2}{J}\right]$$

$$V_B = V_C = P\frac{a}{L_1} - \frac{Pa\left(1-\frac{a}{L_1}\right)}{L_1}\left[\frac{-\frac{a}{L_1}\left[18+12\alpha k_1+6(k_1+k_2)+4\alpha k_1k_2\right]+(9+12k_1+6\alpha k_1-6k_2-4\alpha k_1k_2)}{J}\right]$$

Si $L_1 = L_2 = L$, $I_0 = I_1$, $I_2 = I_3$, $k_1 = k_2 = k$, $\alpha = 1$

$$H_A = H_D = \frac{3Pa\left(1-\frac{a}{L}\right)\left[3(1+2k)-(3+4k)\frac{a}{L}\right]}{hW};\quad H_B = H_C = \frac{3Pa\left(1-\frac{a}{L}\right)\left[(3+2k)\cdot\frac{a}{L}+2k\right]}{hW}$$

$$M_E = M_L = -3Pa\left(1-\frac{a}{L}\right)\frac{\left[3(1+2k)-(3+4k)\frac{a}{L}\right]}{W};\quad M_{FE} = M_{GL} = -Pa\left(1-\frac{a}{L}\right)\frac{\left[\frac{a}{L}(3+2k)^2+2k(3+2k)\right]}{W}$$

$$M_{FG} = M_{GF} = -Pa\left(1-\frac{a}{L}\right)\frac{2k\left[(3+2k)\cdot\frac{a}{L}+2k\right]}{W} \quad ; \quad M_{FB} = M_{GC} = 3Pa\left(1-\frac{a}{L}\right)\frac{\left[(3+2k)\cdot\frac{a}{L}+2k\right]}{W}$$

Esfuerzos de momentos por flexión en las columnas

AE y DL $\longrightarrow M_y = H_A y = -H_D y$ BF y CG $\longrightarrow M_y = H_B y = H_C y$

Momentos en las vigas : EF y LG $\longrightarrow M_x = -H_A h + V_A x \qquad x \leq a$

$M_x = -H_A h + V_A x - P(x-a) \qquad x \geq a$

FG $\longrightarrow M_x = -Pa\left(1-\frac{a}{L}\right)\frac{2k\left[(3+2k)\frac{a}{L}+2k\right]}{W}$

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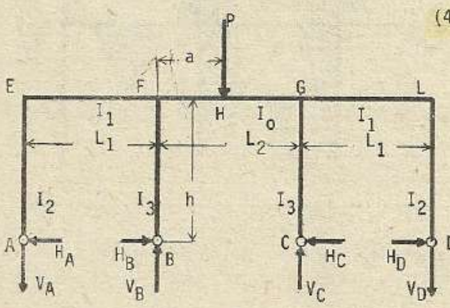
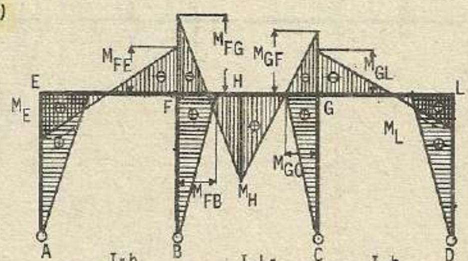
(474)





$k_1 = \frac{I_1 h}{I_3 L_1}$; $\alpha = \frac{I_0 L_1}{I_1 L_2}$; $\alpha k_1 = \frac{I_0 h}{I_3 L_2}$

$$H_A = \frac{3Pa\left(1-\frac{a}{L_2}\right)}{2h}\left[\frac{2k_1}{J} + \frac{1-2\frac{a}{L_2}}{K}\right] \quad \text{otros datos (468)} \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

$$H_B = \frac{Pa\left(1-\frac{a}{L_2}\right)}{2h}\left[\frac{3(3+4k_2)}{J} + \frac{3\left(1-2\frac{a}{L_2}\right)}{K}\right] ; H_C = \frac{Pa\left(1-\frac{a}{L_2}\right)}{2h}\left[\frac{3(3+4k_2)}{J} + \frac{3\left(1-2\frac{a}{L_2}\right)}{K}\right]$$

$$H_D = \frac{3Pa\left(1-\frac{a}{L_2}\right)}{2h}\left[\frac{2k_1}{J} - \frac{1-2\frac{a}{L_2}}{K}\right] ; \quad M_E = \frac{3}{2}Pa\left(1-\frac{a}{L_2}\right)\left[\frac{2k_1}{J} + \frac{1-2\frac{a}{L_2}}{K}\right]$$

$$M_{FE} = -\frac{Pa\left(1-\frac{a}{L_2}\right)}{2}\left[\frac{12k_1(1+2k_2)}{J} + \frac{\left[3+2(k_1+k_2)\right]\left(1-2\frac{a}{L_2}\right)}{K}\right]$$

$$M_{FG} = -\frac{Pa\left(1-\frac{a}{L_2}\right)}{2}\left[\frac{3(3+4k_1+4k_1k_2+4k_2)}{J} - \frac{2(3+k_1+k_2)\left(1-2\frac{a}{L_2}\right)}{K}\right]$$

$$M_{FB} = -\frac{Pa\left(1-\frac{a}{L_2}\right)}{2}\left[\frac{3(3+4k_2)}{J} + \frac{3\left(1-2\frac{a}{L_2}\right)}{K}\right]$$

$$M_{GL} = -\frac{Pa\left(1-\frac{a}{L_2}\right)}{2}\left[\frac{12k_1(1+k_2)}{J} - \frac{\left[3+2(k_1+k_2)\right]\left(1+2\frac{a}{L_2}\right)}{K}\right]$$

$$K = 2(1+2\alpha)(k_1+k_2) + 3(2+\alpha)$$

$$M_{GF} = -\frac{Pa\left(1-\frac{a}{L_2}\right)}{2}\left[\frac{3\left(3+4k_1+4k_1k_2+4k_2\right)}{J} - \frac{3\left(3+k_1+k_2\right)\left(1-2\frac{a}{L_2}\right)}{K}\right]$$

$$M_{GC} = -\frac{Pa\left(1-\frac{a}{L_2}\right)}{2}\left[\frac{3(3+4k_2)}{J} - \frac{3\left(1-2\frac{a}{L_2}\right)}{K}\right]; \quad M_L = \frac{3}{2} Pa\left(1-\frac{a}{L_2}\right)\left[\frac{2k_1}{J} - \frac{1-2\frac{a}{L_2}}{K}\right]$$

$$V_A = P\frac{a}{2L_1}\left(1-\frac{a}{L_2}\right)\left[\frac{6(3k_1+2k_1k_2)}{J} + \frac{2(3+k_1+k_2)\left(1-2\cdot\frac{a}{L_2}\right)}{K}\right]$$

$$V_B = P\left(1-\frac{a}{L_2}\right) + P\frac{a}{2}\left(1-\frac{a}{L_2}\right)\left[\frac{6(3k_1+2k_1k_2)\frac{1}{L_1}}{J} + \frac{2(3+k_1+k_2)\left(1-2\cdot\frac{a}{L_2}\right)\left(\frac{1}{L_1}+\frac{1}{L_2}\right)}{K}\right]$$

$$V_C = P\frac{a}{L_2} + P\frac{a}{2}\left(1-\frac{a}{L_2}\right)\left[\frac{6(3k_1+2k_1k_2)\frac{1}{L_1}}{J} - \frac{2(3+k_1+k_2)\left(1-2\frac{a}{L_2}\right)\left(\frac{1}{L_1}+\frac{1}{L_2}\right)}{K}\right]$$

$$V_D = P\frac{a}{2L_1}\left[\frac{6(3k_1+2k_1k_2)}{J} - \frac{2(3+k_1+k_2)\left(1-2\cdot\frac{a}{L_2}\right)}{K}\right]$$

Si $a = \frac{L}{2}$; $H_A = H_D = \frac{3PL_2k_1}{4Jh}$; $H_B = H_C = \frac{3PL_2(3+4k_2)}{8Jh}$; $M_E = M_L = \frac{3PL_2k_1}{4J}$

$M_{FE} = M_{GL} = -\frac{3PL_2(1+k_2)k_1}{2J}$; $M_{FG} = M_{GF} = \frac{3PL_2(3+4k+4k_1k_2+4k_2)}{8J}$

$M_{FB} = M_{GC} = -\frac{3PL_2(3+4k_2)}{8J}$; $V_A = V_D = \frac{3PL_2(3k_1+2k_1k_2)}{4JL_1}$

$V_B = V_C = \frac{P}{2} + \frac{3PL_2(3k_1+2k_1k_2)}{4JL_1}$

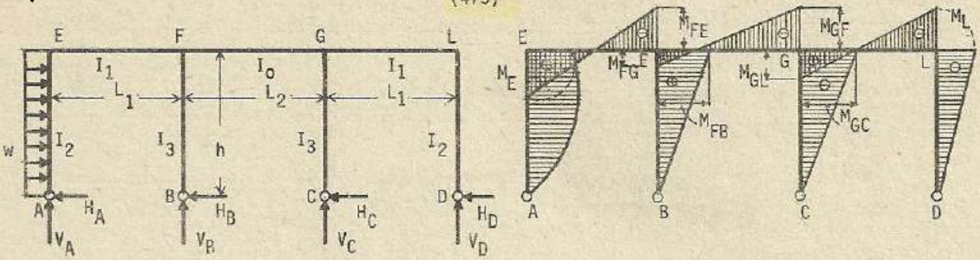
Si $L_1 = L_2 = L$, $k_1 = k_2 = k$, $\alpha = 1$

$H_A = H_D = \frac{3PLk}{4Wh}$; $H_B = H_C = \frac{3PL(3+4k)}{8Wh}$; $M_E = M_L = -\frac{3PLk}{4W}$

$M_{FE} = M_{GL} = -\frac{3PL(1+k)k}{2W}$; $M_{FG} = M_{GF} = -\frac{3PL(4k^2+8k+3)}{8W}$; $M_{FB} = M_{GC} = -\frac{3PL(3+4k)}{8W}$

$V_A = V_D = \frac{3P(2k+3)k}{4W}$; $V_B = V_C = \frac{P}{2} + \frac{3P(2k+3)k}{4W}$; $M_H = \frac{PL(28k^2+36k+9)}{8W}$

(475)



$$k_1 = \frac{I_1 h}{I_3 L_1} \; ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} \; ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} \; ; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

; otros datos (468) (474)

$$(8\alpha k_1 k_2 + 12 k_1 k_2 + 12 k_2 + 12 k_1 + 6\alpha k_1 + 9) = X$$

$$H_A = \frac{wh}{2} + \frac{wh}{8}\left[\frac{6 + (2\alpha+1)(4k_1 - k_2)}{K} - \frac{6k_2 + 2(3+2\alpha)\,k_1 k_2}{X}\right]$$

$$H_B = \frac{wh}{8}\left[\frac{6(1+\alpha) + 5k_2(1+2\alpha)}{K} - \frac{3k_2}{X}\right]; \quad H_C = \frac{wh}{8}\left[\frac{6(1+\alpha) + 5k_2(1+2\alpha)}{K} + \frac{3k_2}{X}\right]$$

$$H_D = \frac{wh}{8}\left[\frac{6 + (2\alpha+1)(4k_1 - k_2)}{K} + \frac{6k_2 + 2(3+2\alpha)\,k_1 k_2}{X}\right]$$

$$V_A = \frac{wh^2}{8L_1}\left[\frac{3(3k_2 + 2k_1k_2 + 2\alpha k_1 k_2)}{X} - \frac{12 + 12k_1\alpha + 4k_1 + 4k_2 - 3\alpha k_2}{K}\right]$$

$$V_B = \frac{wh^2}{8L_1}\left[\frac{12\left(1-\alpha\frac{L_1}{L_2}\right) + k_2\left(4 - 3\alpha - 22\alpha\frac{L_1}{L_2}\right) + 4k_1\left(1 + 3\alpha + 2\alpha\frac{L_1}{L_2}\right)}{K} - \frac{9k_2 + 2(3+3\alpha)\,k_1 k_2}{X}\right]$$

$$V_C = \frac{wh^2}{8L_1}\left[-\frac{9k_2 + 2(3+2\alpha)\,k_1 k_2}{X} - \frac{12\left(1-\alpha\frac{L_1}{L_2}\right) + k_2\left(4 - 3\alpha - 22\alpha\frac{L_1}{L_2}\right) + 4k_1\left(1 + 3\alpha + 2\alpha\frac{L_1}{L_2}\right)}{K}\right]$$

$$V_D = \frac{wh^2}{8L_1}\left[\frac{12 + 4k_1(1+3\alpha) + k_2(4-3\alpha)}{K} + \frac{9k_2 + 2(3+3\alpha)\,k_1 k_2}{X}\right]$$

$$M_E = \frac{wh^2}{8}\left[\frac{6 + (2+\alpha)(4k_1 - k_2)}{K} - \frac{6k_2 + 2(3+2\alpha)\,k_1 k_2}{X}\right]$$

$$M_{FE} = -\frac{wh^2}{8}\left[\frac{6 + 4\alpha k_1 + k_2(5-\alpha)}{K} - \frac{3k_2 + 2\alpha k_1 k_2}{X}\right]; \quad M_{FB} = \frac{wh^2}{8}\left[\frac{6(1+\alpha) + 5k_2(1+2\alpha)}{K} - \frac{3k_2}{X}\right]$$

$$M_{GF} = -\frac{wh^2}{8}\left[\frac{6\alpha + 11\alpha k_2 - 4\alpha k_1}{K} - \frac{2\alpha k_1 k_2}{X}\right]; \quad M_{GL} = \frac{wh^2}{8}\left[\frac{6 + 4\alpha k_1 + k_2(5-\alpha)}{K} + \frac{3k_2 + 2k_1 k_2}{X}\right]$$

$$M_{GC} = -\frac{wh^2}{8}\left[\frac{6(1+\alpha) + 5k_2(1+2\alpha)}{K} + \frac{3k_2}{X}\right]; \quad M_{FG} = \left[\frac{6\alpha + 11\alpha k_2 - 4\alpha k_1}{K} + \frac{2\alpha k_1 k_2}{X}\right]$$

$$M_L = -\frac{wL^2}{8}\left[\frac{6 + (2+\alpha)(4k_1 - k_2)}{K} + \frac{6k_2 + 2(3+2\alpha)\,k_1 k_2}{X}\right]$$

$$K = 2(1+2\alpha)(k_1 + k_2) + 3(2+\alpha)$$

Si $L_1 = L_2 = L$, $I_2 = I_3$, $I_0 = I_1$, donde $k_1 = k_2 = k$ $\quad \alpha = 1$; $20k^2+30k+9=W$

$$H_A = \frac{wh}{2} + \frac{wh}{8}\left[\frac{(3k+2)}{(4k+3)} - \frac{10k^2+6k}{W}\right]; \quad H_B = \frac{wh}{8}\left[\frac{(5k+4)}{(4k+3)} - \frac{3k}{W}\right]$$

$$H_C = \frac{wh}{8}\left[\frac{(5k+4)}{(4k+3)} + \frac{3k}{W}\right]; \quad H_D = \frac{wh}{8}\left[\frac{(3k+2)}{(4k+3)} + \frac{10k^2+6k}{W}\right]$$

$$V_A = \frac{wh^2}{8L}\left[\frac{3\,(4k^2+3k)}{W} - \frac{(17k+12)}{3(4k+3)}\right]; \quad V_B = \frac{wh^2}{8L}\left[\frac{k}{(4k+3)} - \frac{3(4k^2+3k)}{W}\right]$$

$$V_C = \frac{wh^2}{8L}\left[-\frac{3\,(4k^2+3k)}{W} - \frac{k}{(4k+3)}\right]; \quad V_D = \frac{wh^2}{8L}\left[\frac{(17k+12)}{3(4k+3)} + \frac{3(4k^2+3k)}{W}\right]$$

$$M_E = \frac{wh^2}{8}\left[\frac{(3k+2)}{(4k+3)} - \frac{10k^2+6k}{W}\right]; \quad M_{FE} = -\frac{wh^2}{8}\left[\frac{(8k+6)}{3(4k+3)} - \frac{(2k^2+3k)}{W}\right]$$

$$M_{FG} = \frac{wh^2}{8}\left[\frac{(7k+6)}{3(4k+3)} + \frac{2k^2}{W}\right]; \quad M_{FB} = \frac{wh^2}{8}\left[\frac{(5k+4)}{(4k+3)} - \frac{3k}{W}\right]; \quad M_{GF} = -\frac{wh^2}{8}\left[\frac{(7k+6)}{3(4k+3)} - \frac{2k^2}{W}\right]$$

$$M_{GL} = \frac{wh^2}{8}\left[-\frac{(8k+6)}{3(4k+3)} + \frac{(2k^2+3k)}{W}\right]; \quad M_{GC} = -\frac{wh^2}{8}\left[\frac{(5k+4)}{(4k+3)} + \frac{3k}{W}\right]$$

$$M_L = -\frac{wh^2}{8}\left[\frac{(3k+2)}{(4k+3)} + \frac{(10k^2+6k)}{W}\right]$$

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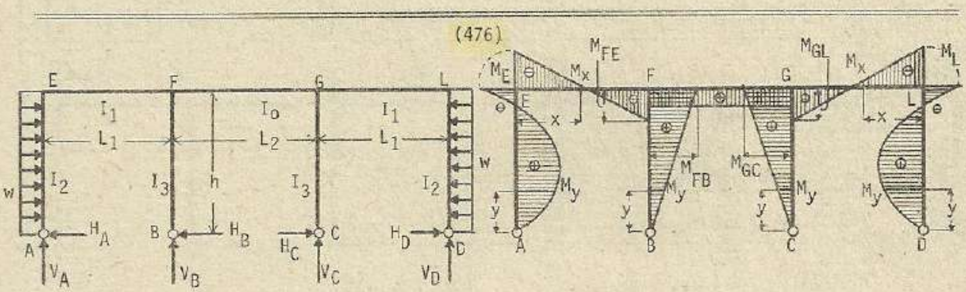
(476)



$k_1 = \frac{I_1 h}{I_3 L_1}$; $\alpha = \frac{I_0 L_1}{I_1 L_2}$; $\alpha k_1 = \frac{I_0 h}{I_3 L_2}$; $k_2 = \frac{I_1 h}{I_2 L_1}$; otros datos (475)

$$H_A = H_D = \frac{3(2k_1k_2\alpha+3k_1k_2+3k_2+2\alpha k_1+4k_1+3)}{2X}\cdot wh \quad ; \quad H_B = H_C = -\frac{3k_2}{4X}\cdot wh$$

$$V_A = V_D = \frac{3(2\alpha k_1k_2+2k_1k_2+3k_2)}{4X}\cdot\frac{wh^2}{L_1}; \quad V_B = V_C = -\frac{3\,(2\alpha k_1k_2+2k_1k_2+3k_2)}{4X}\cdot\frac{wh^2}{L_1}$$

$$M_E = M_L = -\frac{(2\alpha k_1k_2+3k_1k_2+3k_2)}{2X}\cdot wh^2 \quad ; \quad M_{FE} = M_{GL} = \frac{2\alpha k_1k_2+3k_2}{4X}\cdot wh^2$$

$$M_{FG} = M_{GF} = \frac{\alpha k_1k_2}{2X}\,wh^2 \quad ; \quad M_{FB} = M_{GC} = -\frac{3k_2}{4X}\cdot wh^2$$

Esfuerzos de momentos por doblado en las columnas :

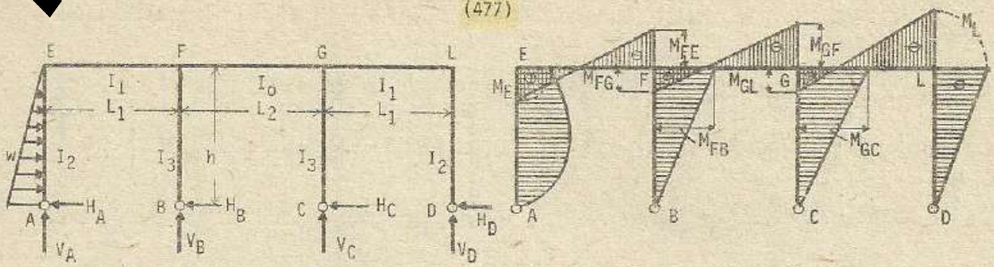
AE y DL ——→ $M_y = H_A y - \frac{wy^2}{2}$; BF y CG ——→ $M_y = H_B y$

Momentos en las vigas : EF y LG ——→ $M_x = H_A h - \frac{wh^2}{2} + V_A x$

$$= H_D h - \frac{wh^2}{2} + V_D D$$

FG : $M = \frac{\alpha k_1k_2}{2X}\,wh^2$ (constante)

(477)



$k_1 = \frac{I_1 h}{I_3 L_1}$; $\alpha = \frac{I_0 L_1}{I_1 L_2}$; $\alpha k_1 = \frac{I_0 h}{I_3 L_2}$; $k_2 = \frac{I_1 h}{I_2 L_1}$; Otros datos (468) (474) (475)

$$H_A = \frac{2}{3} w + \frac{w}{60}\left[\frac{30+(2\alpha+1)\ (20k_1 - 7k_2)}{K} - \frac{14k_2(3+3k_1+2\alpha k_1)}{X}\right]$$

$$H_B = \frac{w}{60}\left[\frac{30(1+\alpha)+27(1+2\alpha)\ k_2}{K} - \frac{21k_2}{X}\right]; \quad H_C = \frac{w}{60}\left[\frac{30\ (1+\alpha)+27(1+2\alpha)k_2}{K} + \frac{21k_2}{X}\right]$$

$$H_D = \frac{w}{60}\left[\frac{30+(2\alpha+1)(20k_1 - 7k_2)}{K} + \frac{14k_2(3+3k_1+2\alpha k_1)}{X}\right]$$

$$V_A = -\frac{wh}{60L_1}\left[\frac{60\ (1+\alpha k_1) + 20\ (k_1+k_2) - 21\alpha k_2}{K} - \frac{7k_2\left\{9+6k_1(1+\alpha)\right\}}{X}\right]$$

$$V_B = \frac{wh}{60L_1}\left\{\frac{60\ (1+\alpha k_1)+20(k_1+k_2)-21\alpha k_2 - \left[20(3-2k_1)\alpha+122\alpha k_2\right]\frac{L_1}{L_2}}{K} - \frac{7k_2\left[9+6k_1(1+\alpha)\right]}{X}\right\}$$

$$V_C = -\frac{wh}{60L_1}\left\{\frac{60\ (1+\alpha k_1)+20(k_1+k_2)-21\alpha k_2 - \left[20(3-2k_1)\alpha+122\alpha k_2\right]\frac{L_1}{L_2}}{K} + \frac{7k_2\left[9+6k_1(1+\alpha)\right]}{X}\right\}$$

$$V_D = \frac{wh}{60L_1}\left\{\frac{60(1+\alpha k_1)+20\ (k_1+k_2)-21\alpha k_2}{K} + \frac{7k_2\left[9+6k_1\ (1+\alpha)\right]}{X}\right\}$$

$$M_E = \frac{wh}{60}\left[\frac{30+(2\alpha+1)\ (20k_1 - 7k_2)}{K} - \frac{14k_2(3-3k_2-2\alpha k_1)}{X}\right]$$

$$M_{FE} = -\frac{wh}{60}\left[\frac{10(3+2k_2) + 20\alpha k_1 + 7k_2(1-\alpha)}{K} - \frac{7k_2(3+2k_1)}{X}\right]$$

$$M_{FG} = \frac{wh}{60}\left[\frac{10(3-2k_1)\ \alpha + 61\alpha k_2}{K} + \frac{14\alpha\ k_1k_2}{X}\right]; \quad M_{GC} = -\frac{wh}{60}\left[\frac{30\ (1+\alpha)+27k\ (12+\alpha)}{K} + \frac{21k_2}{X}\right]$$

$$M_{GL} = \frac{wh}{60}\left[\frac{10(3+2k_2) + 20\alpha k_1 + 7k_2(1-\alpha)}{K} + \frac{7k_2\ (3+2\alpha k_1)}{X}\right]$$

$$M_L = -\frac{wh}{60}\left[\frac{30+(2\alpha+1)\ (20k_1 - 7k_2)}{K} + \frac{14k_2\ (3+3k_1+2\alpha k_1)}{X}\right]$$

$$M_{FB} = \frac{wh}{60}\left[\frac{30\ (\alpha+1) + 27k_2\ (1+2\alpha)}{K} - \frac{21k_2}{X}\right]; \quad M_{GF} = -\frac{wh}{60}\left[\frac{10\ (3-2k_1)\alpha + 61\alpha k_2}{K} - \frac{14\alpha k_1k_2}{X}\right]$$

$$K = 2\ (1+2\alpha)\ (k_1+k_2) + 3\ (2+\alpha)$$

$$X = (8\alpha k_1k_2 + 12k_1k_2 + 12k_1 + 12k_2 + 6\alpha k_1 + 9)$$

$$W = (20k^2 + 30k + 9)$$

Si $L_1 = L_2 = L$, $I_2 = I_3$, $I_0 = I_1$, entonces $k_1 = k_2 = k$ $\qquad \alpha = 1$

$$H_A = \frac{2}{3} w + \frac{w}{60}\left[\frac{13k+10}{4k+3} - \frac{14k\ (5k+3)}{W}\right]; \qquad H_B = \frac{w}{60}\left[\frac{27k+20}{4k+3} - \frac{21k}{W}\right]$$

$$H_C = \frac{w}{60}\left[\frac{27k+20}{4k+3} + \frac{21k}{W}\right] \qquad H_D = \frac{w}{60}\left[\frac{13k+10}{4k+3} + \frac{14k(5k+3)}{W}\right]$$

$$V_A = -\frac{wh}{60L}\left[\frac{79k+60}{3(4k+3)} - \frac{7k(12k+9)}{W}\right] \quad ; \quad V_B = \frac{wh}{60L}\left[-\frac{k}{(4k+3)} - \frac{7k(12k+9)}{W}\right]$$

$$V_C = -\frac{wh}{60L}\left[-\frac{k}{(4k+3)} + \frac{7k(12k+9)}{W}\right] \quad ; \quad V_D = \frac{wh}{60L}\left[\frac{79k+60}{3(4k+3)} + \frac{7k(12k+9)}{W}\right]$$

$$M_E = \frac{wh}{60}\left[\frac{13k+10}{4k+3} - \frac{14k(5k+3)}{W}\right] \quad ; \quad M_{FE} = -\frac{wh}{60}\left[\frac{40k+30}{3(4k+3)} - \frac{7k(2k+3)}{W}\right]$$

$$M_{FG} = \frac{wh}{60}\left[\frac{41k+30}{3(4k+3)} + \frac{14k^2}{W}\right] \quad ; \quad M_{FB} = \frac{wh}{60}\left[\frac{27k+20}{4k+3} - \frac{21k}{W}\right]$$

$$M_{GF} = -\frac{wh}{60}\left[\frac{41k+30}{3(4k+3)} - \frac{14k^2}{W}\right] \quad ; \quad M_{GL} = \frac{wh}{60}\left[\frac{40k+30}{3(4k+3)} + \frac{7k\ (2k+3)}{W}\right]$$

$$M_{GC} = -\frac{wh}{60}\left[\frac{27k+20}{4k+3} + \frac{21k}{W}\right] \quad ; \quad M_L = -\frac{wh}{60}\left[\frac{13k+10}{4k+3} + \frac{14k(5k+3)}{W}\right]$$

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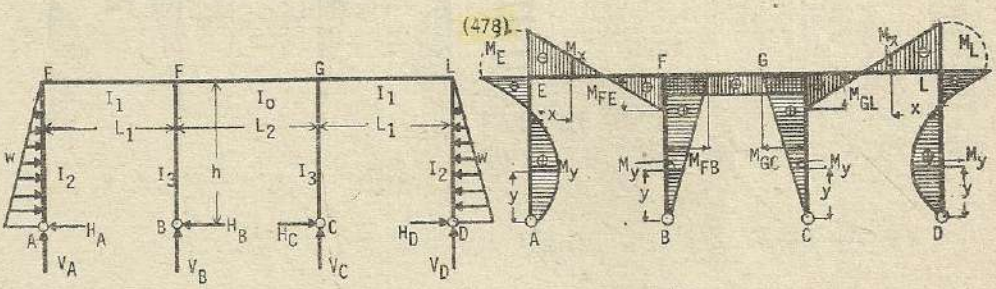
(478)



$$k_1 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} \quad ; \quad k_2 = \frac{I_1 h}{I_2 L_1}$$

$$(8\alpha k_1 k_2 + 12k_1 k_2 + 12k_1 + 12k_2 + 6\alpha k_1 + 9) = X \quad ; \quad (20k^2 + 30k + 9) = W$$

$$H_A = H_D = \frac{22\alpha k_1 k_2 + 33k_1 k_2 + 33k_2 + 20\alpha k_1 + 40k_1 + 30}{5X} w \ ; \quad H_B = H_C = -\frac{7k_2}{10X} \cdot w$$

$$V_A = V_D = \frac{(14\alpha k_1 + 14k_1 + 21)\ k_2}{10X} \cdot \frac{wh}{L_1} \ ; \quad V_B = V_C = -\frac{(14\alpha k_1 + 14k_1 + 21)\ k_2}{10X} \cdot \frac{wh}{L_1}$$

$$M_E = M_L = -\frac{7(2\alpha k_1 + 3k_1 + 3)\ k_2 \cdot wh}{15X} \quad ; \quad M_{FE} = M_{GL} = \frac{7(2\alpha k_1 + 3)\ k_2 \cdot wh}{30X}$$

$$M_{FG} = M_{GF} = \frac{7\alpha k_1 k_2 wh}{15X} \quad ; \quad M_{FB} = M_{GC} = -\frac{7k_2 wh}{10X}$$

Si $L_1 = L_2 = L$, $\quad I_2 = I_3$, $\quad I_o = I$, donde $k_1 = k_2 = k$, $\quad \alpha = 1$

$$H_A = H_D = \frac{55k^2+93k+30}{5W}\,w \; ; \qquad H_B = H_D = -\frac{7k}{10W}\,w \; ; \qquad V_A = V_D = \frac{(28k+21)\,k}{10W}\cdot\frac{wh}{L}$$

$$V_B = V_C = -\frac{(28k+21)k}{10W}\cdot\frac{wh}{L} \; ; \qquad M_E = M_L = -\frac{7(5k+3)kwh}{15W} ; \; M_{FE} = M_{GL} = \frac{7(2k+3)kwh}{30W}$$

$$M_{FG} = M_{GF} = \frac{7k^2wh}{15W} \; ; \quad M_{FB} = M_{GC} = \frac{7kwh}{10W}$$

Momentos por flexión en las columnas :

AE y DL $\longrightarrow$ $M_y = H_Ay + w\cdot\frac{y^3}{3h^2} - \frac{wy^2}{h}$; $\qquad$ BF y CG $\longrightarrow$ $M_y = -H_By$

Momentos en las vigas : EF y LG $\longrightarrow$ $M_x = M_E + V_Ax$

FG $\longrightarrow$ $M_x = \frac{7k^2wh}{15W}$ (constante)

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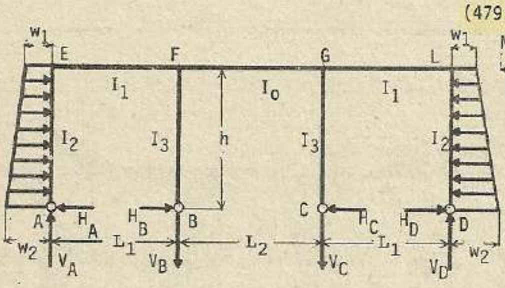

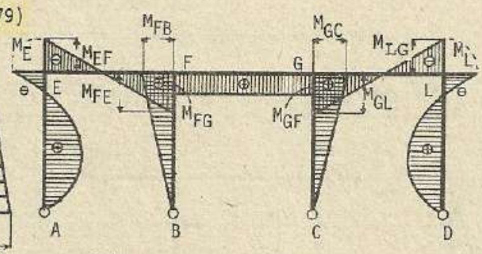

$$k_1 = \frac{I_2L_1}{I_1h} \; ; \qquad k_2 = \frac{I_oL_1}{I_1L_2} \; ; \qquad k_3 = \frac{I_3L_1}{I_1h}$$

$$C = \frac{h^2}{120}\cdot(8w_1+7w_2) \; ; \qquad D = (4+3k_1)\,(4+2k_2 + 3k_3) - 4$$

$$M_E = M_L = -\frac{4(3+2k_2+3K_3)}{D}\cdot C \; ; \quad M_{FE} = M_{GL} = \frac{2(2k_2+3k_3)}{D}\cdot C \; ; \quad M_{FG} = M_{GF} = \frac{4k_2C}{D}$$

$M_{FB} = M_{GC} = \frac{6k_3C}{D}$; Si $L_1 = L_2 = L$; $I_o = I_1$, entonces $k_1 = \frac{I_2L}{I_1h}$; $k_2 = 1$; $k_3 = \frac{I_3L}{I_1h}$

$$D = 3\,(4+3k_1)(2+k_3) - 4 \quad ; \quad C = \frac{h^2}{120}\,(8w_1 + 7w_2)$$

$$M_E = M_L = -\frac{4(5+3k_3)}{D}\cdot C \quad ; \quad M_{FE} = M_{GL} = \frac{2(2+3k_3)}{D}\cdot C$$

$$M_{FG} = M_{GF} = \frac{4C}{D} \quad ; \quad M_{FB} = M_{GC} = \frac{6k_3C}{D}$$

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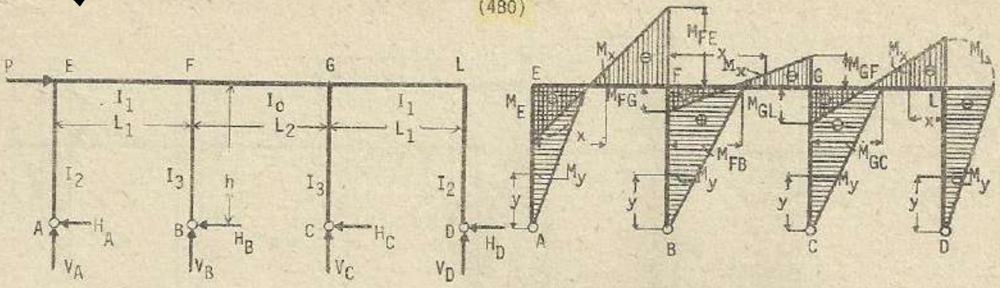


$$k_1 = \frac{I_1 h}{I_3 L_1} \; ; \quad \alpha = \frac{I_0 L_1}{I_1 L_2} \; ; \quad \alpha k_1 = \frac{I_0 h}{I_3 L_2} \; ; \quad k_2 = \frac{I_1 h}{I_2 L_1} \; ; \quad \text{otros datos} \quad \textbf{(474)}$$

$$H_A = H_D = \frac{P}{2} \cdot \frac{(3+2k_1+4\alpha k_1)}{K} ; \qquad H_B = H_C = \frac{P}{2} \cdot \frac{\left[\,3\,(1+\alpha) + 2k_2(1+2\alpha)\,\right]}{2K}$$

$$V_A = -\frac{Ph}{L_1} \cdot \frac{(3+k_1+k_2+3\alpha k_1)}{K} \; ; \qquad V_B = \frac{Ph\left[\,3+k_1+k_2+3\alpha k_1-(3\alpha+4\alpha k_2-2\alpha k_1)\cdot \frac{L_1}{L_2}\,\right]}{KL_1}$$

$$V_C = -\frac{Ph\left[\,3+k_1+k_2+3\alpha k_1 - (3\alpha+4\alpha K_2-2\alpha k_1)\,\right]\frac{L_1}{L_2}}{KL_1} \; ; \qquad V_D = \frac{Ph\,(3+k_1+k_2+3\alpha k_1)}{KL_1}$$

$$M_E = \frac{Ph}{2} \cdot \frac{(3+2k_1+4\alpha k_1)}{K} \; ; \qquad M_{FE} = -\frac{Ph}{2} \cdot \frac{(3+2k_2+2\alpha k_1)}{K} ; \qquad M_{FG} = \frac{Ph}{2} \cdot \frac{(3\alpha+4\alpha k_2-2\alpha k_1)}{K}$$

$$M_{FB} = \frac{Ph}{2} \cdot \frac{\left[3(1+\alpha)+2k_2(1+2\alpha)\right]}{K} \; ; \qquad M_{GF} = -\frac{Ph}{2} \cdot \frac{(3\alpha+4\alpha k_2-2\alpha k_1)}{K}$$

$$M_{GL} = \frac{Ph}{2} \cdot \frac{(3+2k_2+2\alpha k_1)}{K} \; ; \qquad M_{GC} = -\frac{Ph}{2} \cdot \frac{\left[3(1+\alpha)+2k_2(1+2\alpha)\right]}{K} ; \quad M_L = -\frac{Ph}{2} \cdot \frac{(3+2k_1+4\alpha k_1)}{K}$$

Si $L_1 = L_2 = L$, $I_2 = I_3$, $I_0 = I_1$, $k_1 = k_2 = k$, $\alpha = 1$

$$H_A = H_D = \frac{P(2k+1)}{2(4k+3)} \; ; \quad H_B = H_C = \frac{P(k+1)}{(4k+3)} \; ; \quad V_A = -\frac{Ph(5k+3)}{3L(4k+3)} ; \quad V_B = \frac{Phk}{L(4k+3)}$$

$$V_C = -\frac{Phk}{L(4k+3)} \; ; \quad V_D = \frac{Ph\,(5k+3)}{3L\,(4k+3)} \; ; \quad M_E = \frac{Ph(2k+1)}{2(4k+3)} \; ; \quad M_{FE} = -\frac{Ph(2k+3)}{6(4k+3)}$$

$$M_{FG} = \frac{Ph(2k+3)}{6(4k+3)} \; ; \quad M_{FB} = \frac{Ph(k+1)}{(4k+3)} \; ; \quad M_{GF} = -\frac{Ph(2k+3)}{6(4k+3)} \; ; \quad M_{GC} = -\frac{Ph(k+1)}{(4k+3)}$$

$$M_{GL} = \frac{Ph(2k+3)}{6\,(4k+3)} \; ; \qquad M_L = -\frac{Ph(2k+1)}{2(4k+3)}$$

Momentos por flexión en las columnas :

AE ⟶ $M_y = H_A y$; BF ⟶ $M_y = H_B y$; CG ⟶ $M_y = -H_C y$; DC ⟶ $M_y = -H_D y$

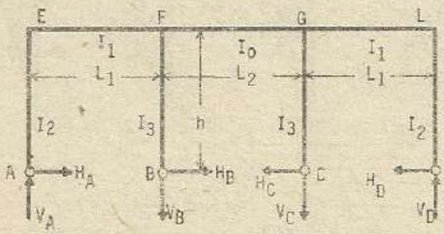
Momentos en las vigas :

EF ⟶ $M_x = H_A h - V_A x$

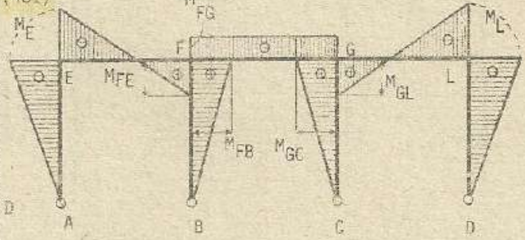
FG ⟶ $M_x = (H_A + H_D)\,h - V_A(L_1+x) + V_B x$

LG ⟶ $M_x = -H_D h + V_D x$

Esfuerzos por cambio de temperatura (caso por ascenso de temperatura)



(481)



$$k_1 = \frac{I_1 h}{I_2 L_1} \quad ; \quad k_2 = \frac{I_1 h}{I_3 L_1} \quad ; \quad \alpha = \frac{L_2}{L_1} \quad ; \quad \beta = \frac{I_0}{I_1} \quad ; \quad V' = (20k_1k_2+12k_1+18k_2+9)$$

$$H_A = H_D = \frac{2\left[2\,\frac{\alpha}{\beta}(1+k_2) + \frac{4}{3}k_2\right] + \alpha\,\frac{\alpha}{\beta}}{\left(\frac{2}{3}k_1+1\right)\left[2\,\frac{\alpha}{\beta}(1+k_2) + \frac{4}{3}k_2\right] - \left[\frac{2}{3}k_2 + \frac{\alpha}{\beta}\left(\frac{2}{3}k_2+1\right)\right]} \cdot \frac{E\varepsilon t I_1}{h^2 L_1}$$

$$H_B = H_C = -\frac{1}{h}\left[H_A h\left(1+\frac{\alpha}{\beta}\right) - V_A L_1\left(\frac{2}{3}+\frac{\alpha}{\beta}\right)\right]$$

$$V_A = V_B = V_C = V_D = \frac{\frac{4}{3}k_2 + \frac{\alpha}{\beta}\left[\alpha\left(\frac{2}{3}k_1+1\right) + 2\left(\frac{2}{3}k_2+1\right)\right]}{\left(\frac{2}{3}k_1+1\right)\left[2\,\frac{\alpha}{\beta}(1+k_2) + \frac{4}{3}k_2\right] - \left[\frac{2}{3}k_2 + \frac{\alpha}{\beta}\left(\frac{2}{3}k_2+1\right)\right]} \cdot \frac{E\varepsilon t I_1}{h L_1}$$

Si $L_1 = L_2 = L \quad ; \quad I_0 = I_1 \quad , \quad \alpha = 1 \quad , \quad \beta = 1$

$$H_A = H_D = \frac{15(4k_2+3)}{V'} \cdot \frac{E\varepsilon t I_1}{h^2} \quad ; \quad H_B = H_C = \frac{15(2k_1+3)}{V'} \cdot \frac{E\varepsilon t I_1}{h^2}$$

$$V_A = V_B = V_C = V_D = \frac{9(2k_1+8k_2+9)}{V'} \cdot \frac{E\varepsilon t I_1}{hL} \quad ; \quad M_E = M_L = -\frac{15(4k_2+3)}{V'} \cdot \frac{E\varepsilon t I_1}{h}$$

$$M_{FE} = M_{GL} = \frac{6(3k_1+2k_2+6)}{V'} \cdot \frac{E\varepsilon t I_1}{h} \quad ; \quad M_{FB} = M_{GC} = -\frac{15(2k_1+3)}{V'} \cdot \frac{E\varepsilon t I_1}{h}$$

$$M_{FG} = M_{GF} = -\frac{12(k_1-k_2)+9}{V'} \cdot \frac{E\varepsilon t I_1}{h}$$

Si $L_1 = L_2 = L \quad ; \quad I_0 = I_1 \quad ; \quad I_2 = I_3 \, , \quad k_1 = k_2 = k \quad ; \quad (20k^2+30k+9) = W$

$$H_A = H_D = \frac{15(4k+3)}{W} \cdot \frac{E\varepsilon t I_1}{h^2} \quad ; \quad H_B = H_C = \frac{15(2k+3)}{W} \cdot \frac{E\varepsilon t I_1}{h^2}$$

$$V_A = V_B = V_C = V_D = \frac{9(10k+9)}{W} \cdot \frac{E\varepsilon t I_1}{hL} \quad ; \quad M_E = M_L = -\frac{15(4k+3)}{W} \cdot \frac{E\varepsilon t I_1}{h}$$

$$M_{FE} = M_{GL} = \frac{6(5k+6)}{W} \cdot \frac{E\varepsilon t I_1}{h} \quad ; \quad M_{FB} = M_{GC} = -\frac{15(2k+3)}{W} \cdot \frac{E\varepsilon t I_1}{h} \quad ; \quad M_{FG} = M_{GF} = -\frac{9}{W} \cdot \frac{E\varepsilon t I_1}{h}$$

Nota : En caso de descenso de temperatura, los signos de los esfuerzos son opuestos a las presentadas.

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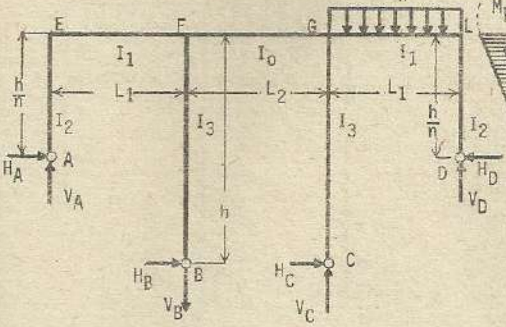

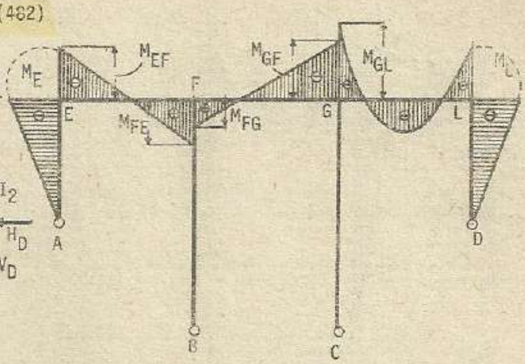

$$k_1 = \frac{I_2 L_1 n}{I_1 h} \quad ; \quad k_2 = \frac{I_0 L_1}{I_1 L_2} \quad ; \quad k_3 = \frac{I_3 L_1}{I_1 h} \quad ; \quad k_4 = \frac{I_3}{n I_2}$$

$$\alpha = 6\left[2(n^2+k_4)(1+2k_2)+k_3(2+3k_2)+2n^2+2n\right] \quad ; \quad \beta = (4+3k_1)(4+2k_2+3k_3) - 4$$

$$M_E = \frac{wL_1^2}{8}\left[\frac{6k_3(n+1+k_2)}{\alpha} - \frac{k_1(6+2k_2+3k_3)}{\beta}\right]$$

$$M_{FB} = \frac{wL_1^2}{8}\left[\frac{6nk_3(n+1+k_2)}{\alpha} - \frac{3k_3(2+k_1)}{\beta}\right] \quad ; \quad M_{GF} = -\frac{wL_1^2 k_2}{4}\left[\frac{3\left[2(n^2+k_4)-k_3(n-1)\right]}{\alpha} + \frac{(2+k_1)}{\beta}\right]$$

$$M_{GL} = -\frac{wL_1^2}{8}\left[\frac{12k_2(n^2+k_4)+6k_3\,(n^2+n+k_2)}{\alpha} + \frac{(2+k_1)\,(2k_2+3k_3)}{\beta}\right]$$

$$M_{GC} = -\frac{wL_1^2}{8}\left[\frac{6nk_3(n+1+k_2)}{\alpha} + \frac{3k_3(2+k_1)}{\beta}\right] \quad ; \quad M_L = -\frac{wL_1^2}{8}\left[\frac{6k_3(n+1+k_2)}{\alpha} + \frac{k_1(6+2k_2+3k_3)}{\beta}\right]$$

$$M_{FG} = \frac{wL_1^2 k_2}{4}\left[\frac{6(n^2+k_4)-3k_3(n-1)}{\alpha} - \frac{(2-k_1)}{\beta}\right]$$

$$M_{FE} = \frac{wL_1^2}{8}\left[\frac{6\left[2k_2(n^2+k_4)+k_3(n^2+n+k_2)\right]}{\alpha} - \frac{(2+k_1)(2k_2+3k_3)}{\beta}\right]$$

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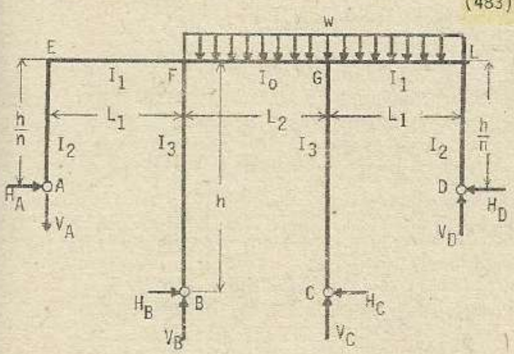

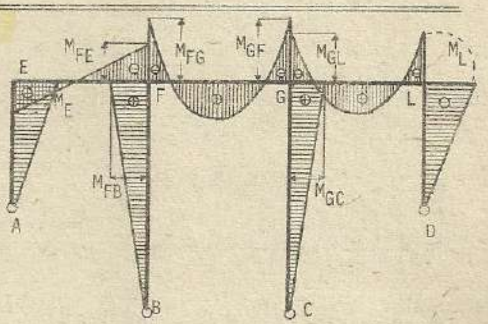

$$k_1 = \frac{I_2 L_1 n}{I_1 h} \quad ; \quad k_2 = \frac{I_0 L_1}{I_1 L_2} \quad ; \quad k_3 = \frac{I_3 L_1}{I_1 h} \quad ; \quad k_4 = \frac{I_3}{n L_2}$$

$$\alpha = 6\left[2(n^2+K_4)(1+2K_2) + K_3(2+3K_2) + 2nK_3(n+1)\right] \quad ; \quad \beta = (4+3K_1)(4+2K_2+3K_3) - 4$$

$$M_{FE} = \frac{wL_1^2}{8}\left[\frac{6\left[2K_2(n^2+K_4)+K_3(n^2+n+K_2)\right]}{\alpha} - \frac{(2-K_1)(2K_2+3K_3)}{\beta}\right] - \frac{wL_2^2\,(1+K_1)}{\beta}$$

$$M_{FG} = \frac{wL_1^2K_2}{4}\left[\frac{6(n^2+K_4)-3K_3(n-1)}{\alpha} - \frac{(2+K_1)}{\beta}\right] - \frac{wL_2^2\left[K_3(4+3K_1)+4(1+K_1)\right]}{4\beta}$$

$$M_{FB} = \frac{wL_1^2}{8}\left[\frac{6nK_3(n+1+K_2)}{\alpha} - \frac{3K_3(2+K_1)}{\beta}\right] + \frac{wL_2^2k_3(4+3K_1)}{4\beta}$$

$$M_E = \frac{wL_1^2}{8}\left[\frac{6K_3(n+1+K_2)}{\alpha} - \frac{K_1(6+2K_2+3K_3)}{\beta}\right] + \frac{K_1wL_2^2}{2\beta}$$

$$M_{GF} = -\frac{wL_1^2K_2}{4}\left[\frac{3\left[2(n^2+K_4) - K_3(n-1)\right]}{\alpha} + \frac{(2+K_1)}{\beta}\right] - \frac{wL_2^2}{4\beta}\cdot\left[K_3(4+3K_1)+4(1+K_1)\right]$$

$$M_{GL} = -\frac{wL_1^2}{8}\left[\frac{12K_2(n^2+k_4)+6K_3(n^2+n+K_2)}{\alpha} + \frac{(2+K_1)(2K_2+3K_3)}{\beta}\right] - \frac{wL_2^2(1+K_1)}{\beta}$$

$$M_{GC} = -\frac{wL_1^2}{8}\left[\frac{6nK_3(n+1+K_2)}{\alpha} + \frac{3K_3(2+K_1)}{\beta}\right] + \frac{wL_2^2K_3(4+3K_1)}{4\beta}$$

$$M_L = -\frac{wL_1^2}{8}\left[\frac{6K_3(n+1+k_2)}{\alpha} + \frac{K_1(6+2K_2+3K_3)}{\beta}\right] + \frac{K_1wL_2^2}{2\beta}$$

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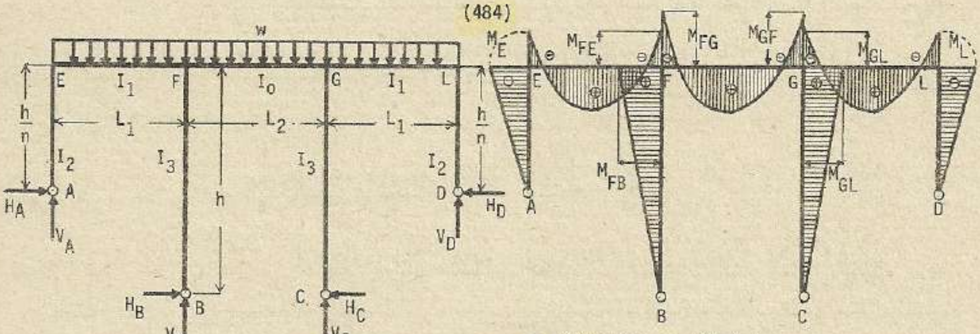

$$\beta = (4+3K_1)(4+2K_2+3K_3)-4$$

$$K_1 = \frac{I_2L_1n}{I_1h} \; ; \quad K_2 = \frac{I_oL_1}{I_1L_2} \quad ; \quad k_3 = \frac{I_3L_1}{I_1h} \; ; \quad K_4 = \frac{I_3}{nI_2}$$

$$M_E = -\frac{wk_1}{4\beta}\left[L_1^2(6+2K_2+3K_3) - 2L_2^2\right] \quad ; \quad M_{FE} = -\frac{w}{4\beta}\left[L_1^2(2+K_1)(2K_2+3K_3) + 4(1+k_1)L_2^2\right]$$

$$M_{FG} = -\frac{w}{4\beta}\left[2K_2(2+K_1)L_1^2 + L_2^2\left[K_3(4+3k_1) + 4(1+k_1)\right]\right]$$

$$M_{FB} = -\frac{wK_3}{4\beta}\left[3L_1^2(2+K_1) - L_2^2(4+3K_1)\right] \quad ; \quad M_{GF} = M_{FG} \quad ; \quad M_{GL} = M_{FE}$$

$$M_{GC} = M_{FB} \quad ; \quad M_L = M_E$$

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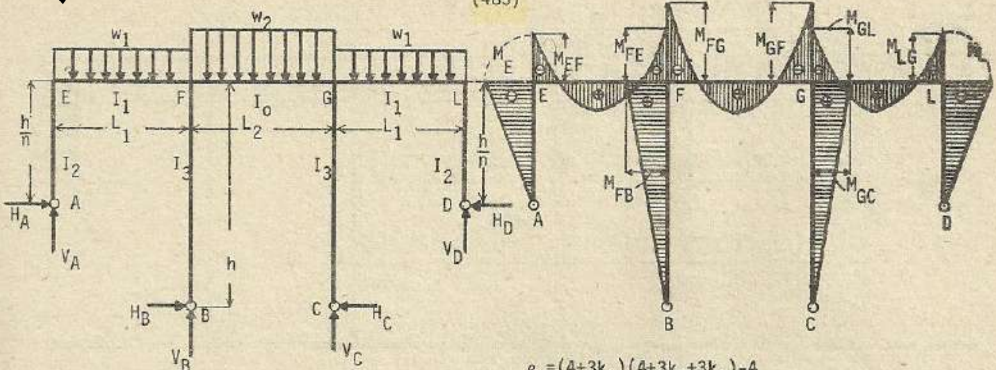

$$\beta = (4+3k_1)(4+3k_2+3k_3)-4$$

$$k_1 = \frac{I_2 L_1 n}{I_1 h} \quad ; \quad k_2 = \frac{I_o L_1}{I_1 L_2} \quad ; \quad k_3 = \frac{I_3 L_1}{I_1 h} \quad ; \quad k_4 = \frac{I_3}{n I_2}$$

$$M_E = -\frac{k_1}{4\beta}\left[w_1 L_1^2(6+2k_2+3k_3)-2w_2L_2^2\right] \quad ; \quad M_{FE} = -\frac{1}{4\beta}\left[w_1L_1^2(2k_2+3k_3)(2+k_1)+ 4w_2L_2^2(1+k_1)\right]$$

$$M_{FG} = -\frac{1}{4\beta}\left[2w_1L_1^2k_2(2+k_1)+w_2L_2^2\left[4(1+k_1)+k_3(4+3k_1)\right]\right] \quad ; \quad M_{GF} = M_{FG} \quad ; \quad M_{GL} = M_{FE}$$

$$M_{FB} = -\frac{k_3}{4\beta}\left[3w_1L_1^2(2+k_1) - w_2L_2^2(4+3K_1)\right] \quad ; \quad M_{GC} = M_{FB} \quad ; \quad M_L = M_E$$

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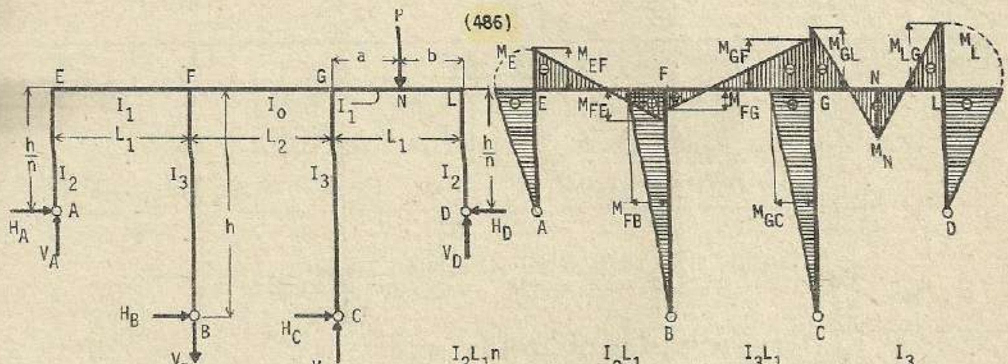
(486)



$$k_1 = \frac{I_2 L_1 n}{I_1 h} \quad ; \quad k_2 = \frac{I_o L_1}{I_1 L_2} \quad ; \quad k_3 = \frac{I_3 L_1}{I_1 h} \quad ; \quad k_4 = \frac{I_3}{n I_2}$$

$$\alpha = 6\left[2(h^2+k_4)(1+2k_2)+k_3(2+3k_2)+2nk_3(n+1)\right] \quad ; \quad \beta = (4+3k_1)(4+2k_2+3k_3)-4$$

$$M_E = \frac{3k_1 Pab}{2L_1^2}\left[\frac{2k_4\left[a(3k_2+n+2)+b(2b+1)\right]}{\alpha} - \frac{a(4+2K_2+3k_3)+2b}{\beta}\right]$$

$$M_{FE} = \frac{Pab}{2L_1^2}\left[\frac{12k_2(n^2+k_4)(a+2b)+6k_3\left[b(3k_2+2n^2+n)+an(n+2)\right]}{\alpha} - \frac{(2k_2+3k_3)\left[2a+b(4+3k_1)\right]}{\beta}\right]$$

$$M_{FG} = \frac{Pabk_2}{L_1^2}\left[\frac{6(n^2+k_4)(2b+a)+9k_3(b-an)}{\alpha} - \frac{b(4+3k_1)+2a}{\beta}\right]$$

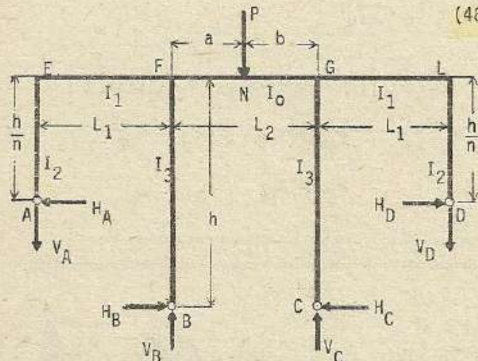
$$M_{FB} = \frac{3Pabk_3}{2L_1^2}\left[\frac{2n\left[a(2+3k_2+n) + b(2n+1)\right]}{\alpha} - \frac{b(4+3k_1)+2a}{\beta}\right]$$

$$M_{GF} = -\frac{Pabk_2}{L_1^2}\left[\frac{6(n^2+k_4)(2b+a)+9k_3(b-an)}{\alpha} + \frac{b(4+3k_1)+2a}{\beta}\right]$$

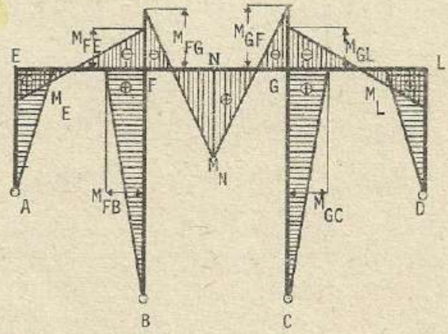
$$M_{GL} = -\frac{Pab}{2L_1^2}\left[\frac{12k_2(a+2b)(n^2+k_4)+6k_3\left[b(3k_2+2n^2+n)+an(n+2)\right]}{\alpha} + \frac{(2k_2+3k_3)\left[2a+b(4+3k_1)\right]}{\beta}\right]$$

$$M_{GC} = -\frac{3Pabk_3}{2L_1^2}\left[\frac{2n\left[a(2+3k_2+n)+b(2n+1)\right]}{\alpha} + \frac{b(4+3k_1)+2a}{\beta}\right]$$

$$M_D = -\frac{3k_1Pab}{2L_1^2}\left[\frac{2k_4\left[a(2+3k_2+n)+b(2n+1)\right]}{\alpha} + \frac{a(4+2k_2+3k_3)+2b}{\beta}\right]$$



(487)



$$k_1 = \frac{I_2L_1n}{I_1h} \; ; \quad k_2 = \frac{I_oL_1}{I_1L_2} \; ; \quad k_3 = \frac{I_3L_1}{I_1h} \; ; \quad k_4 = \frac{I_3}{nI_2}$$

$$\alpha = 6\left[2(n^2+k_4)(1+2k_2)+k_3(2+3k_2+2n^2+2n)\right] \quad ; \quad \beta = (4+3k_1)(4+2k_2+3k_3)-4$$

$$M_E = \frac{3Pab}{L_2^2}\left[\frac{k_3(b-a)(2n+1)}{\alpha} + \frac{k_1(a+b)}{\beta}\right] ; \quad M_{FB} = \frac{3Pabk_3}{2L_2^2}\left[\frac{2n(b-a)(2n+1)}{\alpha} + \frac{(4+3k_1)(a+b)}{\beta}\right]$$

$$M_{FG} = -\frac{3Pab}{2L_2^2}\left[\frac{4(b-a)\left[(n^2+k_4)+k_3(n^2+n+1)\right]}{\alpha} + \frac{(a+b)\left[4(1+k_1)+k_3(4+3k_1)\right]}{\beta}\right]$$

$$M_{FE} = -\frac{3Pab}{L_2^2}\left[\frac{(b-a)\left[2n^2+2k_4+k_3(n+2)\right]}{\alpha} + \frac{2(a+b)(1+k_1)}{\beta}\right]$$

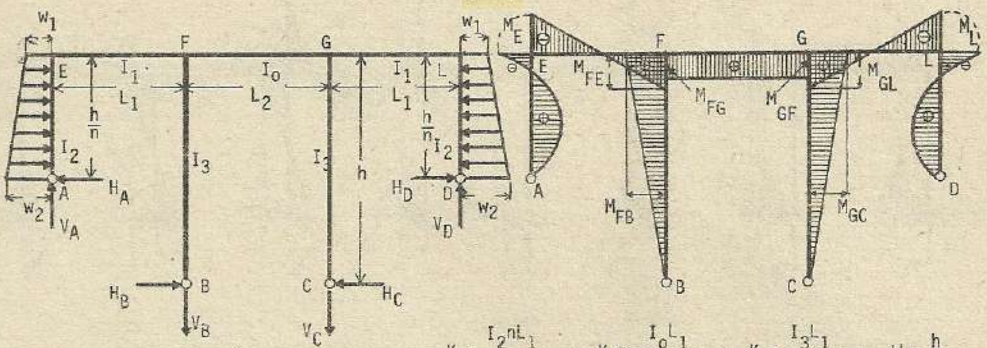
$$M_{GF} = -\frac{3Pab}{2L_2^2}\left[\frac{(a+b)\left[4(1+k_1)+k_3(4+3k_1)\right]}{\beta} - \frac{4(b-a)\left[(n^2+k_4)+k_3(n^2+n+a)\right]}{\alpha}\right]$$

$$M_{GL} = -\frac{3Pab}{L_2^2}\left[\frac{2(a+b)(1+k_1)}{\beta} - \frac{(b-a)\left[2n^2+2k_4+k_3(n+2)\right]}{\alpha}\right]$$

$$M_{GC} = \frac{3Pabk_3}{2L_2^2}\left[\frac{(a+b)(4+3k_1)}{\beta} - \frac{2n(b-a)(2n+1)}{\alpha}\right]$$

$$M_L = \frac{3Pab}{L_2^2}\left[\frac{k_1(a+b)}{\beta} - \frac{k_3(b-a)(2n+1)}{\alpha}\right]$$

(488)



$K_1 = \dfrac{I_2 n L_1}{I_1 h}$; $K_2 = \dfrac{I_o L_1}{I_1 L_2}$; $K_3 = \dfrac{I_3 L_1}{I_1 h}$; $h' = \dfrac{h}{n}$

$\beta = (4+3K_1)(4+2K_2+3K_3)-4$: $\gamma = \dfrac{h'^2}{120}(15w_1+7mh')$: $mh' = w_2 - w_1$

reemplazando término (mh') ; $\gamma = \dfrac{h'^2}{120}(8w_1+7w_2)$

$M_E = M_L = -\dfrac{4(3+2K_2+3K_3)\gamma}{\beta}$; $M_{FG} = M_{GF} = \dfrac{4K_2\gamma}{\beta}$; $M_{FE} = M_{GL} = \dfrac{2(2K_2+3K_3)\gamma}{\beta}$

$M_{FB} = M_{GC} = \dfrac{6K_3\gamma}{\beta}$

Esfuerzos por cambio de temperatura

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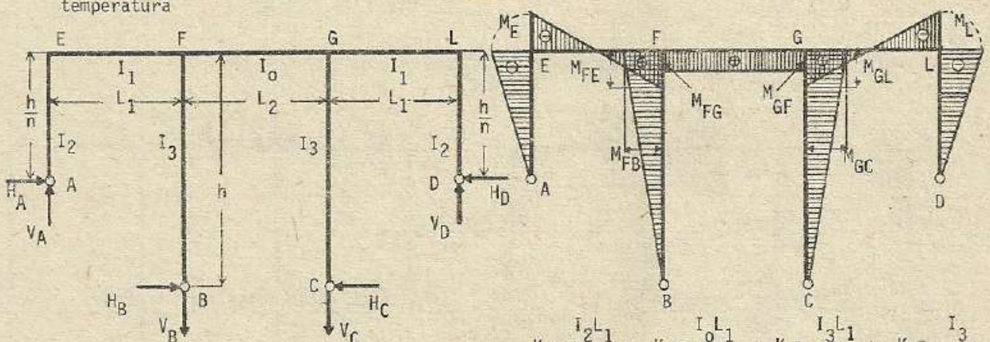


$K_1 = \dfrac{I_2 L_1}{I_1 h}$; $K_2 = \dfrac{I_o L_1}{I_1 L_2}$; $K_3 = \dfrac{I_3 L_1}{I_1 h}$; $K_4 = \dfrac{I_3}{I_2 n}$

$h' = \dfrac{h}{n}$; $\mu = \dfrac{I_2 n}{h}$; δ_1 ; Leve deformación de L_1, dilatación (+) contracción (-)

δ_2 ; Leve deformación de L_2, dilatación (+) contracción (-)

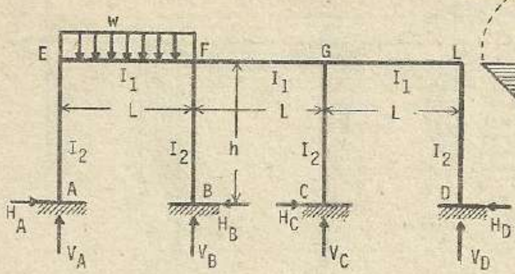
$s_1 = E\delta_1$; $s_2 = E\delta_2$; $\beta = (4+3K_1)(4+2K_2+3K_3)-4$

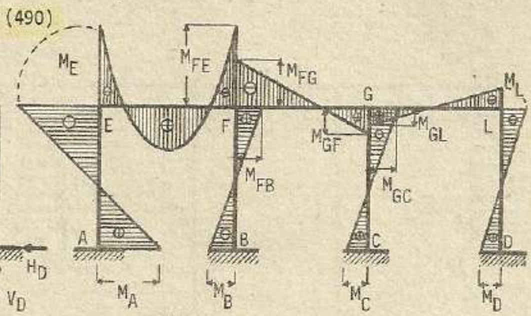
$$M_E = M_L = -\frac{3\mu}{\beta h}\left[3K_3 s_2 + 2n^2(3+2K_2+3K_3)(2s_1+s_2)\right]$$

$$M_{FE} = M_{GL} = \frac{3\mu}{\beta h}\left[n^2(2K_2+3K_3)(2s_1+s_2)+6(K_4+K_3)s_2\right]$$

$$M_{FG} = M_{GF} = \frac{3\mu k_2}{\beta h}\left[2n^2(2s_1+s_2)-(4K_4+3K_3)s_2\right]$$

$$M_{FB} = M_{GC} = \frac{3\mu k_4}{\beta h}\left\{3n_2K_1(2s_1+s_2)+\left[(4+3k_1)(2+k_2)-2\right]s_2\right\}$$





$$K = \frac{I_1 h}{I_2 L} \quad : \quad (36k^2+41k+4) = Y \quad ; \quad (5k^2+10k+4) = Z$$

$$V_A = \frac{(624k^4+2{,}099k^3+2{,}244k^2+816k+64)}{8YZ} \cdot wL \quad ; \quad V_B = \frac{(936k^4+2{,}681k^3+2{,}484k^2+832k+64)}{8YZ} \cdot wL$$

$$V_C = -\frac{(144k^4+339k^3+180k^2+16k)}{8YZ} \cdot wL \quad ; \quad V_D = \frac{(24k^2+79k+44)k^2}{8YZ} \cdot wL$$

$$H_A = \frac{(117k^3+213k^2+95k+8)}{4YZ} \cdot \frac{wL^2}{h} \quad ; \quad H_B = \frac{(198k^3+385k^2+186k+16)}{8YZ} \cdot \frac{wL^2}{h}$$

$$H_C = \frac{(18k^2+5k+2)k}{8YZ} \cdot \frac{wL^2}{h} \quad ; \quad H_D = \frac{(27k^2+23k+3)k}{4YZ} \cdot \frac{wL^2}{h}$$

$$M_B = -\frac{(213k^3+415k^2+198k+16)}{24YZ} \cdot wL^2 \quad ; \quad M_A = \frac{(219k^3+396k^2+178k+16)}{24YZ} \cdot wL^2$$

$$M_C = -\frac{(3k^3-25k^2-10k)}{24YZ} \cdot wL^2 \quad ; \quad M_D = \frac{(69k^3+76k^2+18k)}{24YZ} \cdot wL^2$$

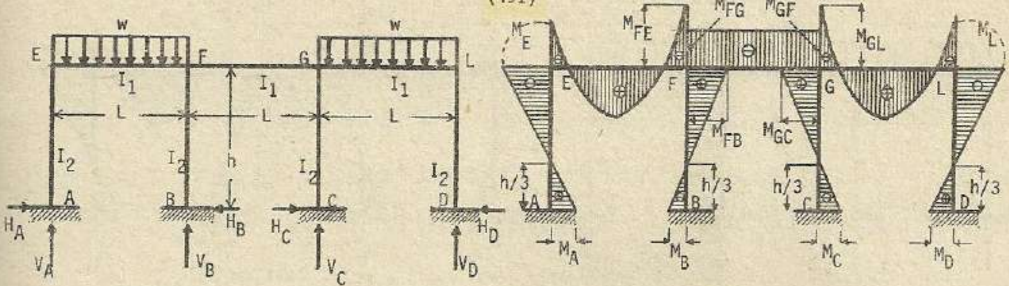
$$M_E = -\frac{(483k^3+882k^2+392k+32)}{24YZ} \cdot wL^2 \quad ; \quad M_{FE} = -\frac{(144k^4+483k^3+519k^2+196k+16)}{12YZ} \, wL^2$$

$$M_{FB} = \frac{(381k^3+740k^2+360k+32)}{24YZ} \cdot wL^2 \quad ; \quad M_{FG} = -\frac{(288k^4+585k^3+298k^2+32k)}{24YZ} \cdot wL^2$$

$$M_{GF} = \frac{(72k^4+195k^3+110k^2+16k)}{24YZ} \cdot wL^2 \quad ; \quad M_{GC} = -\frac{(51k^3+40k^2+16k)}{24YZ} \cdot wL^2$$

$$M_{GL} = \frac{(36k^2+72k+35)k^2}{12YZ} \, wL^2 \quad ; \quad M_E = -\frac{(93k+62)k^2}{24YZ} \cdot wK^2$$

(491)



$$Z = (5k^2+10k+4) \qquad k = \frac{I_1 h}{I_2 L}$$

$$V_A = V_D = \frac{(9k^2+20K+8)}{4Z}\cdot wL \;;\quad V_B = V_C = \frac{(11k^2+20k+8)}{4Z}\cdot wL \;;\quad H_A = H_D = \frac{(1+2K)}{2Z}\cdot\frac{wL^2}{h}$$

$$H_B = H_C = \frac{(2+3K)}{4Z}\cdot\frac{wL^2}{h} \;;\quad M_A = M_D = \frac{(1+2K)}{6Z}\cdot wL^2 \;;\quad M_B = M_C = -\frac{(2+3K)}{12Z}\cdot wL^2$$

$$M_E = M_L = -\frac{(1+2K)}{3Z}\cdot wL^2 \;;\quad M_{FE} = M_{GL} = -\frac{(3k^2+8k+4)}{12Z}\cdot wL^2 \;;\quad M_{FG} = M_{GF} = -\frac{(2+3k)k}{12Z}\cdot wL^2$$

$$M_{FB} = M_{GC} = \frac{(2+3K)}{6Z}\cdot wL^2 \;;$$

Considerando como (x) la distancia, desde los puntos (F) y (G), las ubicaciones de los máximos momentos de (EF) y (GL), tenemos :

$$x_m = \frac{(11k^2+20k+8)}{4Z}\cdot L \quad \therefore \quad M_{máx.} = \frac{(243k^4+760k^3+832k^2+384k+64)}{96\,Z^2}\cdot wL^2$$

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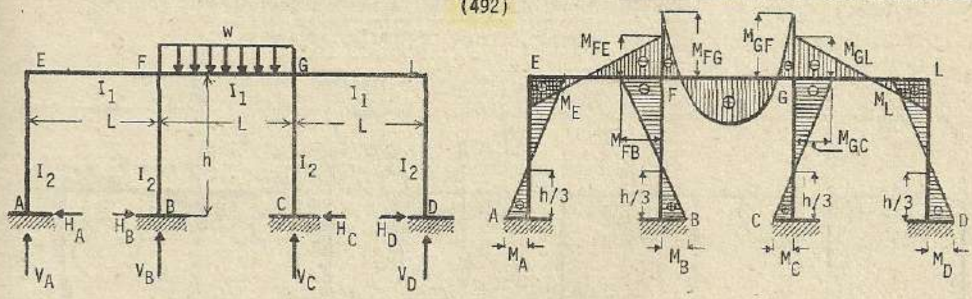
(492)



$$k = \frac{I_1 h}{I_2 L} \;;\quad V_A = V_D = \frac{k(2+k)}{4Z}\cdot wL \;;\quad V_B = V_C = \frac{(11k^2+22k+8)}{4Z}\cdot wL$$

$$H_A = H_D = \frac{k}{4Z}\cdot\frac{wL^2}{h} \;;\quad H_B = H_C = \frac{(1+k)}{2Z}\cdot\frac{wL^2}{h} \;;\quad M_A = M_D = -\frac{k}{12Z}\cdot wL^2$$

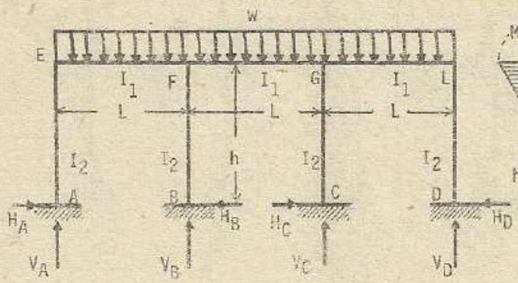
$$M_B = M_C = \frac{(1+k)}{6Z}\cdot wL^2 \;;\quad M_E = M_L = \frac{k}{6Z}\cdot wL^2 \;;\quad M_{FE} = M_{GL} = -\frac{k(4+3k)}{12Z}\cdot wL^2$$

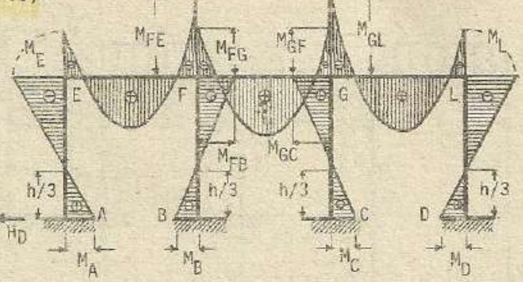
$$M_{FG} = -\frac{(4+8k+3k^2)}{12Z}\cdot wL^2 \;;\quad M_{FB} = M_{GC} = -\frac{(1+k)}{3Z}\cdot wL^2 \;;$$

$$M_{máx.} = \frac{(9k^2+14k+4)}{24\,Z}\cdot wL^2$$ (respecto al punto central de la viga FG)

$$Z = (5k^2+10k+4)$$

(493)





$$k = \frac{I_1 h}{I_2 L} \quad ; \quad Z = (5k^2+10k+4)$$

$$H_A = H_D = \frac{(3k+2)}{4\,Z}\cdot\frac{wL^2}{h} \quad ; \quad H_B = H_C = \frac{k}{4\,Z}\cdot\frac{wL^2}{h} \quad ; \quad V_A = V_D = \frac{4k^2+9k+4}{2\,Z}\cdot wL$$

$$V_B = V_C = \frac{11k^2+21k+8}{2\,Z}\cdot wL \quad ; \quad M_A = M_D = \frac{3k+2}{12\,Z}\cdot wL^2 \quad ; \quad M_B = M_C = -\frac{k}{12\,Z}\cdot wL^2$$

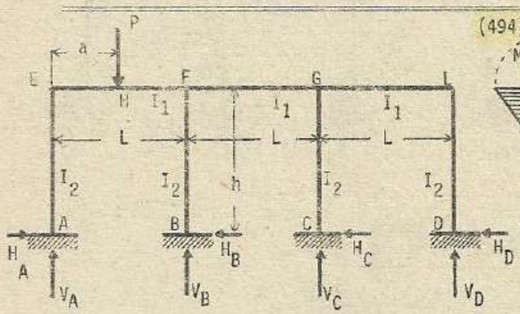
$$M_E = M_L = -\frac{3k+2}{6\,Z}\cdot wL^2 \quad ; \quad M_{FE} = M_{GL} = -\frac{3k^2+6k+2}{6\,Z}\cdot wL^2 \quad ; \quad M_{FG} = M_{GF} = -\frac{3k^2+5k+2}{6\,Z}\cdot wL^2$$

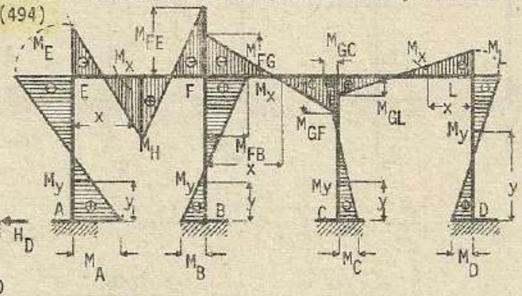
$$M_{FB} = M_{GC} = \frac{k}{6\,Z}\cdot wL^2 \quad ; \quad M_{máx.} = \frac{3k^2+10k+4}{24\,Z}\cdot wL^2$$

(Respecto al punto central de la viga FG .)

Cuando en los pórticos de figuras simétricas, accionan cargas igualmente simétricas, Los esfuerzos de momentos por flexión de las columnas, habitualmente se encierran - dentro de una línea recta, y el punto de inflexión se ubica a 1/3 de la altura, desde la base. Esto es, los esfuerzos de momentos de la parte superior de la columna, es el doble que el de la parte inferior, siendo contrario sus signos.

(494)





$$K = \frac{I_1 h}{I_2 L} \quad ; \quad A = \left(1 - \frac{a}{L}\right) \quad ; \quad Y = (36k^2+41k+4) \quad ; \quad Z = (5k^2+10k+4)$$

$$H_A = 3PaA\left[\frac{(84k^3+160k^2+76k+6) - \frac{a}{L}(51k^3+107k^2+57k+4)}{Y\,Z\,h}\right]$$

$$H_B = 3PaA\left[\frac{(96k^3+166k^2+62k+4) + \frac{a}{L}(6k^3+53k^2+62k+8)}{2\,Y\,Z\,h}\right]$$

$$H_C = 3PaA\left[\frac{-(24k^3+84k^2+54k+4) + \frac{a}{L}(66k^3+173k^2+110k+8)}{2\,Y\,Z\,h}\right]$$

$$H_D = 3PaA\left[\frac{(24k^3+35k^2+18k+2) - \frac{a}{L}(21k^3+47k^2+33k+4)}{Y\,Z\,h}\right]$$

$$V_A = P - \frac{Pa}{2\,Y\,Z\,L}\Big[3(152k^3+354k^2+234k+40)k + 3(229k^3+498k^2+288k+32)\frac{a}{L}$$
$$- (96k^4-619k^3+1{,}048k^2+576k+64)\left(\frac{a}{L}\right)^2\Big]$$

$$V_B = \frac{Pa}{2\,Y\,Z\,L}\Big[3(152k^3+354k^2+234k+40)k + 3(229k^3+498k^2+288k+32)\frac{a}{L}$$
$$- (96k^4+619k^3+1.048k^2+576k+64)\left(\frac{a}{L}\right)^2\Big] + \frac{12kaA\left[k(1+\frac{a}{L}) + (2\frac{a}{L} - \frac{3}{4})\right]}{Y\,L}P$$

$$V_C = PakA\frac{\left[(144k^3+298k^2+78k) + (144k^3+441k^2+384k+48)\frac{a}{L}\right]}{2\,Y\,Z\,L}$$

$$V_D = 3PakA\frac{\left[(8k^3+48k^2+54k+24) + (8k^3-13k^2-64k-48)\frac{a}{L}\right]}{2\,Y\,Z\,L}$$

$$M_A = PaA\frac{\left[(148k^3+270k^2+116k+4) - (77k^3+144k^2+54k-8)\frac{a}{L}\right]}{2\,Y\,X}$$

$$M_E = -PaA\frac{\left[(356k^3+690k^2+340k+32) - (229k^3+498k^2+288k+32)\frac{a}{L}\right]}{2\,Y\,Z}$$

$$M_{FE} = -PaA\frac{\left[(48k^3+144k^2+122k+26)k + (48k^4+195k^3+275k^2+144k+16)\frac{a}{L}\right]}{Y\,Z}$$

$$M_{FG} = -PakA\frac{\left[(96k^3+116k^2-38k-36) + (k+2)(96k^2+161k+52)\frac{a}{L}\right]}{2\,Y\,Z}$$

$$M_{FB} = PaA\frac{\left[(172k^2+282k+88)k + (37k^3+176k^2+184k+32)\frac{a}{L}\right]}{2\,Y\,Z}$$

$$M_B = PaA\frac{\left[(116k^3+216k^2+98k+12) - (19k^3+17k^2-2k+8)\frac{a}{L}\right]}{2\,Y\,Z}$$

$$M_{GF} = PakA\frac{\left[(24k^3+34k^2-46k-36) + (k+2)(24k^2+79k+44)\frac{a}{L}\right]}{2\,Y\,Z}$$

$$M_{GL} = PakA\frac{\left[(12k^3+31k^2+36k+18) + (12k^3+10k^2-37k-36)\frac{a}{L}\right]}{Y\,Z}$$

$$M_{GC} = PakA\frac{\left[-(28k^2+118k+72) + (107k^2+276k+160)\frac{a}{L}\right]}{2\,Y\,Z}$$

$$M_C = PaA\frac{\left[-(44k^3+134k^2+90k+12) + (91k^3+243k^2+170k+24)\frac{a}{L}\right]}{2\,Y\,Z}$$

$$M_L = -PakA\frac{\left[(76k^2+90k+36) - (56k^2+118k+72)\frac{a}{L}\right]}{2\,Y\,Z}$$

$$M_D = PakA\frac{\left[(68k^3+120k^2+72k+12) - (67k^3+164k^2+126k+24)\frac{a}{L}\right]}{2Y\,Z}$$

Si $a = \frac{L}{2}$: $H_A = \frac{3PL(117k^3+213k^2+95k+8)}{8\,Y\,Z\,h}$; $H_B = \frac{3PL(198k^3+385k^2+186k+16)}{16\,Y\,Z\,h}$

$H_C = \frac{3PLk(18k^2+5k+2)}{16\,Y\,Z\,h}$; $H_D = \frac{3PLk(27k^2+23k+3)}{8\,Y\,Z\,h}$

$$V_A = \frac{1{,}152k^4+4{,}037k^3+4{,}436k^2+1{,}632k+128}{16\,Y\,Z}\,P$$

$$V_B = \frac{2{,}088k^4+5{,}783k^3+5{,}156k^2+1{,}680k+128}{16Y\,Z}\,P \quad ; \quad V_C = \frac{3k(144k^3+339k^2+180k+16)}{16Y\,Z}\,P$$

$$V_D = \frac{3k^2(24k^2+79k+44)}{16\,Y\,Z}\,P \quad ; \quad M_A = \frac{(219k^3+396k^2+178k+16)}{16\,Y\,Z}\,PL$$

$$M_E = -\frac{(483k^3+882k^2+392k+32)}{16\,Y\,Z}\,PL \quad ; \quad M_{FE} = -\frac{(144k^4+483k^3+519k^2+196k+16)}{8\,Y\,Z}\,PL$$

$$M_{FG} = -\frac{k(288k^3+585k^2+298k+32)}{16\,Y\,Z}\,PL \quad ; \quad M_{FB} = \frac{(381k^3+740k^2+360k+32)}{16\,Y\,Z}\,PL$$

$$M_B = \frac{(213k^3+415k^2+198k+16)}{16\,Y\,Z}\,PL \quad ; \quad M_{GF} = \frac{k(72k^3+195k^2+110k+16)}{16\,Y\,Z}\,PL$$

$$M_{GL} = \frac{k(36k^3+103k^2+71k)}{8\,Y\,Z}\,PL \quad ; \quad M_{GC} = \frac{k(51k^2+40k+16)}{16\,Y\,Z}\,PL$$

$$M_C = \frac{(3k^3-25k^2-10k)}{16\,Y\,Z}\,PL \quad ; \quad M_L = -\frac{(93k^2+62k)}{16\,Y\,Z}\,PL \quad ; \quad M_D = \frac{(69k^3+76k^2+18k)}{16\,Y\,Z}\,PL$$

Momentos por flexión en las columnas:

AE $\longrightarrow$ $M_y = M_A - H_A y$: BF $\longrightarrow$ $M_y = M_B + H_B y$

CG $\longrightarrow$ $M_y = M_C + H_C y$: DL $\longrightarrow$ $M_y = M_D - H_D y$

Momentos en las vigas : EF $\longrightarrow$ $M_x = M_E + V_A x$ $\quad x \leq a$

$M_x = M_E + V_A x - P(x-a)$ $\quad x \geq a$

FG $\longrightarrow$ $M_x = M_{FG} + (V_A + V_B - P)x$

GL $\longrightarrow$ $M_x = M_L + V_D x$

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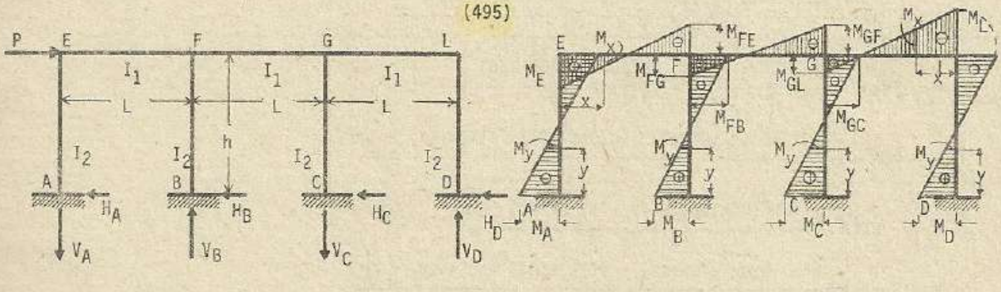
(495)



$$k = \frac{I_1 h}{I_2 L} \quad : \quad Y = (36k^2+41k+4)$$

$$H_A = \frac{(9k^2+8k+1)}{Y}\,P \quad ; \quad H_B = \frac{(18k^2+25k+2)}{2\,Y}\,P \quad ; \quad H_C = \frac{(18k^2+25k+2)}{2\,Y}\,P$$

$$H_D = \frac{(9k^2+8k+1)}{Y}\,P \quad ; \quad V_A = \frac{3k(5k+4)}{2\,Y}\cdot\frac{Ph}{L} \quad ; \quad V_B = \frac{9k^2}{2\,Y}\cdot\frac{Ph}{L} \quad ; \quad V_C = \frac{9k^2}{2Y}\cdot\frac{Ph}{L}$$

$$V_D = \frac{3k(5k+4)}{2\,Y}\cdot\frac{Ph}{L} \quad ; \quad M_A = -\frac{(9k^2+10k+2)}{2\,Y}\,Ph \quad ; \quad M_B = -\frac{(9k^2+13k+2)}{2\,Y}\,Ph$$

$M_C = \frac{(9k^2+13k+2)}{2\,Y} \cdot Ph$; $M_D = \frac{(9k^2+10k+2)}{2\,Y} \cdot Ph$; $M_E = \frac{3(3k+2)k}{2\,Y} \cdot Ph$

$M_{FE} = -\frac{3(k+1)k}{Y} \cdot Ph$; $M_{FG} = \frac{3(k+2)k}{2\,Y} \cdot Ph$; $M_{FB} = \frac{3(3k+4)k}{2\,Y} \cdot Ph$

$M_{GF} = \frac{3(k+2)k}{2\,Y} \cdot Ph$; $M_{GL} = \frac{3(k+1)k}{Y} \cdot Ph$; $M_{GC} = -\frac{3(3k+4)k}{Y} \cdot Ph$

$$M_L = -\frac{3(3k+2)}{2\,Y} \cdot Ph$$

Momentos por flexión en las columnas:

AE $\longrightarrow M_y = M_A + H_A y$: BF $\longrightarrow M_y = M_B + H_B y$

CG $\longrightarrow M_y = M_C - H_C y$: DL $\longrightarrow M_y = M_D - H_D y$

Momentos en las vigas:

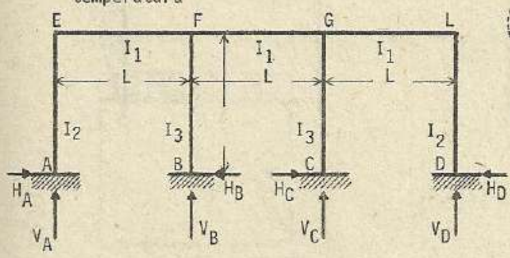
EF $\longrightarrow M_x = M_E - V_A x$

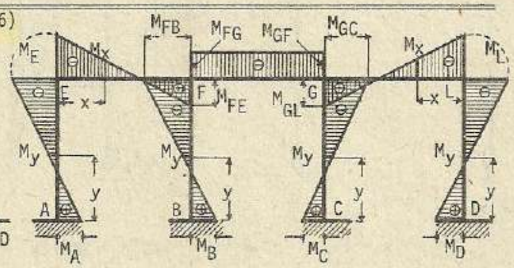
FG $\longrightarrow M_x = M_{FG} - (V_A - V_B)x$

GL $\longrightarrow M_x = M_L + V_D x$

Esfuerzos por cambio de temperatura

(496)





$k_1 = \frac{I_1 h}{I_2 L}$: $k_2 = \frac{I_1 h}{I_3 L}$: $C = (4k_1+6k_2+5k_1K_2+4)$; $Z = (5k^2+10k+4)$

$H_A = H_D = 3E\varepsilon t I_1 \frac{(19k_1+6k_2+20k_1k_2+4)}{h^2 k_1 C}$; $H_B = H_C = 6E\varepsilon t I_1 \frac{(k_1+9k_2+5k_1k_2+1)}{h^2 k_1 C}$

$V_A = V_B = V_C = V_D = 9E\varepsilon t I_1 \frac{(k_1+4k_2+6)}{hLC}$; $M_A = M_D = 3E\varepsilon t I_1 \frac{(9k_1+6k_2+10k_1k_2+4)}{hk_1 C}$

$M_B = M_C = 3E\varepsilon t I_1 \frac{(2k_1+8k_2+5k_1k_2+2)}{hk_2 C}$; $M_E = M_L = -30E\varepsilon t I_1 \frac{(k_2+1)}{hC}$

$M_{FE} = M_{GL} = 3E\varepsilon t I_1 \frac{(3k_1+2k_2+8)}{h\,C}$; $M_{FG} = M_{GF} = -6E\varepsilon t I_1 \frac{(1+k_1-k_2)}{h\,C}$

$$M_{FB} = M_{GC} = -15E\varepsilon t I_1 \frac{(2+k_1)}{h\,C}$$

Si $I_2 = I_3$, $k_1 = k_2 = k$

$H_A = H_D = 3E\varepsilon t I_1 \frac{(20k^2+25k+4)}{h^2 kZ}$; $H_B = H_C = 6E\varepsilon t I_1 \frac{(5k^2+10k+1)}{h^2 k\,Z}$

$V_A = V_B = V_C = V_D = 9E\varepsilon t I_1 \frac{(5k+6)}{hL\,Z}$; $M_A = M_D = 3E\varepsilon t I_1 \frac{(10k^2+15k+4)}{hk\,Z}$

$$M_B = M_C = 3E\varepsilon t I_1 \frac{(5k^2+10k+2)}{hk\,Z} \quad ; \quad M_E = M_L = 30E\varepsilon t I_1 \frac{(1+k)}{h\,Z}$$

$$M_{FE} = M_{GL} = 3E\varepsilon t I_1 \frac{(5k+8)}{h\,Z} \quad ; \quad M_{FG} = M_{GF} = -15E\varepsilon t I_1 \frac{(k+2)}{h\,Z}$$

$$M_{FB} = M_{GC} = -6E\varepsilon t I_1 \frac{1}{h\,Z}$$

Esfuerzos de momentos por flexión o doblado en las columnas :

AE y DL $\longrightarrow M_y = M_A - H_A y = M_D - H_D y$: BF y CG $\longrightarrow M_y = M_B - H_B y = M_C - H_C y$

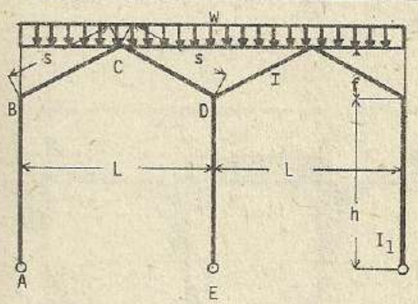
Momentos en las vigas :

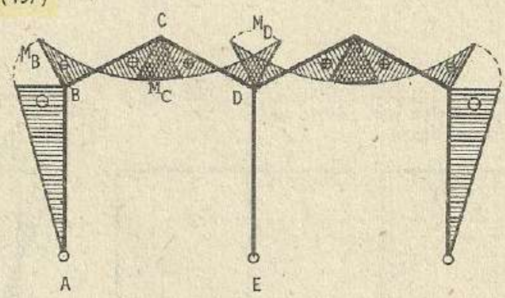
EF y LG $\longrightarrow M_x = M_E + V_A x = M_L + V_D x$

FG $\longrightarrow M = M_{FG} = M_{GF}$ (constante)

Nota : El problema desarrollado corresponde al caso de ascenso de la temperatura, en caso contrario, los signos serán contrarios a las presentadas.

(497)

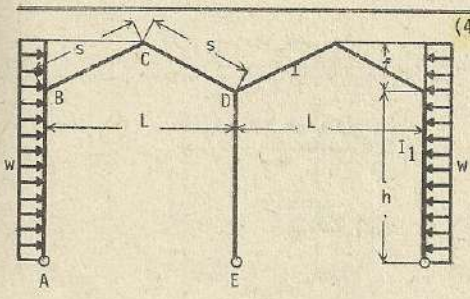


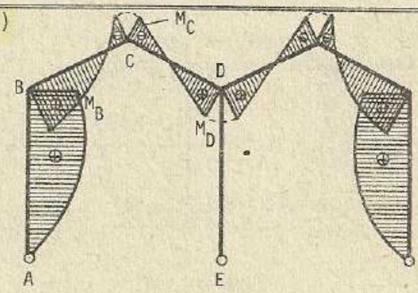


$$k = \frac{I\,h}{I_1 s} \quad ; \quad n = \frac{f}{h} \quad ; \quad \delta = 4(3+2k) + n(12+7n)$$

$$M_B = \frac{wL^2(1+n)}{\delta} \quad ; \quad M_C = \frac{wL^2}{16} \cdot \frac{8(1+k) - n(2+3n)}{\delta} \quad ; \quad M_D = -\frac{wL^2}{8} \cdot \frac{8(1+k) + n(2+n)}{\delta}$$

(498)





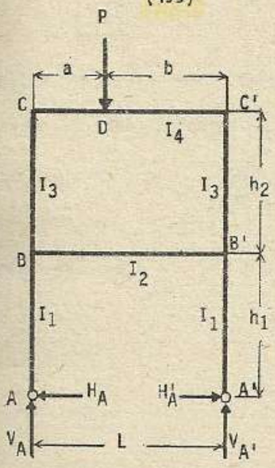
$$k = \frac{I\,h}{I_1 s} \quad ; \quad n = \frac{f}{h} \quad ; \alpha = 6+7n \quad ; \quad \gamma = 6(1+k)+n \quad ; \delta = 4(3+2k) + n(12+7n)$$

$$\beta = 2(3+n) + 5k$$

$$M_B = \frac{wf^2}{8}\left[\frac{26+37n}{\delta}\right] + \frac{wh^2}{2}\left[\frac{n\alpha - 2k}{\delta}\right] \quad ; \quad M_C = -\frac{wf^2}{8}\left[\frac{40+n(21+4n)+31k}{\delta}\right] + \frac{wh^2}{2}\left[\frac{4n\beta+k(2+5n)}{\delta}\right]$$

$$M_D = \frac{wf^2}{8}\left[\frac{2(19+17k) + n(13+2n)}{\delta}\right] + \frac{wh^2}{4}\left[\frac{2n\gamma+h(2+3n)}{\delta}\right]$$

(499)



$$k_1=\frac{I_2h_1}{I_1L} \quad ; \quad k_2=\frac{I_2h_2}{I_3L} \quad ; \quad \alpha=\frac{a}{L} \quad ; \quad \lambda=\frac{I_2}{I_4}$$

$$m=(3k_1+2k_1k_2+3k_2)(k_2+2\lambda)+k_1k_2 \quad : \quad n=1+6k_2+\lambda$$

$$V_A=P(1-\alpha) \quad ; \quad V_{A'}=P\alpha \quad ; \quad H_A=H_{A'}=\frac{M_{AB}}{h_1}$$

$$M_{BA}=M_{B'A'}=\frac{3Pa(1-\alpha)}{2}\cdot\frac{k_2\lambda}{m}$$

$$M_{BC}=\frac{Pa(1-\alpha)}{2}\left[\frac{k_2(3+2k_1)}{m}-\frac{1-2\alpha}{n}\right]$$

$$M_{B'C'}=\frac{Pa(1-\alpha)}{2}\left[\frac{k_2(3+2k_1)}{m}+\frac{1-2\alpha}{n}\right]$$

$$M_{BB'}=M_{BA}-M_{BC} \quad ; \quad M_{B'B}=M_{B'A'}-M_{B'C'}$$

$$M_C=-\frac{Pa(1-\alpha)\lambda}{2}\left[\frac{2(3k_1+2k_1K_2+3k_2)}{m}+\frac{1-2\alpha}{n}\right]$$

$$M_{C'}=-\frac{Pa(1-\alpha)\lambda}{2}\left[\frac{2(3k_1+2k_1k_2+3k_2)}{m}-\frac{1-2\alpha}{n}\right]$$

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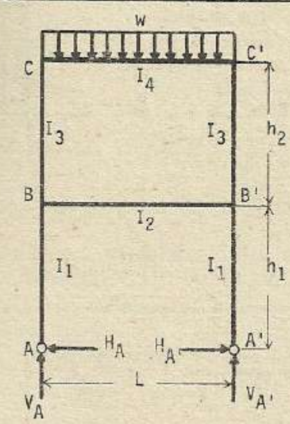
(500)



$$k_1=\frac{I_2h_1}{I_1L} \quad ; \quad k_2=\frac{I_2h_2}{I_3L} \quad : \text{ otros datos (499)}$$

$$V_A=V_A'=\frac{wL}{2} \quad ; \quad H_A=H_{A'}=\frac{M_{BA}}{h_1}$$

$$M_{BA}=M_{B'A'}=\frac{wL^2}{4}\cdot\frac{k_2\lambda}{m} \quad ; \quad M_{BB'}=M_{B'B}=-\frac{wL^2}{6}\cdot\frac{k_1k_2\lambda}{m}$$

$$M_{BC}=M_{B'C'}=M_{BA}-M_{BB'}$$

$$M_C=M_{C'}=-\frac{wL^2}{6}\cdot\frac{\lambda(3k_1+2k_1k_2+3k_2)}{m}$$

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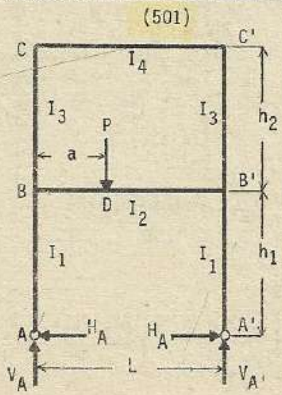
(501)



$$K_1=\frac{I_2h_1}{I_1L} \quad ; \quad k_2=\frac{I_2h_2}{I_3L} \quad ; \text{ otros datos (499)}$$

$$V_A=P(1-\alpha) \quad ; \quad V_{A'}=P\alpha \quad ; \quad H_A=H_{A'}=\frac{M_{BA}}{h_1}$$

$$M_{BA}=M_{B'A'}=-\frac{3Pa(1-\alpha)}{2}\cdot\frac{k_2(k_2+2\lambda)}{m}$$

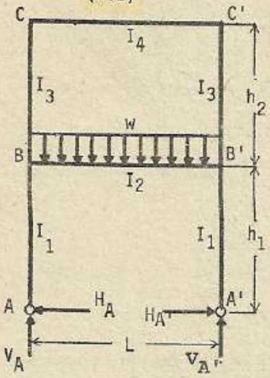
$$M_{BC}=\frac{Pa(1-\alpha)}{2}\left[\frac{2k_1(2k_2+3\lambda)}{m}+\frac{1-2\alpha}{n}\right]$$

$$M_{B'C'}=\frac{Pa(1-\alpha)}{2}\left[\frac{2k_1(2k_2+3\lambda)}{m}-\frac{1-2\alpha}{n}\right]$$

$$M_{BB'}=M_{BA}-M_{BC} \quad : \quad M_{B'B}=M_{B'A'}-M_{B'C'}$$

$$M_C = -\frac{Pa(1-\alpha)}{2}\left[\frac{2k_1k_2}{m} - \frac{1-2\alpha}{n}\right]; \quad M_{C'} = -\frac{Pa(1-\alpha)}{2}\left[\frac{2k_1k_2}{m} + \frac{1-2\alpha}{n}\right]$$

(502)



$$k_1 = \frac{I_2h_1}{I_1L} \quad ; \quad k_2 = \frac{I_2h_2}{I_3L} \quad ; \text{ otros datos (499)}$$

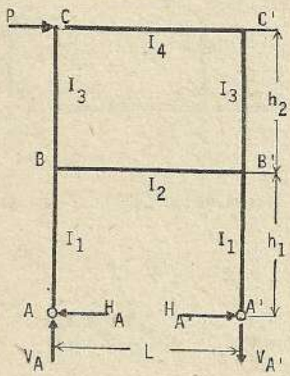
$$V_A = V_{A'} = \frac{wL}{2} \quad ; \quad H_A = H_{A'} = \frac{M_{BA}}{h_1}$$

$$M_{BA} = M_{B'A'} = -\frac{wL^2}{4}\cdot\frac{k_2(k_2+2\lambda)}{m} \quad ; \quad M_{BB'} = M_{B'B} = M_{BA} - M_{BC}$$

$$M_{BC} = M_{B'C'} = \frac{wL^2}{6}\cdot\frac{k_1(2k_2+3\lambda)}{m}$$

$$M_C = M_{C'} = -\frac{wL^2}{6}\cdot\frac{k_1k_2}{m}$$

(503)



$$K_1 = \frac{I_2h_1}{I_1L} \quad ; \quad K_2 = \frac{I_2h_2}{I_3L} \text{ : otros datos (499): } \beta = \frac{h_1}{h_2}$$

$$V_A = V_{A'} = -\frac{ph_2(1+\beta)}{L} \quad ; \quad H_A = -H_{A'} = \frac{P}{2} \quad ; \quad M_{BA} = P\frac{h}{2}$$

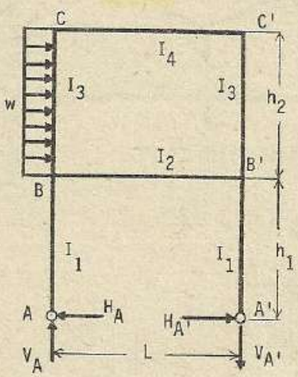
$$M_{B'A'} = -P\frac{h}{2} \quad ; \quad M_{BC} = -\frac{Ph_2}{2}\cdot\frac{(3k_2+\lambda-\beta)}{n}$$

$$M_{B'C'} = \frac{Ph_2}{2}\cdot\frac{(3k_2+\lambda-\beta)}{n} \quad ; \quad M_{B'B} = M_{B'A'} - M_{BC}$$

$$M_{B'B} = M_{B'A} - M_{B'C'} \quad ; \quad M_C = \frac{Ph_2}{2}\cdot\frac{(3k_2+1+\beta)}{n}$$

$$M_{C'} = -\frac{Ph_2}{2}\cdot\frac{(3k_2+1+\beta)}{n}$$

(504)



$$k_1 = \frac{I_2h_1}{I_1L} \quad ; \quad K_2 = \frac{I_2h_2}{I_3L} \quad ; \quad \beta = \frac{h_1}{h_2}$$

Otros datos (499)

$$V_A = -V_{A'} = -\frac{wh_2^2(1+2\beta)}{2L} \quad ; \quad H_A = \frac{M_{BA}}{h_1} \quad ; \quad H_{A'} = \frac{M_{B'A}}{h}$$

$$M_{BA} = \frac{wh_2^2}{8}\left[4\beta - \frac{k_2(k_2+3\lambda)}{m}\right]; \quad M_{B'A'} = -\frac{wh_2^2}{8}\left[4\beta + \frac{k_2(k_2+3\lambda)}{m}\right]$$

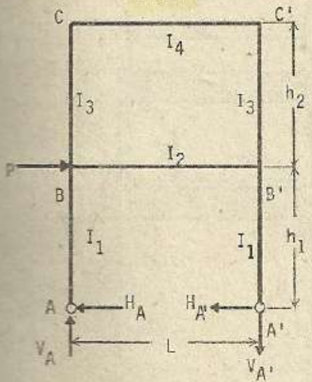
$$M_{BB'} = \frac{wh_2^2}{4}\left[\frac{4k_2(1+3\beta)+\lambda(1+2\beta)}{n} + \frac{k_1k_2(k_2+3\lambda)}{3m}\right]$$

$$M_{B'B} = -\frac{wh_2^2}{4}\left[\frac{4k_2(1+3\beta)+\lambda(1+2\beta)}{n} - \frac{k_1k_2(k_2+3\lambda)}{3m}\right]$$

$$M_{BC} = M_{BA} - M_{BB'} \quad ; \quad M_{B'C'} = M_{B'A'} - M_{B'B}$$

$$M_C = \frac{wh_2^2}{4}\left[\frac{2k_2+1+2\beta}{n} - \frac{k_2(6k_1+2k_1k_2+3k_2)}{6m}\right]; \quad M_{C'} = -\frac{wh_2^2}{4}\left[\frac{2k_2+1+2\beta}{n} - \frac{k_2(6k_1+2k_1k_2+3k_2)}{6m}\right]$$

(505)



$$k_1 = \frac{I_2 h_1}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h_2}{I_3 L} \quad ; \quad \text{otros datos (499)}$$

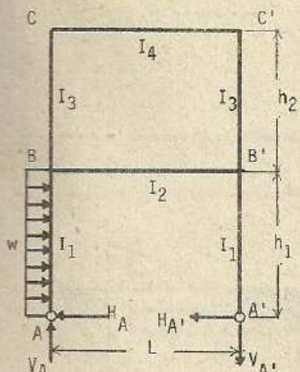
$$V_A = V_{A'} = -P\,\frac{h_1}{L} \quad ; \quad H_A = -H_{A'} = -\frac{P}{2}$$

$$M_{BA} = -M_{B'A'} = +P\,\frac{h_1}{2} \quad ; \quad M_{BC} = -M_{B'C'} = P\,\frac{h_1}{2n}$$

$$M_{BB'} = -M_{B'B} = P\,\frac{h_1}{2}\cdot\frac{6k_2+\lambda}{n}$$

$$M_C = -M_{B'B} = P\,\frac{h_1}{2n} = M_{BC} = -M_{B'C'}$$

(506)



$$k_1 = \frac{I_2 h_1}{I_1 L} \quad ; \quad K_2 = \frac{I_2 h_2}{I_3 L} \quad ; \quad \text{otros datos (499)}$$

$$V_A = -V_{A'} = -w\,\frac{h_1^2}{2L} \quad ; \quad H_A = \frac{w h_1}{2} + \frac{M_{BA}}{h_1} \quad ; \quad H_{A'} = \frac{M_{B'A'}}{h_1}$$

$$M_{BB'} = \frac{w h_1^2}{8}\left[\frac{2(6k_2+\lambda)}{n} - \frac{k_1 k_2 (k_2+2\lambda)}{m}\right]$$

$$M_{B'B} = -\frac{w h_1^2}{8}\left[\frac{2(6k_2+\lambda)}{n} + \frac{k_1 k_2 (k_2+2\lambda)}{m}\right]$$

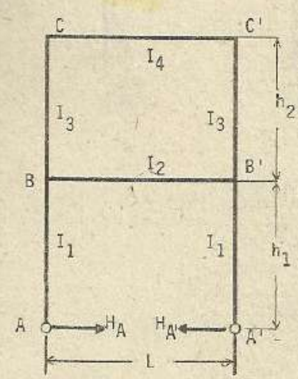
$$M_{BC} = \frac{w h_1^2}{8}\left[\frac{2}{n} - \frac{k_1(2k_2+3\lambda)}{m}\right] ; \; M_{BA} = M_{BB'} + M_{BC}$$

$$M_{B'C'} = -\frac{w h_1^2}{8}\left[\frac{2}{n} + \frac{k_1(2k_2+3\lambda)}{m}\right] ; \quad M_{B'A'} = M_{B'B} + M_{B'C'}$$

$$M_C = \frac{w h_1^2}{8}\left[\frac{2}{n} + \frac{k_1 k_2}{m}\right] \quad ; \quad M_{C'} = -\frac{w h_1^2}{8}\left[\frac{2}{n} - \frac{k_1 k_2}{m}\right]$$

Esfuerzos debido al cambio de temperatura

(507)



$$k_1 = \frac{I_2 h_1}{I_1 L} \quad ; \quad k_2 = \frac{I_2 h_2}{I_3 L} \quad ; \quad \text{otros datos (499)}$$

$$H_A = H_{A'} = \frac{M_{BA}}{h_1} \quad ; \quad M_{BB'} = M_{B'B} = -\frac{3E\varepsilon t I_2}{h_1}\cdot\frac{k_2(2\lambda+k_2)}{m}$$

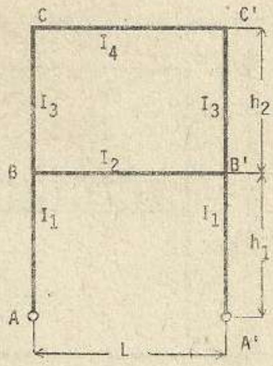
$$M_{BA} = M_{B'A'} = -\frac{3E\varepsilon t I_2}{h_1}\cdot\frac{k_2(k_2+2) + \lambda(2k_2+3)}{m}$$

$$M_{BC} = M_{B'C'} = -\frac{3E\varepsilon t I_2}{h_1}\cdot\frac{(2k_2+3\lambda)}{m}$$

$$M_C = M_{C'} = \frac{3E\varepsilon t I_2}{h_1}\cdot\frac{k_2}{m}$$

Cuidado : En caso de descenso de temperatura los signos de los esfuerzos son contrarios a las presentadas.

Los esfuerzos por cambio de temperatura pueden también resolverse según :



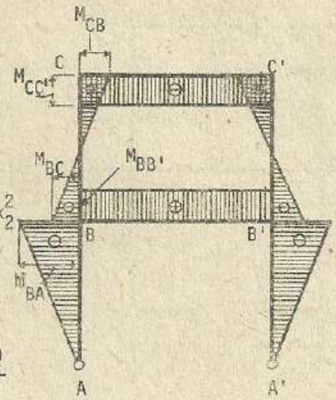
$$k_1 = \frac{I_2 h_1}{I_1 L} \; ; \; k_2 = \frac{I_3 h_1}{I_1 h_2}$$

$$k_3 = \frac{I_4 h_1}{I_1 L}$$

$$\mu = 2(2k_2+k_3)(3+2k_1+4k_2) - 4k_2^2$$

$$M_C = \frac{6E\varepsilon Ltk_2k_3\, I_1}{\mu h_1^2}$$

$$M_{BC} = -\frac{6E\varepsilon LtI_1k_2(3k_2+2k_3)}{\mu h_1^2}$$



$$M_{BB'} = -\frac{6E\varepsilon LtI_1k_1(2k_2+k_3)}{\mu h_1^2} \quad ; \quad M_{BA} = -\frac{6E\varepsilon LtI_1\left[(2k_2+k_3)(k_1+2k_2) - k_2^2\right]}{\mu h_1^2}$$

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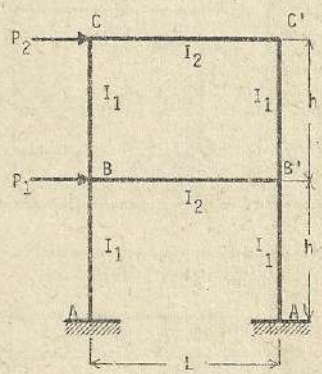
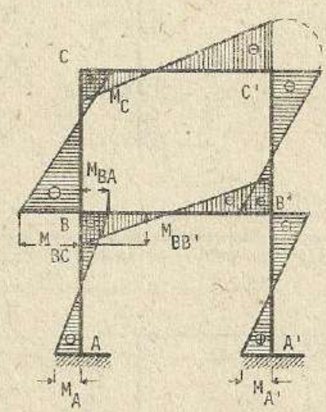


$$k = \frac{I_2 h}{I_1 L}$$



$$M_A = -\frac{(3k+1)hP_2 + (18k^2+12k+1)h(P_1+P_2)}{2(36k^2+18k+1)}$$

$$M_{BA} = \frac{-(3k+1)hP_2 + 6k(3k+1)h(P_1+P_2)}{2(36k^2+18k+1)} \; ; \; M_{BB'} = \frac{6k(3k+1)hP_2 + 3k(6k+1)h(P_1+P_2)}{2(36k^2+18k+1)}$$

$$M_{BC} = -\frac{(3k+1)(6k+1)hP_2 - 3kh(P_1+P_2)}{2(36k^2+18k+1)} \; ; \; M_C = \frac{9k(2k+1)hP_2 + 3kh(P_1+P_2)}{2(36k^2+18k+1)}$$

Si $P_1 = P_2 = P$; $k = 1$, $\left(\frac{h}{L} = \frac{I_1}{I_2}\right)$

$M_A = -\frac{3}{5}Ph$; $M_{BA} = \frac{2}{5}Ph$; $M_{B'B} = \frac{3}{5}Ph$; $M_{BC} = -\frac{1}{5}Ph$; $M_C = \frac{3}{10}Ph$

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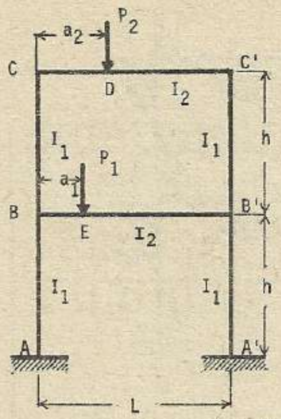


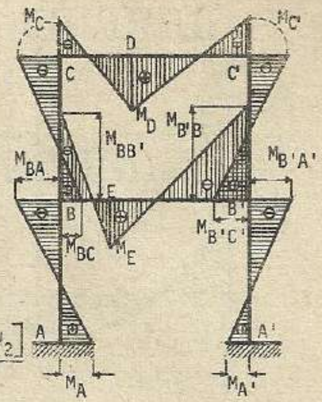
$$k = \frac{I_2 h}{I_1 L}$$

$$N_1 = \frac{6}{L} \cdot \frac{P_1 a_1 (L-a_1)}{L^2}$$

$$N_2 = \frac{6}{L} \cdot \frac{P_2 a_2 (L-a_1)}{L^2}$$

$$M_A = M_{A'} = \frac{(k+2)N_1 - N_2}{3(k^2+6k+7)}$$

$$M_{BA} = M_{B'A'} = -\frac{2\left[(k+2)N_1 - N_2\right]}{3(k^2+6k+7)}$$



$$M_{BB'} = M_{B'B} = -\frac{(4k+3)N_1 + kN_2}{3(k^2+6k+7)} \quad ; \quad M_{BC} = M_{B'C'} = \frac{(2k+3)N_1 + (k+2)N_2}{3(k^2+6k+7)}$$

$$M_C = M_{C'} = -\frac{kN_1 + (2k+7)N_2}{3(k^2+6k+7)}$$

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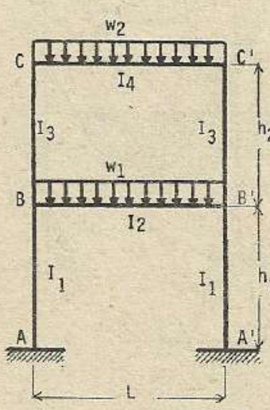


$$k'_1 = \frac{I_1}{h_1} \; ; \; k'_2 = \frac{I_2}{L} \; ; \; k'_3 = \frac{I_3}{h_2} \; ; \; k'_4 = \frac{I_4}{L}$$

$$\alpha = \frac{k'_3}{k'_1} \; ; \; \beta_1 = \frac{k'_2}{k'_1} \; ; \; \beta_2 = \frac{k'_4}{k'_3}$$

$$A = 12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)$$

$$M_{AB} = \frac{(2+\beta_2)w_1L^2 - w_2L^2}{A} \quad ; \quad M_{BA} = \frac{2(2+\beta_2)w_1L^2 - 2w_2L^2}{A}$$

$$M_{BB'} = -\frac{(4+2\beta_2+3\alpha+2\alpha\beta_2)w_1L^2 + \beta_1 w_2L^2}{A}$$

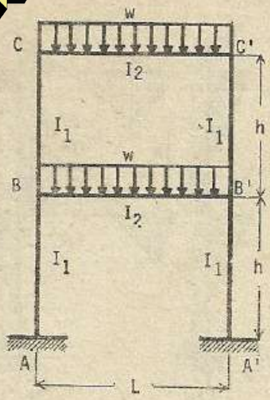
$$M_{BC} = \frac{(3\alpha+2\alpha\beta_2)w_1L^2 + (2+\beta_1)w_2L^2}{A} \quad ; \quad M_{CB} = \frac{\alpha\beta_2 w_1L^2 + (4+2\beta_1+3\alpha)w_2L^2}{A}$$

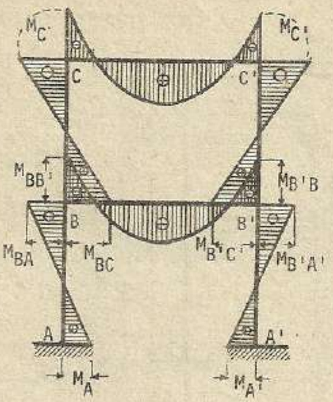
$$M_{CC'} = \frac{\alpha\beta_2 w_1L^2 + (4+2\beta_1+3\alpha)w_2L^2}{A}$$

Si $w_1 = w_2 = w$, $\beta_1 = \beta_2 = \beta$, $\alpha = 1$; $B = 12(7+6\beta+\beta^2)$

$$M_{AB} = \frac{(1+\beta)wL^2}{B} \quad ; \quad M_{BA} = \frac{2(1+\beta)wL^2}{B} \quad ; \quad M_{BB'} = -\frac{(7+5\beta)wL^2}{B}$$

$$M_{BC} = \frac{(5+3\beta)\,wL^2}{B} \quad ; \quad M_{CB} = \frac{(7+3\beta)wL^2}{B} \quad ; \quad M_{CC'} = -\frac{(7+3\beta)wL^2}{B}$$





Si k = 1 , $\frac{h}{L} = \frac{I_1}{I_2}$: $M_A = \frac{1}{84} wL^2$; $M_{BA} = -\frac{2}{84} wL^2$; $M_{BB'} = -\frac{6}{84} wL^2$

$$M_{BC} = \frac{4}{84} wL^2 \quad ; \quad M_C = -\frac{5}{84} wL^2$$

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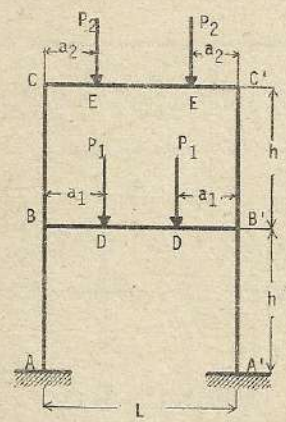


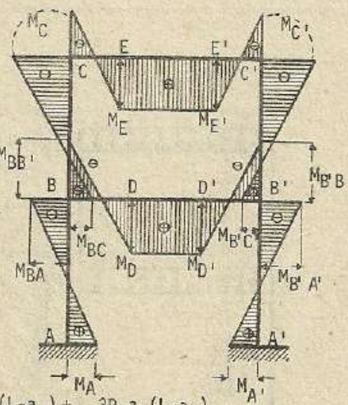
k = 1

$$N_1 = 3P_1a_1 \frac{(L - a_1)}{L}$$

$$N_2 = 3P_2a_2 \frac{(L - a_2)}{L}$$

$$M_A = \frac{3P_1a_1(L-a_1) - P_2a_2(L-a_2)}{14L}$$

$$M_{BA} = -2M_A$$



$$M_{BB'} = -\frac{11P_1a_1(L-a_1) + P_2a_2(L-a_2)}{14L} \quad ; \quad M_{BC} = \frac{5P_1a_1(L-a_1) + 3P_2a_2(L-a_2)}{14L}$$

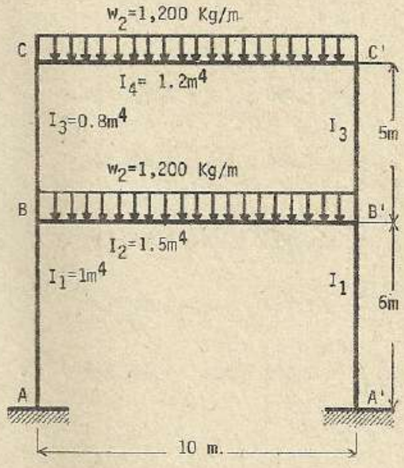
$$M_C = -\frac{P_1a_1(L-a_1) + 9P_2a_2(L-a_2)}{14L}$$

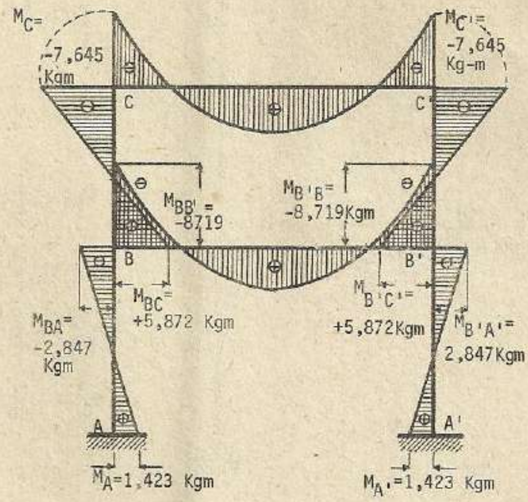
Si $P_2 = P_1 = P$, $a_2 = a_1 = \frac{L}{3}$

$M_A = \frac{2}{63} PL$; $M_{BA} = -\frac{4}{63} PL$; $M_{BB'} = -\frac{12}{63} PL$; $M_{BC} = \frac{8}{63} PL$

$$M_C = -\frac{10}{63} PL$$

Aplicación numérica





$$k'_1 = \frac{I_1}{h_1} = \frac{1}{6} \; ; \quad k'_2 = \frac{I_2}{L} = \frac{1.5}{10} \; ; \quad k'_3 = \frac{I_3}{h_2} = \frac{0.8}{5} \; ; \quad k'_4 = \frac{I_4}{L} = \frac{1.2}{10}$$

$$\beta_1 = \frac{k'_2}{k'_1} = \frac{1.5}{10} \times \frac{6}{1} = 0.9 \; ; \quad \beta_2 = \frac{k'_4}{k'_3} = \frac{1.2}{10} \times \frac{5}{0.8} = \frac{3}{4} \; ; \quad \alpha = \frac{k'_3}{k'_1} = \frac{0.8}{5} \times \frac{6}{1} = \frac{4.8}{5}$$

$$M_A = \frac{(2+\beta_2)w_1L^2 - w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = \frac{(2+\frac{3}{4}) \times 1,200x10x10-1,200x10x10}{12(4+2x0.9+2x\frac{3}{4}+0.9x\frac{3}{4}+3x\frac{4.8}{5}+2x\frac{4.8}{5}x\frac{3}{4})} = \frac{21,0000}{147.54}$$

$$= 1,423 \text{ kg m}$$

$$M_{BA} = \frac{2(2+\beta_2)w_1L^2 - 2w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = \frac{2(2+\frac{3}{4})1,200x10^2 - 2x1,200x10^2}{147.54} = 2,847 \text{ kgm}$$

$$M_{BB'} = -\frac{(4+2\beta_2+3\alpha+2\alpha\beta_2)w_1L^2 + \beta_1 w_2 L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = -\frac{(4+2\frac{3}{4}+3x\frac{4.8}{5}+2\frac{4.8}{5}x\frac{3}{4})x1200x10^2-0.9x12x10^4}{147.54}$$

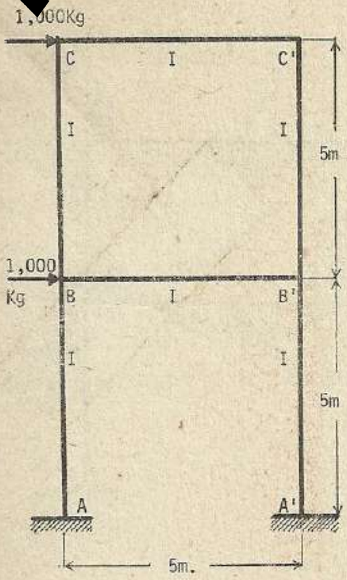
$$= -8,719 \text{ Kg m}$$

$$M_{BC} = \frac{(3\alpha+2\alpha\beta_2)w_1L^2+(2+\beta_1)w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = \frac{(3x\frac{4.8}{5}+2x\frac{4.8}{5}x\frac{3}{4})1x200x10^2+(2+0.9)x1200x10^2}{147.54}$$

$$= 5,872 \text{ Kg m}$$

$$M_{CB} = \frac{\alpha\beta_2w_1L^2+(4+2\beta_1+3\alpha)w_2L^2}{12(4+2\beta_1+2\beta_2+\beta_1\beta_2+3\alpha+2\alpha\beta_2)} = \frac{\frac{4.8}{5}x\frac{3}{4}x1200x10^2+(4+2x0.9+3x\frac{4.8}{5})1,200x10^2}{147.54}$$

$$= 7,645 \text{ Kg m}$$



$$k = \frac{I_1 h}{I_1 L} = \frac{I \times 5}{I \times 5} = 1$$

$$M_A = -\frac{3}{5}\ Ph = -\frac{3}{5} \times 1{,}000 \times 5 = -3{,}000 \text{ Kgm}$$

$$M_{BA} = -\frac{2}{5}\ Ph = \frac{2}{5} \times 1{,}000 \times 5 = 2{,}000 \text{ Kgm}$$

$$M_{BB'} = \frac{3}{5}\ Ph = \frac{3}{5} \times 1{,}000 \times 5 = 3{,}000 \text{ Kgm}$$

$$M_{BC} = -\frac{1}{5}\ Ph = -\frac{1}{5} \times 1{,}000 \times 5 = -1{,}000 \text{ Kgm}$$

$$M_C = \frac{3}{10}\ Ph = \frac{3}{10} \times 1{,}000 \times 5 = 1{,}500 \text{ Kgm}$$

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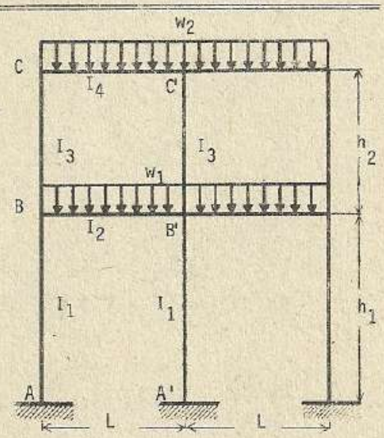

$$k'_1 = \frac{I_1}{h_1} \quad ; \quad k'_2 = \frac{I_2}{L} \quad ; \quad k'_3 = \frac{I_3}{h_2} \quad ; \quad k'_4 = \frac{I_4}{L}$$

$$\beta_1 = \frac{k'_2}{k'_1} \qquad \beta_2 = \frac{k'_4}{k'_3} \qquad \alpha = \frac{k'_3}{k'_1}$$

$$C = 14\left[4\ (1+\beta_1+\beta_2+\beta_1\ \beta_2) + \alpha\ (3+4\ \beta_2)\right]$$

$$M_{BA} = \frac{4(1+\beta_2)\ w_1 L^2 - 2 w_2 L^2}{C}$$

$$M_{BB'} = \frac{\left[4\ (1+\beta_2) + \alpha\,(3+4\beta_2)\right] w_1 L^2 + 2\ \beta_1 w_2 L^2)}{C}$$

$$M_{B'B} = \frac{\left[4\ (1+1.5\beta_1+\beta_2+1.5\ \beta_1\beta_2) + \alpha(3+4\beta_2)\right] w_1 L^2 - \beta_1 w_2 L^2}{C}$$

$$M_{BC} = \frac{\alpha\ (3+4\beta_2)\ w_1 L^2 + 2\ (1+\beta_1)\ w_2 L^2}{C} \quad ; \quad M_{CB} = \frac{2\alpha\beta_2 w_1 L^2 + (4+4\beta_1+3\alpha)\ w_2 L^2}{C}$$

$$M_{C'C} = \frac{\left[4\ (1+\beta_1+1.5\beta_2+1.5\beta_1\beta_2) + \alpha(3+6\ \beta_2)\right] w_2 L^2 - \alpha\ \ \beta_2\ w_1 L^2}{C}$$

$$M_A = \frac{1}{2}\ M_{BA} \quad ; \quad M_{CC'} = -\ M_{CB}$$